READ ME FIRST

- Don't spend too much time on any one problem. This exam will take approximately 60 minutes.
- Amount of time spent on a problem is not necessarily proportional to the points.
- Scan through the entire test and do the easy problems first.
- If something is not clear, ASK.
- BE NEAT. We cannot give you points for something that we can't read.
- Write down your assumptions.
- Don't just write your answer, show how you got them.
- This is a CLOSED BOOK, CLOSED NOTES exam. You are allowed one 8.5x11 "cheat sheet".

1	20 points	Quaternions	
2	20 points	Integration	
3	20 points	Rotation	
4	20 points	Morphing	
5	20 points	Forward Kinematics	
	100 points	GRAND TOTAL	

1. **Quaternions (20 points)**
   (a) **5 points**

      Find the unit quaternion $\hat{q}_a$ that corresponds to R_y^{90}.

   (b) **5 points**

      Find the unit quarternion $\hat{q}_b$ that corresponds to a rotation of 90 degrees about the axis $[1, 1, 1]$.

   (c) **10 points**

      Find the unit quarternion $\hat{q}_c$ halfway between $\hat{q}_a$ and $\hat{q}_b$ using spherical linear interpolation.

2. **Integration (20 points)**

The velocity of a 2D point P: (P_x, P_y) is known to be V: $[1+x^2, 1+y^2]$. A point P_0 starts off at $(1, 1)$.

(a) **10 points**

Where is the point after 1 Euler integration step using h = 0.5?

(b) **10 points**

Where is the point after another Euler integration step?

3. **Rotation (20 points)**

An object moves from point A: (a_x, a_y, a_z) to point B: (b_x, b_y, b_z) in a straight line.

(a) **5 points**

Express the position of this object, $P(t)$, as a function of time parameterized from 0 to 1.

(b) **10 points**

The object is also suppose to rotate about the axis defined by the line from point A to point B as it moves from A to B. If the desired number of rotations is k, express the rotation angle θ as a function of time, i.e. $\theta(t)$, as t goes from 0 to 1.

(c) **5 points**

If instead of moving from A to B in a straight line, the path from A to B is defined by the curve: $P(t) = dt^2 + et + f$. We still want the object to rotate about the axis along the path from A to B. What is the axis of rotation? Hint: it varies over time.

4. **Morphing (20 points)**

   Analyze the line-pair using the Beier and Neely algorithm. At time=0, a line goes from $(10, 10)$ to $(10, 70)$. The corresponding line at time=10 goes from $(10, 10)$ to $(10, 30)$.
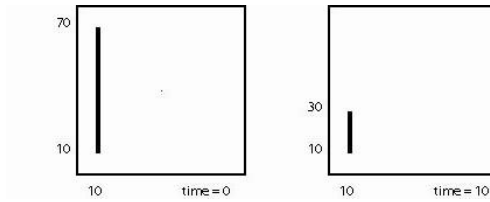
   (a) **5 points**

   Where is the corresponding line for the intermediate grid at time=5.

   (b) **5 points**

   Which pixel in the initial (time=0) frame corresponds to the intermediate grid pixel at $(50, 20)$?

   (c) **5 points**

   Which pixel in the final (time=10) frame corresponds to the intermediate grid pixel at $(50, 20)$?

   (d) **5 points**

   If the pixels in the inital frame are all blue, and the pixels in the final frame are all red, what is the color of the intermediate grid pixel at $(50, 20)$?

5. **Forward Kinematics (20 points)**

   In the figure below, joint A is at $(0,0)$. Find the position of the end-effector E given L1, L2, θ_A, θ_B. Express your answer in terms of the given variables.