CMPS 201
Midterm 2
Review Problems

Study problems 1, 2 and 3 on the midterm 1 review sheet, as well as the posted solutions to homework assignments 4, 5 and 6.

1. Recall the n^{th} harmonic number was defined to be $H_n = \sum_{k=1}^{n}\left(\frac{1}{k}\right) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} + \frac{1}{n}$. Use induction to prove that

$$\sum_{k=1}^{n} kH_k = \frac{1}{2}n(n+1)H_n - \frac{1}{4}n(n-1)$$

for all $n \geq 1$. (Hint: Use the fact that $H_n = H_{n-1} + \frac{1}{n}$.)

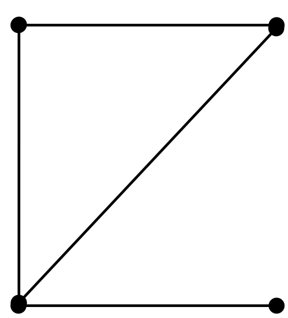
2. Suppose we are given 4 gold bars (labeled 1, 2, 3, 4), one of which *may* be counterfeit: gold-plated tin (lighter than gold) or gold-plated lead (heavier than gold). Again the problem is to determine which bar, if any, is counterfeit and what it is made of. The only tool at your disposal is a balance scale, each use of which produces one of three outcomes: tilt left, balance, or tilt right.

   a. Use a decision tree argument to prove that at least 2 weighings must be performed (in worst case) by any algorithm that solves this problem. Carefully enumerate the set of possible verdicts.
   b. Determine an algorithm that solves this problem using 3 weighings (in worst case). Express your algorithm as a decision tree.
   c. Find an adversary argument that proves 3 weighings are necessary (in worst case), and therefore the algorithm you found in (b) is best possible. (Hint: study the adversary argument for the min-max problem discussed in class to gain some insight into this problem. Further hint: put some marks on the 4 bars and design an adversary strategy that, on each weighing, removes the fewest possible marks, then show that if the balance scale is only used 2 times, not enough marks will be removed.)

   Remark: parts (a) and (b) above are realistic exam problems, while part (c) is probably too long. It is still worth some of your time however, and will probably be a homework problem due Thursday, the week after the exam.

3. For each graph G pictured below, determine whether it is a yes or no instance of the Hamiltonian Cycle problem (HC) discussed in class. In each case, find the corresponding instance $f(G)$ of the Travelling Salesman problem (TSP) under the mapping $f: HC \rightarrow TSP$ discussed in class. (See lecture notes from 11-16-17, pages 4 through 8, and especially the example starting on page 6.) Verify in each case that f carries a yes instance to a yes instance, or a no instance to a no instance.

   a. $n = |V(G)| = 4$



b. $n = |V(G)| = 6$