CMPE 177 7-29-09

PST

MIDTERM 2 : 5:00 - 6:10

BREAK : 6:10 - 6:20

lecture : 6:20 - 7:30

- EULER TRAIL: A TRAIL THAT INCLUDES EVERY EDGE.
- EULER TOUR: A CLOSED EULER TRAIL.

THM (EULER)

A CONNECTED GRAPH IS EULERIAN IFF EVERY VERTEX HAS even DEGREE.

<u>Corollary</u>

A conn. graph G is Eulerian iff $E(G)$ partitions into disjoint cycles.

<u>Proof</u>: Suff. to show:

$E(G)$ partitions into disjoint cycles iff all vertices have even degree.

($\Rightarrow$) Last time

($\Leftarrow$) Suppose each vertex has even degree. Use induction on $|E(G)|$.

<u>Base</u>: $|E(G)| = 1$. Then G must be . Indeed $E(G)$ consists of one cycle.

INDUCTION STEP

LET $|E(G)| > 1$. ASSUME RESULT HOLDS FOR ALL CONN. GRAPHS WITH FEWER EDGES, i.e. ASSUME: IF H IS CONN. & HAS ALL EVEN DEGREES, THEN $E(H)$ PARTITIONS INTO DISJOINT CYCLES.

SINCE G IS CONN., NO VERTEX HAS DEGREE 0. SINCE ALL DEGREES ARE EVEN, ALL DEGREES ARE ≥ 2. BY EARLIER LEMMA, G MUST CONTAIN A CYCLE C. LET

$$H_1, H_2, \ldots, H_K$$

BE THE CONN. COMPONENTS OF

$$G-C=\left(V(G),\ E(G)-E(C)\right)$$

EACH H_i IS CONN, HAS FEWER EDGES THAN G, AND HAS ALL EVEN VERTEX DEGREES (SINCE even -2 is even.) $\therefore$ BY INDUCTION HYP. EACH $E(H_i)$ PARTITIONS INTO CYCLES $(i=1,2,\ldots,k)$. THEN SO ALSO DOES

$$E(G)=E(C)\cup E(H_1)\cup\ldots\cup E(H_k)$$

i.e. $E(G)$ ~~is~~ PARTITIONS INTO DISJOINT CYCLES. ///.

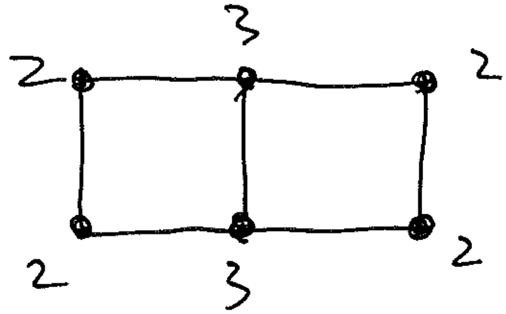
NOTE:

EULERIAN GRAPHS ARE IN SOME SENSE THE 'OPPOSITE' OF TREES:
ALL EDGES ARE CYCLE EDGES, i.e.
NO BRIDGES.

HOWEVER!

E(G) PARTITIONS INTO CYCLES
$\not\Leftrightarrow$ ALL EDGES BELONG TO A CYCLE.

EX.



GOAL: DETERMINE AN EULER TOUR:

$v_0 \, e_1 \, v_1 \, e_2 \, v_2 \, \cdots\cdots \, e_{m-1} \, v_{m-1} \, e_m \, v_0$

Fleury's Algorithm

1.) Pick any $v_0 \in V(G)$

2.) for $i \leftarrow 1$ to $m = |E(G)|$

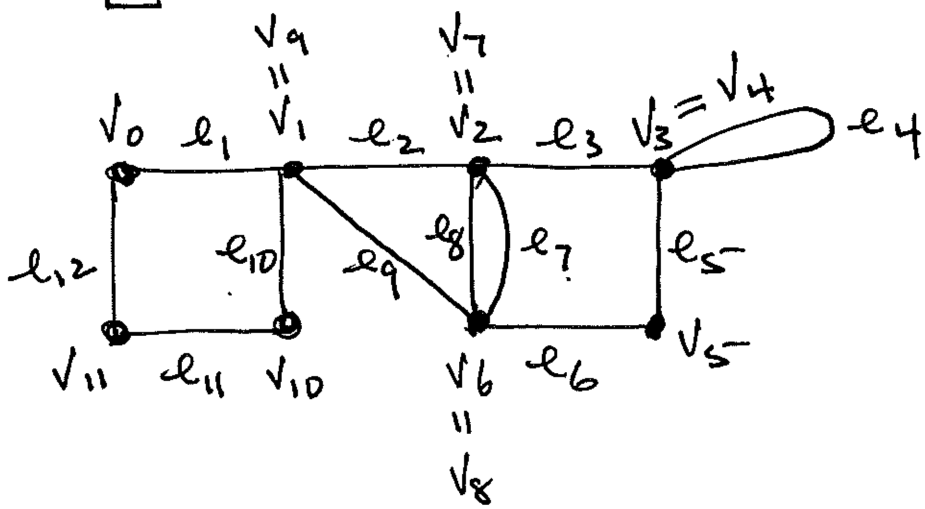
3.) Pick $e_i \in G - \{e_1, \dots, e_{i-1}\}$ s.t.

(i) e_i is incident with v_{i-1}

(ii) if possible e_i is a non-bridge in $G - \{e_1, \dots, e_{i-1}\}$

4.) $v_i \leftarrow$ 'other end' of e_i

Ex.



Read proof of validity p. 91

THM

A CONN. GRAPH IS SEMI-EULERIAN IFF IT HAS EXACTLY TWO VERTICES OF ODD DEGREE

EXERCISE: PROVE THIS

HINT: INSERT A NEW EDGE AND APPLY EULER'S THM.

(3.3) Hamiltonian Graphs

Let G be connected.

<u>Defn.</u> A <u>Hamiltonian Path</u> in G is a path which includes every vertex. A <u>Hamiltonian Cycle</u> is a closed Hamiltonian path.

<u>Defn</u>

- G is <u>Hamiltonian</u> iff it contains a Hamiltonian cycle.
- G is <u>semi-Hamiltonian</u> iff it contains a (non-closed) Hamiltonian path, but no Hamiltonian cycle.

Ex.

FACT:

G is HAMILTONIAN IFF ITS UNDERLYING SIMPLE GRAPH is HAMILTONIAN.
(PROVIDED $n = |V(G)| \geq 3$.)

$\therefore$ USUALLY ASSUME G is SIMPLE.

$\boxed{\text{THERE is NO (KNOWN) EASY CRITERION FOR DECIDING WHEN A GRAPH is HAMILTONIAN!}}$

MOST THEOREMS ARE OF THE FORM: If G HAS 'ENOUGH' EDGES, THEN G IS HAMILTONIAN.