2.1 TREES : DEFINITIONS

DEFN:

A GRAPH IS CALLED ACYCLIC IF IT CONTAINS NO CYCLE SUBGRAPH. SUCH A GRAPH IS ALSO CALLED A FOREST.

A CONNECTED ACYCLIC GRAPH IS CALLED A TREE.

EX.

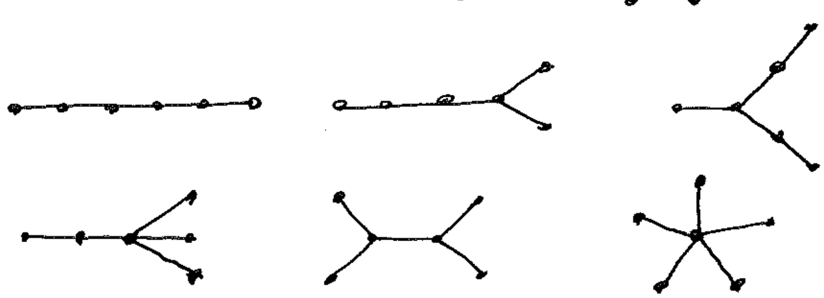
$n=1$

$n=2$ 

$n=3$ 

$n=4$ 

$n=5$ 

$n=6$ 

SEE FIGURES 2.1 & 2.2 P. 47-48.

LEMMA.
IF THE VERTICES OF G ALL HAVE DEGREE AT LEAST 2, THEN G CONTAINS A CYCLE.

CONSEQUENTLY, ANY TREE (WITH $n>1$) MUST CONTAIN SOME VERTICES OF DEGREE 1, WHICH WE CALL LEAVES.

PROOF.
WE ASSUME G IS SIMPLE SINCE OTHERWISE THE CONCLUSION IS TRIVIALLY TRUE.

LET $x_0, x_1 \in V(G)$ BE ADJACENT. CHOOSE x_2 TO BE ANY VERTEX ADJACENT TO x_1 OTHER THAN x_0. (THIS IS POSSIBLE BY OUR HYPOTHESIS.) PICK $x_3, x_4, \ldots$ SIMILARLY, AND IN GENERAL CHOOSE x_{i+1} TO BE ANY VERTEX ADJACENT TO x_i OTHER THAN x_{i-1}.

EXTEND THIS PATH UNTIL SOME VERTEX x_k IS REPEATED. (THIS MUST HAPPEN SINCE THERE ARE ONLY FINITELY MANY VERTICES.) THE SEGMENT OF THIS PATH FROM x_k TO x_k IS THE REQUIRED CYCLE:

$$x_k \to x_{k+1} \to \cdots \to x_k .$$

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