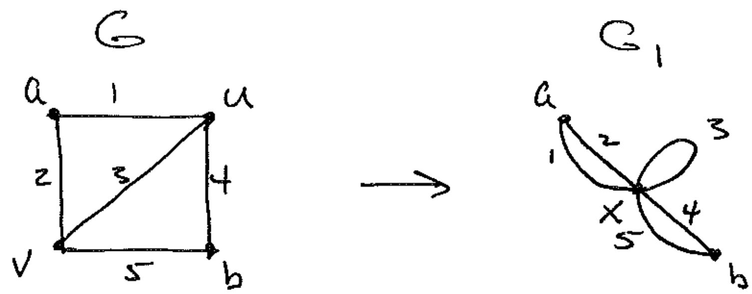
1.8 Fusion

Let u, v be distinct vertices in G. We <u>fuse</u> or <u>identify</u> u, v to obtain a new graph G_1 by replacing them by a single new vertex x such that every edge which was incident with either u or v in G is incident with x in G_1.

<u>Ex</u>



$$A(G) = \begin{array}{c|cccc} & a & u & v & b \\ \hline a & 0 & 1 & 1 & 0 \\ u & 1 & 0 & 1 & 1 \\ v & 1 & 1 & 0 & 1 \\ b & 0 & 1 & 1 & 0 \end{array} \qquad A(G_1) = \begin{array}{c|ccc} & a & x & b \\ \hline a & 0 & 2 & 0 \\ x & 2 & 1 & 2 \\ b & 0 & 2 & 0 \end{array}$$

There is a simple relationship between the adjacency matrices $A(G)$ and $A(G_1)$.

1.) Replace u's row (column) by the sum of u's row (column) and v's row (column).
2.) Delete v's row and column.

IN THE PREVIOUS EXAMPLE :

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \xrightarrow{(1)} \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{pmatrix} \xrightarrow{(2)} \begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

THE FOLLOWING ALGORITHM FINDS THE NUMBER OF CONNECTED COMPONENTS IN A GRAPH. ITS TWO MAIN STEPS ARE FUSION, AND REPLACEMENT OF A GRAPH BY ITS UNDERLYING SIMPLE GRAPH. EACH OF THESE STEPS CAN BE TRANSLATED TO A COMPUTATION ON THE ADJACENCY MATRIX.

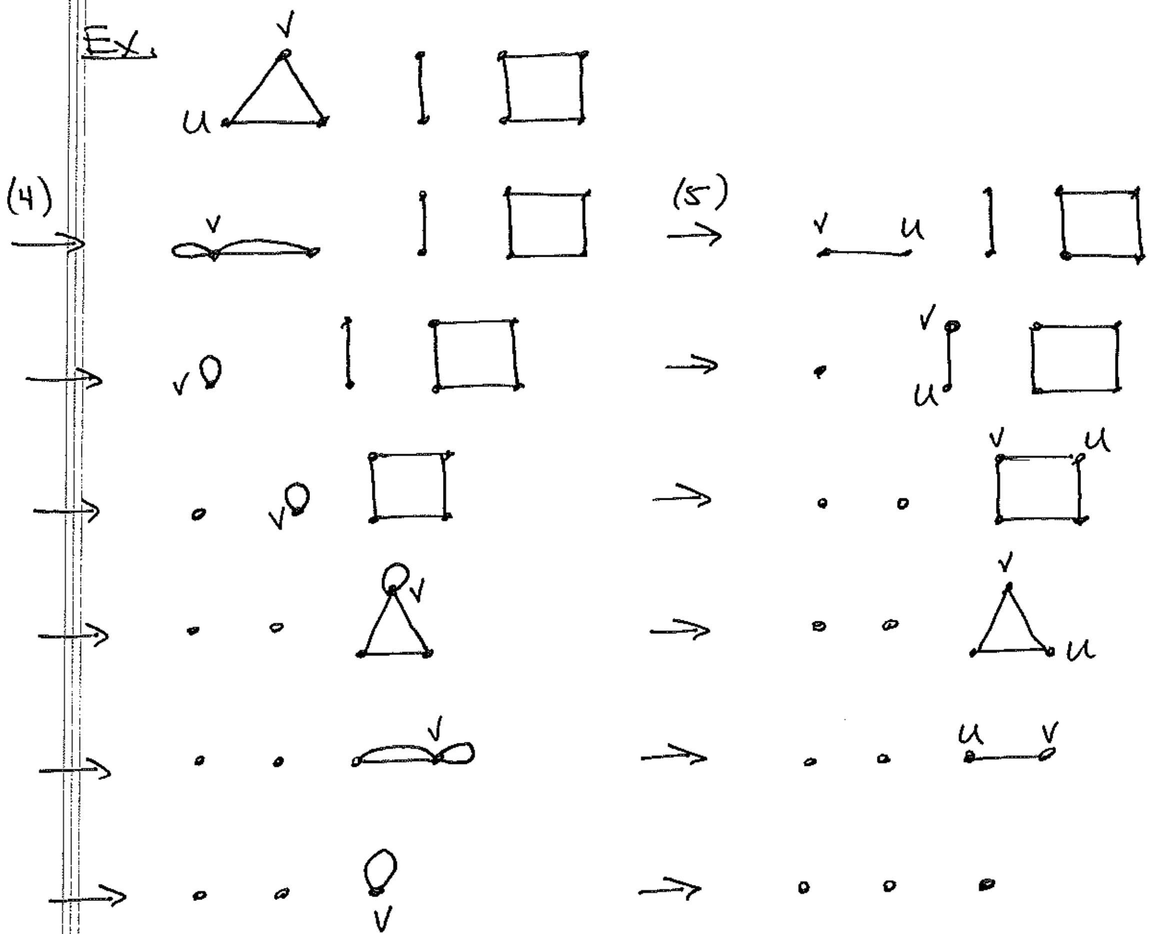
ALGORITHM

1.) REPLACE G BY ITS UNDERLYING SIMPLE GRAPH
2.) WHILE THERE IS A NON-ISOLATED VERTEX V
3.) WHILE THERE IS A VERTEX U ADJACENT TO V
4.) FUSE U TO V, CALL THE NEW VERTEX V
5.) REPLACE THE GRAPH BY ITS UNDERLYING SIMPLE GRAPH.

WHEN THIS ALGORITHM IS COMPLETE, G WILL HAVE BEEN REPLACED BY A NULL GRAPH (i.e. ONE WITH NO EDGES.) THE NUMBER OF VERTICES IN THIS GRAPH IS THE NUMBER OF CONNECTED COMPONENTS IN G.

EXCERCISE: PROVE THIS.

Ex.



EXCERCISE!
WRITE DOWN THE CORRESPONDING MATRIX OPERATIONS IN THIS EXAMPLE.