CMPE 177 7-15-09

MIDTERM 1: 5:00 - 6:10

BREAK: 6:10 - 6:20

lecture: 6:20 - 7:30

I will POST:
- CHAP 3, 4?, 5?
- HW 4 PROBLEMS

Recall:

KRUSKAL

1.) $F \leftarrow \phi$

2.) while $|F| < n-1$

3.) choose $e \in E' - F$ SUCH THAT

(i) $(V, F \cup \{e\})$ is ACYCLIC

(ii) $w(e)$ is min. SUBJECT TO (i)

4.) $F \leftarrow F \cup \{e\}$

THM

When KRUSKAL is COMPLETE $T = (V, F)$ is A M.W.S.T.

PROOF: ALREADY SHOWED T IS A SP. TREE. SUPPOSE S IS SOME SP. TREE IN G, OTHER THAN T.

MUST show $w(T) \leq w(S)$.

LET $e_1, e_2, e_3, \ldots, e_{n-1}$ BE THE EDGES IN THE ORDER THAT THEY WERE CHOSEN BY KRUSKAL.

LET e_k BE THE 1ST EDGE IN T which is NOT in S, i.e.

$$\{e_1, \ldots, e_{k-1}\} \subseteq E(S) \text{ BUT } e_k \notin E(S).$$

LET $H = S + e_k$. BY THE TREENESS THM, e_k BELONGS TO A UNIQUE CYCLE IN H, CALL IT C.

C MUST CONTAIN AN EDGE e WHICH DOES NOT BELONG TO T. (OTHERWISE T WOULD CONTAIN THE CYCLE C.)

Let $R = H - e = (S + e_K) - e$.

OBSERVE R is CONNECTED, AND SINCE $|E(R)| = n-1$, R is ANOTHER SP. TREE IN G.

NOTE $w(e_K) \le w(e)$. why? OTHERWISE KRUSKAL WOULD HAVE CHOSEN e ON K^{TH} ITERATION INSTEAD OF e_K.

(OBSERVE $(V, \{e_1, \ldots, e_{K-1}, e\})$ is Acyclic SINCE $\{e_1, \ldots, e_{K-1}, e\} \subseteq E(S)$.)

$\therefore$ $W(R) \le W(S)$. ALSO R CONTAINS ONE MORE EDGE OF T THAN S DOES SINCE $\{e_1, \ldots, e_{K-1}, e_K\} \subseteq E(R)$.

NOW REPEAT THIS PROCESS WITH R IN PLACE OF S, TO OBTAIN YET ANOTHER SP. TREE R_2 WITH ONE MORE EDGE IN COMMON WITH T, AND $w(R_2) \leq w(R)$

CONTINUING WE OBTAIN A SEQ. OF SP. TREES

$$\underset{\substack{\parallel \\ R_0}}{S}, \underset{\substack{\parallel \\ R_1}}{R}, R_2, R_3, \ldots\ldots T$$

AND

$$w(T) \leq \ldots\ldots \leq w(R_2) \leq w(R) \leq w(S).$$

$\therefore\ w(T) \leq w(S)$

AS REQUIRED.

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AS BEFORE $G = (V, E)$ IS CONN.

AND $E' = E - \{\text{loops}\}$.

PRIM

1.) choose $v \in V$

2.) choose $e \in E'$ s.t. $w(e)$ is min
AMONGST ALL EDGES INCIDENT WITH v

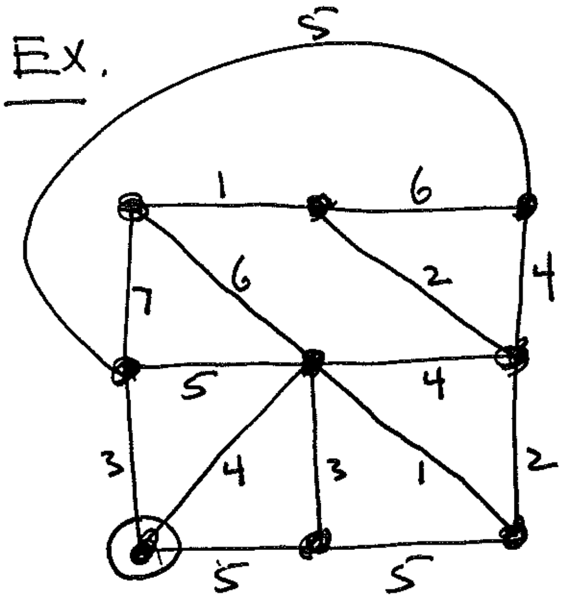
3.) $F \leftarrow \{e\}$

4.) while $|F| < n-1$

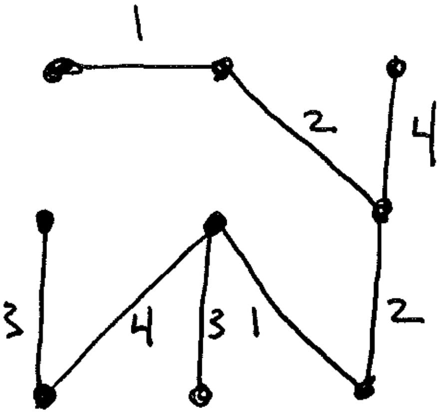
5.) choose $e \in E' - F$ s.t.
(i) e has exactly one end in $G[F]$
(ii) $w(e)$ is min SUBJECT TO (i).

6.) $F \leftarrow F \cup \{e\}$

Ex.



G



$w(T) = 20$

Rmks

1.) Observe on each iteration of loop 4-6 the graph $G[F]$ is conn. since we always add an edge incident with the preceding subgraph.

2.) $G[F]$ is acyclic since we always create a new leaf, so the new edge is not part of a cycle. Thus $G[F]$ is a tree on each iteration of loop 4-6.

3.) NOTE step (5) is ALWAYS POSSIBLE SINCE $G[F]$ cannot include all n vertices until $|F| = n-1$ (BY TREE-NESS THM.) $\therefore$ AT least 1 edge SATISFYING 5(i) EXISTS AS LONG AS $|F| < n-1$.

4.) when Prim is complete, $|F| = n-1$, so THAT $T = G[F]$ is A SP. TREE IN G.

THM

when Prim is complete, $T = G[F]$ is A M.W.S.T. IN G.

PROOF:

ABOVE RMKS. show T is A SP. TREE IN G.

LET S BE ANY OTHER SP. TREE in G, must show $w(T) \le w(S)$.

let $\{e_1, e_2, \ldots, e_{n-1}\} = F$ BE THE EDGES OF T IN THE ORDER ADDED BY Prim. LET

$$T_i = G[\{e_1, \ldots, e_i\}] \quad (1 \le i \le n-1).$$

In PARTICULAR $T_{n-1} = T$.