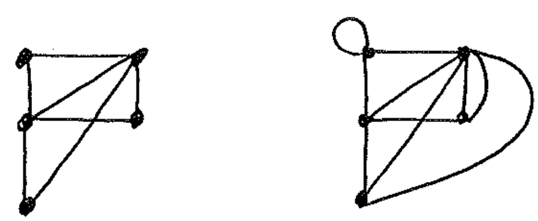
1.5 SUBGRAPHS

DEFN

LET G AND H BE GRAPHS. WE SAY H IS A SUBGRAPH OF G IF $V(H) \subseteq V(G)$ AND $E(H) \subseteq E(G)$. WE SOMETIMES WRITE $H \subseteq G$ TO INDICATE THIS.

H IS CALLED A SPANNING SUBGRAPH OF G IF $H \subseteq G$ AND $V(H) = V(G)$

EX. H G



IF $e \in E(G)$ WE DENOTE BY $G-e$ THE SUBGRAPH OF G OBTAINED BY DELETING THE EDGE e FROM G.

Similarly if $F \subseteq E$ THEN $G-F$ DENOTES THE SUBGRAPH OF G OBTAINED BY DELETING ALL EDGES IN F.

IF $v \in V(G)$ THEN $G - v$ DENOTES THE SUBGRAPH OBTAINED BY DELETING THE VERTEX v, ALONG WITH ALL EDGES INCIDENT WITH v.

IF $U \subseteq V(G)$ THEN $G - U$ DENOTES THE SUBGRAPH OBTAINED BY DELETING ALL VERTICES IN U, ALONG WITH ALL THEIR INCIDENT EDGES.

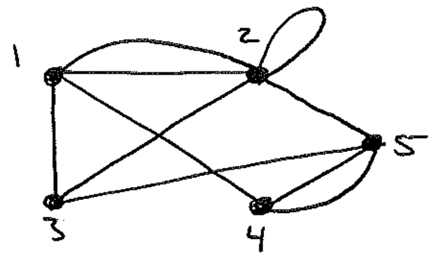
THE <u>UNDERLYING SIMPLE GRAPH</u> OF G IS OBTAINED BY DELETING ALL LOOPS, AND ALL BUT ONE EDGE IN EACH GROUP OF PARALLEL EDGES.

<u>EX</u> ABOVE $H \subseteq G$ IS THE UNDERLYING SIMPLE GRAPH.

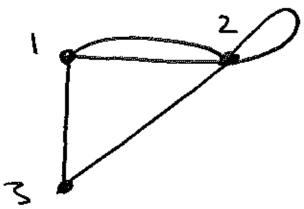
<u>DEFN.</u>
LET $U \subseteq V(G)$. THE SUBGRAPH OF G <u>INDUCED BY U</u>, DENOTED $G[U]$ IS THE GRAPH WITH VERTEX SET U, AND WITH EDGE SET CONSISTING OF ALL EDGES OF G WHOSE ENDS ARE IN U.
BOTH OF

EX

G



$G[\{1,2,3\}]$

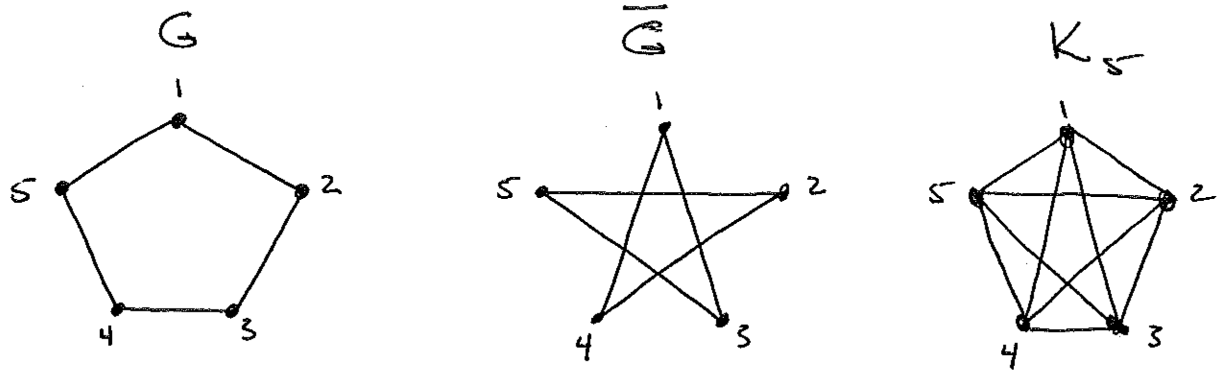


SIMILARLY WE CAN DEFINE THE SUBGRAPH INDUCED BY A SET OF EDGES $F \subseteq E(G)$.

DEFN:

LET G BE A SIMPLE GRAPH. THE COMPLEMENT OF G, DENOTED $\overline{G}$, IS THE GRAPH WITH $V(\overline{G}) = V(G)$ SUCH THAT $u, v \in V(G)$ ARE ADJACENT IN $\overline{G}$ IF AND ONLY IF THEY ARE NOT ADJACENT IN G.

EX.

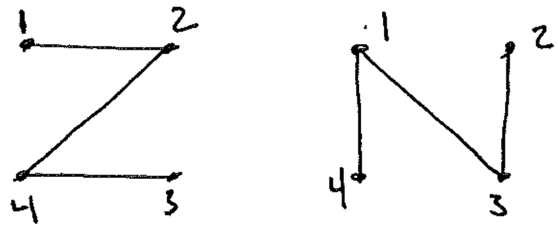


OBSERVE THAT $E(G) \cap E(\overline{G}) = \emptyset$ AND $E(G) \cup E(\overline{G}) = E(K_n)$ WHERE $n = |V(G)|$.

NOTATION: WRITE $G_1 \cong G_2$ TO MEAN G_1 IS ISOMORPHIC TO G_2.

DEFN.
A SIMPLE GRAPH G IS CALLED SELF-COMPLEMENTARY IF $G \cong \overline{G}$.

EX.



$1 \to 2$
$2 \to 3$
$3 \to 4$
$4 \to 1$

EX. PREVIOUS.
ISOMORPHISM: $1 \to 1$, $2 \to 3$, $3 \to 5$, $4 \to 2$, $5 \to 4$.

PROBLEM 1.5.3 (P.23) ASKS THAT YOU PROVE THE FOLLOWING. IF G IS SELF-COMPLEMENTARY, THEN EITHER $n \equiv 0 \pmod 4$ OR $n \equiv 1 \pmod 4$ WHERE $n = |V(G)|$.

IN PARTICULAR OBSERVE THAT NO GRAPH ON 6 OR 7 VERTICES CAN BE SELF-COMPLEMENTARY.

EXCERCISE
FIND A GRAPH ON 8 VERTICES WHICH IS SELF-COMPLEMENTARY.