3.4 Travelling Salesman Problem

Given a weighted graph G

(1) Determine whether or not G is Hamiltonian.

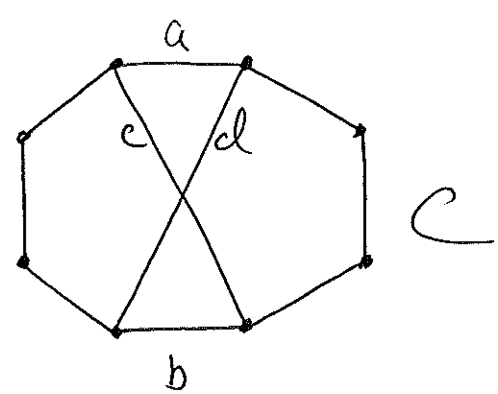
AND

(2) if so, find an optimal (i.e. lowest weight) Hamiltonian cycle in G.

As we've seen there is no simple (necessary and sufficient) criterion for (1). There is also no known efficient (i.e. polynomial time) algorithm for (2).

If G is a complete graph then G is known to be Hamiltonian, so (1) is moot.

In this case there is an efficient algorithm for finding a reasonable (i.e. not necessarily optimal, but pretty good) solution to (2).



TWO OPTIMAL-METHOD

GIVEN A HAMILTONIAN CYCLE C IN A WEIGHTED COMPLETE GRAPH, AND EDGES c, d AS ABOVE:

$$\text{if } w(c)+w(d) < w(a)+w(b)$$
$$C \leftarrow C-\{a,b\}+\{c,d\}.$$

REPEAT THIS PROCEDURE UNTIL NO FURTHER IMPROVEMENTS CAN BE MADE.

EXERCISE

CREATE AN EXAMPLE WHERE THE TWO-OPTIMAL METHOD DOES NOT FIND THE OPTIMAL HAMILTONIAN CYCLE.

i.e. A WEIGHTED COMPLETE GRAPH, WITH AN INITIAL HAMILTONIAN CYCLE. TRACE THE ALGORITHM TO FIND AN IMPROVED CYCLE, THEN DISPLAY AN EVEN BETTER CYCLE.

READ: CLOSEST INSERTION ALGORITHM.