CMPE 177
APPLIED GRAPH THEORY & ALGORITHMS
WINTER 2001

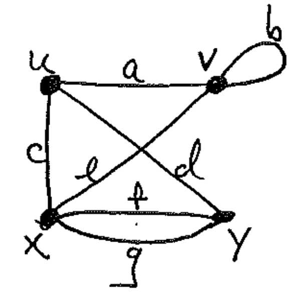
1.1 BASIC DEFINITIONS

DEFN.
A GRAPH $G = (V, E)$ IS A PAIR OF FINITE SETS

(1) $V = V(G)$ A (NON-EMPTY) SET WHOSE ELEMENTS ARE CALLED VERTICES OR NODES.
(2) $E = E(G)$ A (POSSIBLY EMPTY) SET WHOSE ELEMENTS ARE CALLED EDGES. EACH EDGE CORRESPONDS TO AN UNORDERED PAIR OF VERTICES.

EX. $V = \{u, v, x, y\}$
$E = \{a, b, c, d, e, f\}$

$a \Leftrightarrow (u,v)$
$b \Leftrightarrow (v,v)$ LOOP
$c \Leftrightarrow (u,x)$
$d \Leftrightarrow (u,y)$
$e \Leftrightarrow (x,v)$
$f \Leftrightarrow (x,y)$ } MULTIPLE EDGE
$g \Leftrightarrow (x,y)$ }



WE SAY EDGE a JOINS VERTICES u, v, AND THAT u, v ARE THE END VERTICES OF a.

WE ALSO SAY VERTEX u IS <u>ADJACENT</u> TO VERTEX v, VERTEX u IS <u>INCIDENT</u> WITH EDGE a, AND EDGE a IS <u>ADJACENT</u> TO EDGE c.

AN EDGE SUCH AS b WITH IDENTICAL END VERTICES IS CALLED A <u>LOOP</u>.

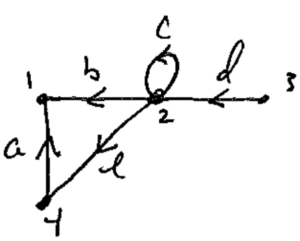
TWO EDGES SUCH AS f AND g WITH THE SAME END VERTICES ARE CALLED <u>MULTIPLE EDGES</u>, (ALSO <u>PARALLEL EDGES</u>.)

A GRAPH WITH NO LOOPS AND NO MULTIPLE EDGES IS CALLED A <u>SIMPLE GRAPH</u> (ALT. DEFN.)

(NOTE: SOME AUTHORS USE THE TERM MULTI-GRAPH.)

IF EACH EDGE INSTEAD CORRESPONDS TO AN ORDERED PAIR OF VERTICES, WE SPEAK OF A <u>DIRECTED GRAPH</u>.

<u>EX.</u>



$V(G) = \{1, 2, 3, 4\}$
$E(G) = \{a, b, c, d, e\}$

$a \leftrightarrow (4,1)$, $b \leftrightarrow (2,1)$, $c \leftrightarrow (2,2)$, $d \leftrightarrow (3,2)$, $e \leftrightarrow (2,4)$.