An Optimization-Based Framework for Simultaneous Plant-Controller Redesign

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In this paper we develop a framework for the redesign of computer-controlled, closed-loop, mechanical systems for improved dynamic performance. A central notion which underlies the redesign framework is that, in order to achieve the best possible performance from a constrained closed-loop system, the plant and controller should be designed simultaneously. The framework is presented as the formulation and solution of a progression of optimization problems which establish the limits of performance of the dynamic system under various conditions of interest, thereby enabling the engineer to systematically establish the various redesign possibilities. Using a second order linear dynamic system and a nonlinear controller as an example, we demonstrate the application of the framework and substantiate the idea that in order to achieve the best possible performance from a constrained closed-loop system, the plant and controller should be redesigned simultaneously. We then show how the redesign framework can be used to select the best control strategy for a robotic manipulator from a dynamic performance standpoint. Finally, in order to demonstrate that the redesign framework yields solutions which the engineer can implement with confidence, we present the experimental verification of the numerical solution of a manipulator redesign optimization problem.

1 Introduction

In order to motivate the problems and issues addressed in this paper, consider the following design context. Given an existing feedback-controlled dynamic system which consists of a nominal plant structure (or configuration), a nominal set of plant parameters which characterize the plant structure, a nominal feedback controller structure and a nominal set of controller parameters which characterize the controller structure, the engineer would like to redesign this controlled dynamic system in order to improve its dynamic performance.

To improve the performance of the given feedback-controlled dynamic system, the engineer can redesign the system, in order of preference, in the following ways:

(1) The engineer can redesign the controller by either selecting new controller parameters or by choosing a different controller structure. Since the systems of interest to us are computer controlled, redesigning the controller does not involve hardware changes, making this the most desirable action.

(2) The engineer may change a plant parameter (or group of parameters) so that, in effect, some components are re-sized. In general, since the plant has changed, the controller will also need to be modified. Changing the plant parameters is usually more difficult than changing the controller (since new components must either be bought or made) but is not as serious as changing the plant structure (and hence changing the entire plant).

(3) The engineer changes the *plant structure*. This is the least preferable and most costly change.

Because the dynamic performance of a closed-loop system is due, through constraints, to both the plant and controller, it is often difficult to determine which of the above changes to make. There is therefore a need for a systematic framework to help the engineer make the correct set of design and/or redesign decisions. Such a redesign framework is developed in Section 3.

Successful redesign of the existing controlled dynamic system depends crucially on a thorough understanding of the performance capabilities of the dynamic system and the resolution of performance related issues such as the following:

(1) the determination of the limits of performance of the controlled dynamic system under various conditions of interest.

(2) for a given controller structure, the determination of the (best) combination of plant parameters and controller parameters that optimizes dynamic performance under given constraints.

(3) from a given set of competing controller structures, the determination of the best controller structure from a dynamic performance standpoint.

While a large part of control research deals with the formulation, implementation, and robustness of feedback controller structures (or strategies), there is a relatively miniscule amount of work which addresses the resolution of the above important issues which arise in the design of feedback controlled dynamic systems for performance. The aforementioned redesign frame-

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work will allow the engineer to systematically and quantitatively understand the performance capabilities of the controlled dynamic system, enable him (or her) to resolve performance-related issues such as those mentioned above, and, as a consequence, redesign the existing plant and controller to achieve improved dynamic performance.

The dynamic performance of a feedback-controlled dynamic system is limited by plant design, controller design, and system constraints. Often, improved system performance can be obtained if the plant and controller are redesigned simultaneously under a given set of system constraints. Currently, no unified framework exists for simultaneous plant-controller design which takes important system constraints into account. For example, most controller design methods ignore actuator constraints in the design process, requiring ad-hoc redesign if numerical simulations of the closed-loop system yield constraint violations.

The framework presented in Section 3 yields plant and controller design parameters which meet performance requirements but do not violate constraints. The framework indicates possible redesigns of a nominal design based on the solution of a series of optimization problems and is applicable to computer-controlled, closed-loop, multibody mechanical systems. Since constraints are taken into account a priori as part of the problem formulation, the resulting redesign solutions can be implemented with confidence. (The experimental verification in Section 5 validates the last statement.) An extremely important class of constraints, addressed in the present paper, are the bounds on the actuator efforts.

In some respects, the framework can be viewed as an extension of optimal control theory. In fact, one of the optimization problems we solve is the optimal control problem. However, optimal control theory [1-2] is concerned with finding the best control strategy for a *fixed* plant while our method focuses on optimizing controller design and plant design simultaneously, Also, the application of optimal control theory often results in a control strategy which is open-loop while we concentrate exclusively on utilizing closed-loop controllers, reaping the well-known benefits of feedback control.

Other researchers have used optimization as an aid in the design of closed-loop controllers. Boyd [3] has applied factorization [4] to finding the optimal feedback controller by solving a convex programming problem. As in our method, Boyd minimizes dynamic performance measures and explicitly levies constraints. However, Boyd's methods are applicable only to linear systems with linear controllers and fixed plants while our method may be applied to nonlinear system and plants with free parameters. Optimal closed-loop controllers may also be designed using H_∞ control theory [5-6], but only for linear systems with fixed plants.

Numerous researchers (see, for instance, [7]) have applied optimization to the design of dynamic systems, some recent applications being [8-9]. However, most concentrate on optimizing plant design and ignore the role of the controller in determining closed-loop response. Some notable exceptions are researchers in the field of large space structures who optimize plant and controller design simultaneously [10-11]. While applicable to only linear systems, their work does share our design philosophy.

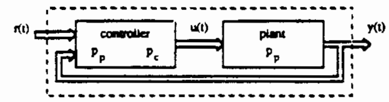

Fig. 1 Block diagram of the closed-loop system

Our framework is applicable to computer-controlled, closed-loop, nonlinear, multibody mechanical systems, like robotic manipulators and magnetic tape drives. For brevity, such systems will be called closed-loop systems and, for the purposes of this paper, consist of a plant and controller, tied together with a feedback loop (Fig. 1).

The contents of the paper are as follows. The plant-controller redesign problem is formally defined in Section 2. The corresponding redesign framework is then developed in Section 3. Section 4 demonstrates the application of the redesign framework to improve the performance of a second order single-input, single-output system while Section 5 uses the redesign framework to address a problem of considerable engineering interest: the selection of the best control strategy for a robotic manipulator from a dynamic performance standpoint. Finally, Section 6 summarizes the present work and draws relevant conclusions.

2 Problem Definition

In this paper, *plant* refers to the uncontrolled dynamic system of interest and *closed-loop system* refers to the combination of the plant and controller, tied together with a feedback loop. The controller in Fig. 1 is shown to be a function of both the "controller" parameters and the plant parameters in order to include those controllers which use information about the plant to compute a control input (e.g., observer-based controllers).

Suppose that a nominal closed-loop system exists with inadequate performance. The nominal design can be described by four quantities: the plant structure (or configuration), the controller structure, a vector of parameters describing the plant, and a vector of parameters describing the controller. The plant structure is defined analytically by the following equations:

$$\dot{x}(t) = \mathbf{f}(\mathbf{x}(t),\mathbf{u}(t),\mathbf{p}_p,t),$$

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t),\mathbf{u}(t),\mathbf{p}_p,t). \quad (1)$$

Since the plant structure is known, the functions $\mathbf{f}(\cdot)$ and $\mathbf{h}(\cdot)$ are known. The controller is defined analytically by the following equation:

$$\mathbf{u}(t) = \mathbf{c}(\mathbf{y}(t),\mathbf{r}(t),\mathbf{p}_p,\mathbf{p}_c,t). \quad (2)$$

The nominal controller structure, and thus the function $\mathbf{c}(\cdot)$, is known. Parameter vectors representing the nominal plant structure and nominal controller structure are given by $\mathbf{p}_p$ and $\mathbf{p}_c$, respectively.

Constraints and requirements on the closed-loop system response can be represented by the dynamic equalities and inequalities

$$\mathbf{S}_e(x(t),\mathbf{u}(t),\mathbf{p}_p,\mathbf{p}_c,t) = \mathbf{0},$$
$$\mathbf{S}_i(\mathbf{x}(t),\mathbf{u}(t),\mathbf{p}_p,\mathbf{p}_c,t) \leq \mathbf{0}, \quad (3)$$

Nomenclature

$\mathbf{c}(\cdot)$ = controller function vector
$\mathbf{f}(\cdot)$ = plant dynamic equation vector
$\mathbf{h}(\cdot)$ = output function vector
J = objective function
$\mathbf{p}_c$ = controller parameter vector
$\mathbf{p}_p$ = plant parameter vector
$\mathbf{r}(t)$ = external reference input vector
$\mathbf{S}_e(\cdot)$ = closed-loop system equality constraint vector
$\mathbf{S}_i(\cdot)$ = closed-loop system inequality constraint vector
t = time
t_f = final time
$\mathbf{u}(t)$ = control-input vector
$\mathbf{u}^*(t)$ = optimal control-input vector
$\mathbf{x}(t)$ = dynamic system state vector
$\dot{x}(t)$ = $\mathbf{x}(t)$ differentiated with respect to time
$\mathbf{y}(t)$ = output vector

Table 1 Outline of the optimization problems. For all, it is necessary that the problem pass if a redesign solution exists with only the free variables changed. Only in problems three and four is it also sufficient that the problems pass for such a redesign solution to exist.

		controller		plant	
problem #	sufficient	parameters	structure	parameters	structure
1	no	-	free	free	fixed
2	no	-	free	fixed	fixed
3	yes	free	fixed	free	fixed
4	yes	free	fixed	fixed	fixed

where the subscript e denotes equality constraints and the subscript i denotes inequality constraints. Usually, the equality constraints will include the initial and/or final conditions

$$\mathbf{x}(0) = \mathbf{x}_0,$$

$$\mathbf{x}(t_f) = \mathbf{x}_f. \tag{4}$$

The closed-loop dynamic performance is explicitly measured by an objective (or cost) function of the form

$$J = \mathbf{K}(\mathbf{x}_f, \mathbf{p}_p, \mathbf{p}_c, t_f) + \int_0^{t_f} \mathbf{L}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{p}_p, \mathbf{p}_c, t)\,\mathbf{d}t. \tag{5}$$

where $\mathbf{K}(\cdot)$ and $\mathbf{L}(\cdot)$ are general, nonlinear functions. Most traditional time-domain performance measures—like rise time, settling time, and minimum time—can be expressed in this form. Without loss of generality, we will assume that the objective function must either be reduced or minimized in order to improve performance.

We can now state the general redesign problem as follows: How should a given nominal closed-loop system defined by $\mathbf{f}(\cdot)$, $\mathbf{h}(\cdot)$, $\mathbf{p}_p$, $\mathbf{p}_c$, and $\mathbf{c}(\cdot)$ be redesigned so that the performance, as measured by the objective function J, is satisfactorily improved while the system constraints $\mathbf{S}_e(\cdot)$ and $\mathbf{S}_i(\cdot)$ are not violated?

In order to resolve the general redesign problem posed above, the designer has to make one of the following design decisions, which are arranged in order of increasing cost:

(1) Change the controller parameters.
(2) Change the controller structure.
(3) Change the plant parameters and (as a result) the controller parameters.
(4) Change the plant structure.

The redesign framework described in the next section provides a rational basis for making the appropriate redesign decision.

3 Framework Description

We use optimization theory to study redesign issues for nonlinear closed-loop systems for the following reasons:

(1) In redesign, we look to better the dynamic performance of a nominal closed-loop system. If we know the *best* performance the system can achieve (i.e., its performance limit), we will know if it is possible to achieve the desired performance improvement. Optimization theory provides a natural way to determine performance limits.
(2) Optimization provides a consistent framework with which to pose problems containing constraints, nonlinearities, and dynamic performance measures. Different problems of interest may be formulated and their solutions compared within the same framework.
(3) General optimization algorithms exist which are able to solve the problems of interest. See, for example, [12–15].
(4) Designers are, in general, familiar with optimization. For example, optimal control theory is common in control system design.

Perhaps the most important reason for using optimization theory is that, in contrast to most commonly used approaches [1,2], the feedback controller parameters (or gains) are obtained with all important constraints embedded *a priori* in the formulation of the (optimization) problem. An important consequence of this fact is that the performance of the actual system, under the "action" of these controller gains, closely matches the predicted simulation results, i.e., the optimization yields "realistic" controller gains. [The preceding statement can be corroborated by comparing the simulation results of Fig. 5(*a*) with the experimental results of Fig. 5(*b*)]. Furthermore the incorporation of constraints *a priori* in the formulation of the optimization problem also enables us to compare different *feedback control* strategies, acting on a given plant, from a performance standpoint.

Our framework is based on examining the solution of four optimization problems, each lending insight into a different aspect of redesign. In each, the objective function (5) is minimized subject to the dynamic system (1) and the constraints (3). The problems differ in the set of variables that can be chosen to minimize the objective function and in whether the controller structure is specified or free. The solution of each optimization problem yields the best performance available from the dynamic system, i.e., its performance limit, under the given set of conditions indicated in the last four columns of Table 1. As will become clear in the sequel, knowledge of the performance limits, under the various conditions of Table 1, is the key to making rational and proper redesign decisions. The four optimization problems, which have been numbered for future reference, are described below.

Problem No. 1: For the given plant structure, determine the combination of control input $\mathbf{u}(t)$ and plant parameters vector $\mathbf{p}_p$ which yields the best performance, i.e., minimizes the objective function (5).

The solution to this problem determines whether any redesign solution exists with the same plant structure (1) as the nominal system. The optimal control $\mathbf{u}^*(t)$ obtained as the solution to this problem is the best over all controllers: no controller, regardless of structure, can perform better. However, the optimal control $\mathbf{u}^*(t)$ will most likely be open-loop (as opposed to closed-loop) and will not actually be implemented. Rather, the solution indicates the best performance achievable by *any* controller, regardless of structure, acting on any plant with the given structure.

If the performance of the system after this test proves not good enough, then the plant structure *must* be redesigned. However, if the performance is good enough, no definitive information is known. It is possible, though not guaranteed, that adequate performance can be achieved without redesigning the plant structure since in practice a closed-loop controller, and not the optimal control, will be used. In other words, the test is necessary but not sufficient to ensure the existence of a redesign solution with the given plant structure.

Problem No. 2: For the given plant structure and given (nominal) plant parameters, determine the control input $\mathbf{u}(t)$ which yields the best performance.

The solution to this problem determines whether any redesign solution exists with the same plant as the nominal system. The control input is free but the plant—both in structure and in parameter values—is fixed, so that a standard optimal control problem is solved. The resulting optimal control solution $\mathbf{u}^*(t)$ indicates the best performance achievable by any controller acting on the nominal plant.

If the performance of the optimal solution is not good enough, then the plant *must* be redesigned: no controller acting on the nominal plant can achieve adequate performance. As in problem one, if the performance of the optimal solution is good enough, no definitive information is gained. It is possible, though not guaranteed, that adequate performance can be

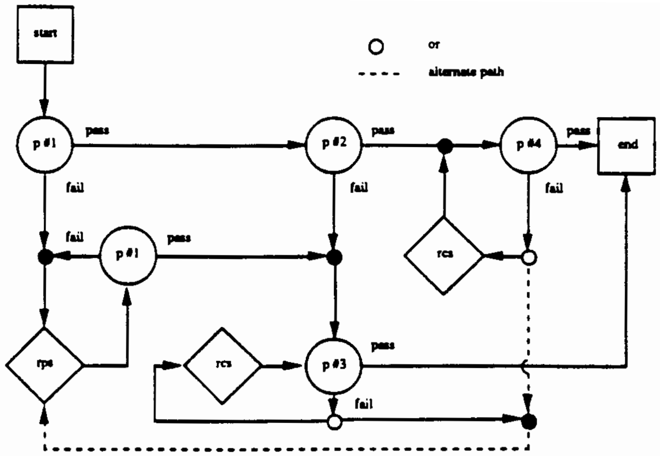

Fig. 2 Redesign flowchart

achieved by redesigning only the controller. This test is necessary but not sufficient to ensure the existence of a nominal redesign solution with the same plant structure and parameters as the nominal plant. It is hoped that a nominal design which passed problem one but fails problem two can achieve adequate performance without redesign of the plant structure.

Problem No. 3: For the given plant structure and a given *feedback* controller structure of the form (2), determine the plant parameter vector $\mathbf{p}_p$ and controller parameter vector $\mathbf{p}_c$ which yields the best performance.

The solution to this problem determines whether any redesign solution exists with the given plant structure and a given feedback controller structure. The plant parameters and controller parameters are variable, but the controller structure and plant structure are fixed. If the optimal solution provides adequate performance, then the parameters found may be used in the redesign. If the performance is inadequate, then either the controller structure or the plant structure must be redesigned. This test is necessary and sufficient to ensure the existence of a redesign solution with the same plant structure and controller structure as the nominal system.

Problem No. 4: For the given plant structure with specified plant parameter $\mathbf{p}_p$ and a given *feedback* controller structure, determine the controller parameter vector $\mathbf{p}_c$ which yields the best performance.

The solution to this problem determines whether any redesign solution exists with the nominal feedback controller structure and given plant. The controller structure, plant structure, and plant parameters are fixed. Only parameters describing the controller are variable. If the performance of the optimal system is good enough then, like problem three, the analytical redesign is complete. If not, then some redesign other that that of the controller parameters must be done. This test is *necessary* and *sufficient* to ensure the existence of a redesign solution with the same plant structure, plant parameter values, and controller structure as the nominal system.

The four problems are summarized in Table 1. Figure 2 shows one way, but by no means the only way, that the problems may be used in redesign. In the discussion below, the aim is to reduce or minimize the objective function in order to improve performance. In Fig. 2, p #1 denotes problem 1 defined and discussed above; similarly p #i, (i = 2, 3, 4), denotes the appropriate problem defined and discussed above. The abbreviation rps in Fig. 2 denotes "redesign plant structure" and rcs denotes "redesign controller structure."

In Fig. 2, the redesign process begins with a nominal design that does not meet the desired performance specifications. Problem one (p #1) is first solved; if the optimal objective function value found in problem one is lower than the (desired) performance specification, the solution is said to "pass" and problem two is solved. If the optimal objective function value is higher than the performance specification, the problem is said to "fail" and, iteratively, the plant structure is redesigned (rps) and problem one retried until a structure is found which passes. After a structure that passes problem one is found, problem three is solved.

If problem one was passed with the nominal plant structure, problem two is solved. Problem four is solved if problem two passes, else problem three is solved. If either problem three or problem four passes, the redesign is complete. However, if either fails, the controller structure is iteratively redesigned (rcs) and the problem retried until passed. Because it is possible that no controller structure can be found which passes, an alternate action to redesigning the controller structure is designing the plant structure.

The redesign flowchart is arranged so that the simplest change which yields a successful redesign is found. For example, if a nominal design exists which requires changing only controller gains in order to satisfactorily improve system performance, problems one and two would pass and new controller gains would be assigned in the solution of problem four. If a nominal design exists which requires a new controller structure to satisfactorily improve performance, problems one and two would pass, problem four would fail, the controller structure would be redesigned, and new controller parameters appropriate to the new controller structure would be assigned in resolving problem four.

Two examples will be used to demonstrate the application and utility of the redesign framework. In the next section, the redesign framework of figure two is applied to a nondimensionalized second-order spring-mass-damper plant. The example shows that the redesign optimization problems are solvable and lead to better closed-loop dynamic performance. Then, in Section 5, we use the redesign framework to shed light on a problem of considerable engineering interest at the present time: the selection of the best control strategy for a robotic manipulator from a dynamic performance standpoint. The latter problem is addressed in the broader context of simultaneous plant-controller design of robotic manipulators for improved performance.

4 Example 1: Redesign of a Second-Order System

A spring-mass-damper plant can be described by the state vector $\mathbf{x}(t) = [x_1(t)\ x_2(t)]^T$, where $x_1(t)$ is position and $x_2(t)$ is velocity. If all states are available for output, the nominal plant structure is given by dynamic equations

$$\mathbf{f}(\cdot) = \begin{bmatrix} x_2(t) \\ -(k/m)x_1(t) - (c/mx_2(t) + (1/m)u(t) \end{bmatrix}$$

$$h(\cdot) = x_1(t) \tag{6}$$

where m is mass, c is the damping coefficient, and k is the spring constant. The mass of the plant is fixed at $m = 1$ so that the plant parameter vector is

$$\mathbf{p}_p = [k\ c]^T. \tag{7}$$

The control input commanded by the controller is linear state feedback, proportional-derivative (*pd*) control given by

$$\mathbf{u}'(t) = nr(t) - k_1x_1(t) - k_2x_2(t) \tag{8}$$

where k_1 and k_2 are constant feedback gains and n is a reference gain selected such that the closed-loop system has zero steady-state error in response to a step input (i.e., $n = k + k_1$). If the control input commanded by the controller is subject to saturation, with maximum $\mathbf{u}_{max}$, the nominal controller structure is

$$\mathbf{c}(\cdot) = \begin{Bmatrix} \mathbf{u}'(t) \text{ for } |\mathbf{u}'(t)| < \mathbf{u}_{max} \\ \mathbf{u}_{max}\ sgn\ (\mathbf{u}'(t)) \text{ for } |\mathbf{u}'(t)| \geq \mathbf{u}_{max} \end{Bmatrix} \tag{9}$$

Table 2 Summary of results for Example 1. Parameters not selected in a problem are indicated by a "-".

parameter	nominal	problem #1	problem #2	problem #3
k	1.00	1.91	1.00	1.93
c	2.00	0.219	2.00	0.186
k_1	4.00	-	-	13.7
k_2	2.46	-	-	6.36
t_{s1}	-	0.985	1.59	-
t_{f1}	-	1.31	1.77	-
t_{s2}	-	4.41	5.02	-
t_{f2}	-	5.26	5.40	-
J	2.02	1.31	1.66	1.32

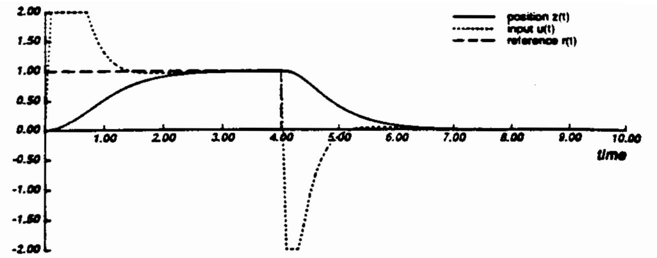

Fig. 3(*a*) Nominal closed-loop system for Example 1

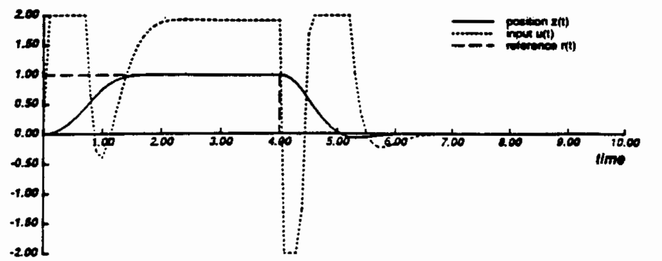

Fig. 3(*b*) Redesigned closed-loop system for Example 1

and the nominal controller parameter vector is

$$\mathbf{p}_c = [k_1\ k_2]^T. \tag{10}$$

Note that the actuator constraints are embedded *a priori* in the feedback controller structure.

The performance of candidate redesigns will be compared by their ability to follow the reference input:

$$\mathbf{r}(t) = \begin{cases} 1 \text{ for } 0 \le t < t_r \\ 0 \text{ for } t_r \le t < \infty, \end{cases} \tag{11}$$

where $t_r = 4$ is the command switch time. The system starts from rest initial conditions, levying equality constraints

$$\mathbf{S}_e(\cdot) = [x_1(0)\ x_2(0)]^T. \tag{12}$$

We choose the performance measure to be the integral absolute error (IAE) criteria

$$J = \int_0^\infty |x(t) - r(t)|\ dt \tag{13}$$

The IAE is a measure of the ability of the closed-loop system to track reference commands. It is similar to the more familiar integral time absolute error (ITAE) criteria for pole placement [16], but without the time weighting. The ITAE criteria places more emphasis on the error at later times; the IAE criteria places equal emphasis on the error at all times. If the system tracks the reference perfectly, the objective function value is zero.

The closed-loop parameter values (7) and (10) for the nominal system, summarized in the first column of Table 2, have been chosen such that the plant and the closed-loop system are critically damped. The response of the nominal system to the reference input (11) and the resultant control input $\mathbf{u}(t)$ are shown in Fig. 3(*a*). We address the problem where the dynamic performance of the nominal system is inadequate and redesign is required. Since the nominal value of the objective function is $J_{nom} = 2.02$, we decide to set a performance goal of $J_{\max} = 1.50$ which represents a 25 percent decrease in the nominal performance measure value (or a 25 percent improvement in performance).

As seen in the previous section, the process of rational redesign will involve the resolution of the following performance related questions:

(1) For the given plant structure and given actuator constraints, what is the best performance attainable if the plant parameters can vary within their specified limits.

(2) What is the best performance attainable for the given plant structure, given (nominal) plant parameters and given actuator constraints.

(3) For the given plant structure, a given nominal *feedback* control strategy (state feedback in our case) and given actuator constraints,

(*a*) what is the best performance attainable by the appropriate combination of plant parameter and controller parameters and

(*b*) what are the values of the plant parameters and controller parameters which yield the best performance.

In addition, we will pose the following useful question:

(4) How "good" is the proposed state-feedback control strategy from a performance standpoint.

The solutions of Problems one, two, and three of the redesign framework will yield, respectively, quantitative answers to questions one, two, and three above. Comparison of the solution of Problem three with that of Problem one will yield the answer to question four above.

The solution of Problem one will tell us whether it might be possible to achieve the desired performance improvement without redesigning the plant structure. (As mentioned in the previous section if the problem "fails," then the plant structure must be redesigned; if the problem "passes," then it *might* be possible to obtain the desired performance improvement.) The solution of Problem two will inform the designer whether it might be possible to achieve the desired performance improvement with the given plant structure and the given nominal plant parameters. (If problem two "fails," then the plant parameters must be varied from their nominal values in order to obtain the desired performance improvement.) Finally, the solution of Problem three will inform the designer whether or not (s)he can obtain the desired performance improvement by a suitable combination of plant parameters and controller parameters for the given nominal feedback control strategy and will also provide the values of the plant parameters and the controller parameters that yield the best performance.

With the above background, we now discuss the solution of our redesign problem in more detail.

In accordance with the redesign framework of Fig. 2, we first solve problem one: choose $\mathbf{u}(t)$ and $\mathbf{p}_p$ to minimize (13) subject to (6) and (12). The optimal control is found to be piecewise constant:

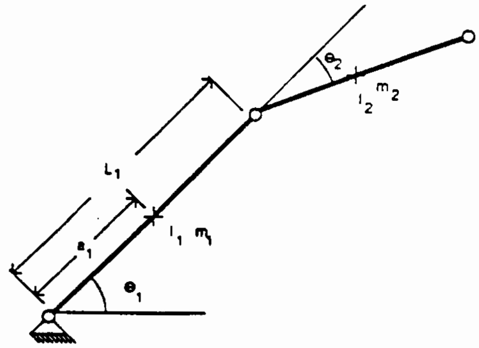

Fig. 4 A two-degree-of-freedom planar manipulator

$$\mathbf{u}^*(t) = \begin{Bmatrix} \mathbf{u}_{max} \text{ for } 0 \le t < t_{s1} \\ -\mathbf{u}_{max} \text{ for } t_{s1} \le t < t_{f1} \\ \text{k for } t_{f1} \le t < t_r \\ -\mathbf{u}_{max} \text{ for } t_r \le t < t_{s2} \\ \mathbf{u}_{max} \text{ for } t_{s2} \le t < _{f2} \\ 0 \text{ for } t_{f2} \le t \end{Bmatrix} \quad (14)$$

where t_{s1}, t_{f1}, t_{s2}, t_{f2} are switch times. The optimal values for the plant parameters and switch times are summarized in Table 2. The optimal value for the objective function in problem one, $J_1 = 1.31$, is lower than the performance goal J_{max} and problem one is said to "pass" (Fig. 2).

Since problem one passed, problem two is solved: choose $\mathbf{u}(t)$ to minimize (13) subject to (6) and (12). The optimal control has the same form (14) as the optimal control found in problem one, but with the optimal switch times shown in Table 2. The objective value for problem two, $J_2 = 1.66$, is greater than the performance goal so the problem fails, indicating that no redesign solution exists with the same plant parameters and plant structure as the nominal system. We must redesign the plant. Since problem one passed, it is hoped that only the plant parameters, and not the plant structure, need be redesigned.

We now solve problem three: choose $\mathbf{p}_p$ and $\mathbf{p}_c$ to minimize (13) subject to (6), (9), and (12). Note that we have retained the nominal controller structure. The optimal solution to problem three shows that the objective function value $J_3 = 1.32$ is lower than the performance goal; the problem passes and the redesign is complete. Figure 3(*b*) shows the plant position $x_1(t)$ and input $\mathbf{u}(t)$ in response to the reference input (11) for the improved, redesigned system.

Examining the last row of Table 2, we see that the performance J_3 (= 1.32) obtained by a suitable combination of plant parameters and controller parameters for the state-feedback control strategy (problem 3) is very close to the best performance J_1 (= 1.31) obtainable by the dynamic system (problem 1). This implies that the state-feedback control strategy is an extremely good control strategy for the given dynamic system and the given application. The example therefore also demonstrates how the framework can be used to assess the "goodness," from a performance standpoint, of a selected feedback control strategy—a state feedback strategy in the present case.

5 Example 2: Application to Robot Manipulator Control

The second example applies the redesign optimization to an experimental robot to show how two controllers interact differently with the same plant. We will address the following three important but largely unanswered issues which arise in the design of robotic systems for dynamic performance:

(1) Given two widely used control strategies which are representative of those used in robotic applications—a simple proportional-derivative control strategy (at each joint) and a more complex feedback linearization control strategy—determine which of these control strategies yields better performance for the given task.

(2) Should a manipulator be counterweighted in order to improve its performance?

(3) What combination of controller parameters and plant parameters yields the best performance for each of the above control strategies?

Table 3(*a*) Component mass properties of the manipulator

component	mass (kg)	mass center (m)	inertia (kg·m²)
link 1	0.230	0.150	0.0015
link 2	0.200	0.130	0.0013
motor 1	0.0	0.0	0.00064
motor 2	1.600	0.305	0.00064
counterweight 1	m_{c1}	-0.10	0.0
counterweight 2	m_{c2}	-0.10	0.0

Table 3(*b*) Other fixed parameters of the manipulator

$\Theta_0 = [0\ 0\ 0\ 0]$	$\Theta_d = [10°\ 0\ 90°\ 0]$
$\tau_{max1} = 0.5$ N-m	$\tau_{max2} = 0.5$ N-m
$r_{c1} = -0.10$ m	$r_{c2} = -0.10$ m

The resolution of the first issue will demonstrate how one chooses a suitable control strategy for robotic manipulator applications. The resolution of the second issue will demonstrate how one chooses a suitable configuration from a dynamic performance standpoint. Furthermore, we will also demonstrate how one can determine that combination of plant parameters and controller parameters (for a given feedback control strategy) which yields the best performance for the given task. We will resolve all the above issues by solving Problem three in our redesign framework. Finally we will present experimental verification of the "best" solution from a performance standpoint.

The nominal plant is a two-link planar robot, shown in Fig. 4. The robot is direct-driven by two motors; motor 1, grounded to the robot base, drives link 1 and motor 2, grounded to link 1, drives link 2. The links rotate in the (horizontal) plane perpendicular to gravity.

In Fig. 4, I_i, m_i, a_i, and L_i are , respectively, the composite principal moment of inertia, composite mass, composite center of mass, and length of link i (see Table 3)[1]. For example, the composite mass of link 1 consists of the sum of the mass of link 1 and the mass of motor 2, which is grounded to link 1. The robot has two degrees of freedom, which shall be taken as the joint angles $\theta_1(t)$ and $\theta_2(t)$. For simplicity, define the following:

$$p_1 = I_2 + m_2 a_2^2 \qquad p_3 = I_1 + m_1 a_2^2 + m_2 l_1^2$$

$$p_2(t) = m_2 a_2 l_2 \cos(\theta_2(t)) \qquad p_4(t) = m_2 a_2 l_2 \sin(\theta_2(t))$$

$$d_{11}(t) = p_1 + 2p_2(t) + p_3 \qquad d_{21}(t) = d_{12}(t)$$

$$d_{12}(t) = p_1 + p_2(t) \qquad d_{22} = p_1$$

$$v_1(t) = -p_4(t)[\dot{\theta}_2^2(t) + 2\dot{\theta}_1(t)\dot{\theta}_2(t)] \qquad v_2(t) = p_4(t)\dot{\theta}_1^2(t)$$

$$D(t) = \begin{pmatrix} d_{11}(t) & d_{12}(t) \\ d_{21}(t) & d_{22}(t) \end{pmatrix} \qquad V(t) = \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix}$$

$$\ddot{\Theta}(t) = \begin{pmatrix} \ddot{\theta}_1(t) \\ \ddot{\theta}_2(t) \end{pmatrix} \qquad T(t) = \begin{pmatrix} \tau_1(t) \\ \tau_2(t) \end{pmatrix}$$

[1]All values in Table 3 correspond to an actual robot for which we will later verify some of the analytical results.

The robot dynamic equations (or plant structure) can then be expressed as

$$T(t) = D(t)\ddot{\Theta}(t) + V(t), \tag{15}$$

so that

$$\ddot{\Theta}(t) = D^{-1}(t)[T(t) - V(t)]. \tag{16}$$

Note that the robot dynamic Eqs. (16) are nonlinear (in the so-called joint velocities) due to $V(t)$ and that the dynamics of each link are coupled due to $V(t)$ and the nondiagonal $D(t)$.

The dynamic task of the robot is to move from one specified point to another in the workspace and the dynamic performance of the robot will be measured by the time the robot takes in making the move (the move time t_f). Two position controllers will be studied. The first controller is a local proportional-derivative (PD) controller with structure

$$u_1'(t) = \tau_1(t) = -k_{11}\theta_1(t) - k_{12}\dot{\theta}_1(t)$$

$$u_2'(t) = \tau_2(t) = -k_{21}\theta_2(t) - k_{22}\dot{\theta}_2(t) \tag{17}$$

The PD controller is local in that it treats each link as a separate system: control input $u_1'(t)$ is computed using only information about link 1 and control input $u_2'(t)$ is computed using only information about link 2. However, because the link dynamics are coupled, either control input working alone will affect both links. Interactions between the links are treated as disturbances by the controller, and rejected like any other disturbance.

The second controller to be studied is a feedback linearization (FL) controller [17], which uses an input transformation to decouple the dynamic system (16). Explicitly, the transformation is

$$[u_1'(t) u_2'(t)]^T = D^{-1}(t)[T(t) - V(t)]. \tag{18}$$

Since $D(t)$ and $V(t)$ are functions of the states of the system, the input transformation is also a function of the states. Under the transformation (18), the system (16) becomes

$$\ddot{\theta}_1(t) = u_1'(t),$$

$$\ddot{\theta}_2(t) = u_2'(t). \tag{19}$$

Now, each control input is computed using information about both links, but control input $\mathbf{u}_1'(t)$ affects link 1 only and input $\mathbf{u}_2'(t)$ affects link 2 only. The transformed system is then controlled by the linear control law

$$u_1'(t) = -k_{11}\theta_1(t) - k_{12}\dot{\theta}_1(t),$$

$$u_2'(t) = -k_{21}\theta_2(t) - k_{22}\dot{\theta}_2(t). \tag{20}$$

Note that the torque input $T(t)$ can be obtained by combining Eqs. (15), (19), and (20).

The motors which drive the links are subject to the following constraints

$$|\tau_1(t)| \leq \tau_{max1},$$

$$|\tau_2(t) \leq \tau_{max2}. \tag{21}$$

As mentioned earlier, the above actuator limits are taken into account a priori in the determination of the controller parameters (or gains).

The two controllers are different in several ways. The PD controller treats the interactions between links as disturbances and uses the actuator torques to reject the disturbances. The FL controller explicitly calculates the interaction torques and adjusts the link input torques to counteract the interactions. Both controllers work well when the maximum torque limits τ_{max1} and τ_{max2} are high: the PD controller has plenty of torque to reject interactions, and the FL controller has enough torque to explicitly remove interactions. However, when torque limits are not large enough and the motors saturate, the controllers

Table 4 Local optimal for Example 2

system		controller gains				objective
controller	counterwt	k_{11}	k_{12}	k_{21}	k_{22}	move time t_f
PD	no	795	122	1020	60.6	0.613
PD	yes	332	57.2	1160	102	0.830
FL	no	994	156	3310	269	0.857
FL	yes	2010	288	4670	238	0.859

respond differently and can yield significantly different performance, as will be seen. The first redesign question will therefore be to determine which of the two control strategies yields better performance for the given task and the given constraints.

The second redesign question which we will pose is the following: should the robot be counterweighted in order to decrease its move time? A counterweighted robot has fewer dynamic interactions and no nonlinear interactions between links as can be seen by letting the centers of mass a_i go to zero in the robot dynamic Eqs. (15). Thus, both controllers will have fewer interactions to counteract, and may yield faster dynamic response. However, counterweights add mass and inertia to the links, and the torque-limited motors might not be able to overcome the additional load.

To answer the redesign questions, we solve an optimization problem in which controller gains and counterweight masses are chosen to minimize the move time. This is an example of problem three (see Table 1) in which plant parameters (counterweight masses) and controller parameters (controller gains) are designed simultaneously. Counterweights of mass m_{c1} and m_{c2} will be distances r_{c1} and r_{c2} from the link rotation axis for links 1 and 2, respectively. Formally, the problem is to choose the controller gains and the plant parameters k_{11}, k_{12}, k_{21}, k_{22}, m_{c1}, and m_{c2} to minimize

$$J = t_f \tag{22}$$

subject to the initial and final conditions

$$[\theta_1(0)\ \dot{\theta}_1(0)\ \theta_2(0)\ \dot{\theta}_2(0)] = \Theta_0,$$

$$[\theta_1(t_f)\ \dot{\theta}_1(t_f)\ \theta_2(t_f)\ \dot{\theta}_2(t_f)] = \Theta_d, \tag{23}$$

the torque limit (21), the plant structure (16), and the controller structure (17) or (18). Fixed parameters in the problem, including the link mass properties, are shown in Table 4.

The problem was formulated and solved using a penalty function approach. The objective (22) and the final state constraint (23) were replaced with the objective

$$J = t_f + 10\|\Theta_f - \Theta_d\| \tag{24}$$

where Θ_f is the actual state at t_f. For each controller, local optima of interest exist for the robot with and without counterweights. Thus, we will discuss four cases: the system with

(1) PD controller and no counterweight,
(2) PD controller and counterweight,
(3) FL controller and no counterweight,
(4) FL controller and counterweight.

The optimal solutions for the four cases are summarized in Table 4. The move time t_f for each of the four possible cases are given in the last column of Table 4. The following conclusions can be drawn from an examination of Table 4:

(1) The combination of a simple PD control strategy and an uncounterweighted manipulator yields significantly better performance than any of the other cases.
(2) The simple PD control strategy yields (*a*) significantly better performance than the more complex feedback linearization control strategy for the uncounterweighted case and (*b*) yields slightly better performance than the FL control strategy for the counterweighted case.
(3) The uncounterweighted design yields better performance

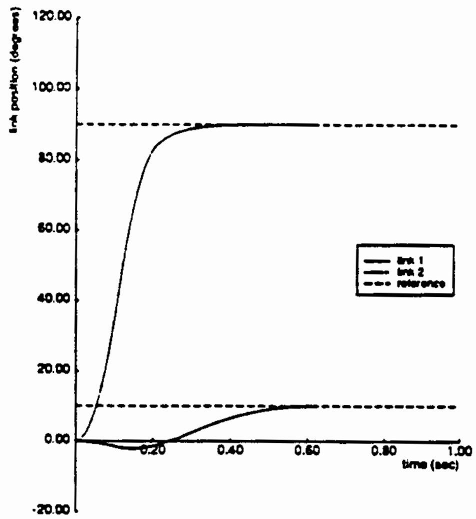

Fig. 5(*a*) Simulation results for pd controller with no counterweight and gains as in Table 4

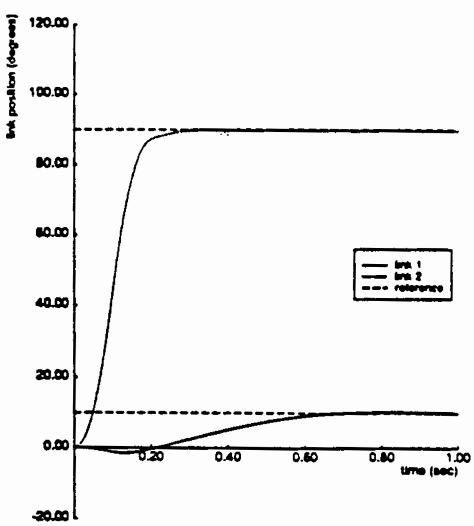

Fig. 5(*b*) Experimental results for pd controller with no counterweight and gains as in Table 4

than the counterweighted design. Therefore for the present example it is not advantageous to use a counterweight.

In summary, the example demonstrates that, in the present context, the simpler mechanical design combined with the simpler control strategy yields better dynamic performance. Therefore adding complexity either in terms of control strategy or mechanical configuration does not yield any advantage from a performance standpoint. One must bear in mind, however, that the above conclusions are dependent on the task specifications and the objective function. Perhaps the most important conclusion is that we have demonstrated how the redesign framework can be used to draw definitive and useful conclusions about the performance of feedback-controlled dynamic systems.

In order to demonstrate the validity of the redesign optimization results, a simple experiment was conducted. The robot described by (16) with the torque limits and component mass properties in Table 3 was controlled using the PD controller (17) with the optimal controller gains given in row one of Table 4. The position of both links (as measured by optical encoders) versus time is presented in Fig. 5(*b*).

The response of the simulated system, Fig. 5(*a*), is in close agreement with the experimental results, Fig. 5(*b*), despite the use of a model that ignores such potentially important effects as joint friction, controller sampling rate, and actuator dynamics. The model agrees well with the experiment because the most significant constraint, motor saturation, was correctly modeled and then incorporated into the formulation of the plant-controller redesign problem. The experimental value of the performance measure (about 0.65 sec) is within 10 percent of the simulation value and, more importantly, the simulation response captures all of the significant features of the experimental response, proving that the optimization based framework yields controllers that actually deliver the desired or predicted performance.

6 Summary and Conclusions

The dynamic performance of a closed-loop dynamic system is limited by plant design, controller design, and system constraints such as the bounds on actuator efforts. With this fact in mind, we have developed an optimization-based redesign framework which enables the designer to improve the performance of a closed-loop dynamic system by the simultaneous redesign of plant and controller. The framework is presented as the formulation and solution of a progression of optimization problems which establish the limits of performance of the dynamic system under various conditions of interest, thereby enabling the designer to systematically establish the various redesign possibilities. Using a second order linear dynamic system and a nonlinear controller as an example, we demonstrated the application of the framework as well as one of the important underlying ideas of the present work, viz., that in order to achieve the best possible performance from a constrained closed-loop system, the plant and controller should be redesigned simultaneously.

The system constraints which exert a critical influence on dynamic performance are those imposed by the bounds on actuator efforts. Constraints on actuator efforts are not normally taken into account in most analytical *feedback* control system design techniques for linear and nonlinear systems, leading to solutions which do not accurately predict the performance of the actual dynamic system. Consequently these solutions cannot be used to compare competing control strategies (for the purpose of selecting the best control strategy from a dynamic performance standpoint). Nor can these solutions be implemented with confidence on the actual system. Using a planar two-degree-of-freedom manipulator as an example, we have shown that the solutions obtained from the application of the proposed redesign framework do not suffer from these major shortcomings. The robotic manipulator example also demonstrated the nonobvious result that, for the particular task, the more complex feedback linearization control strategy frequently proposed in the literature does not lead to better dynamic performance than a simple PD control strategy and can even lead to significantly poorer performance then the PD control strategy.

A natural extension of the present work would be to study typical performance robustness issues [6] within the redesign framework.

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