# Protein folding: not just another optimization problem 

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## Outline of Talk

What is Bioinformatics?
\& What is a protein?
The folding problem and variants on it:

- Local structure prediction
- Fold recognition with HMMs
- What is a null model?
- Why use the reverse-sequence null?
- Two approaches to statistical significance.
- What distribution do we expect for scores?
- Fitting the distribution.
- Comparative modeling
- "Ab initio" methods
- Contact prediction


## What is Bioinformatics?

Bioinformatics: using computers and statistics to make sense out of the mountains of data produced by high-throughput experiments.
\& Genomics: finding important sequences in the genome and annotating them.
E Phylogenetics: "tree of life".
\& Systems biology: piecing together various control networks.
\& DNA microarrays: what genes are turned on under what conditions.
\& Proteomics: what proteins are present in a mixture.
\& Protein structure prediction.

## What is a protein?

\& There are many abstractions of a protein: a band on a gel, a string of letters, a mass spectrum, a set of 3D coordinates of atoms, a point in an interaction graph, ... .
\& For us, a protein is a long skinny molecule (like a string of letter beads) that folds up consistently into a particular intricate shape.
\& The individual "beads" are amino acids, which have 6 atoms the same in each "bead" (the backbone atoms: N , H, CA, HA, C, O).
The final shape is different for different proteins and is essential to the function.
\& The protein shapes are important, but are expensive to determine experimentally.

## Folding Problem

The Folding Problem:
If we are given a sequence of amino acids (the letters on a string of beads), can we predict how it folds up in 3-space?


## Fold-recognition problem

The Fold-recognition Problem:
Given a sequence of amino acids $A$ (the target sequence) and a library of proteins with known 3-D structures (the template library),
figure out which templates $A$ match best, and align the target to the templates.
\& The backbone for the target sequence is predicted to be very similar to the backbone of the chosen template.
\& Progress has been made on this problem, but we can usefully simplify further.

## Remote-homology Problem

The Homology Problem:
Given a target sequence of amino acids and a library of protein sequences,
figure out which sequences $A$ is similar to and align them to $A$.
\& No structure information is used, just sequence information. This makes the problem easier, but the results aren't as good.
\& This problem is fairly easy for recently diverged, very similar sequences, but difficult for more remote relationships.

## New-fold prediction

What if there is no template we can use?
We can try to generate many conformations of the protein backbone and try to recognize the most protein-like of them.
Search space is huge, so we need a good conformation generator and a cheap cost function to evaluate conformations.

## Secondary structure Prediction

\& Instead of predicting the entire structure, we can predict local properties of the structure.
What local properties do we choose?
\& We want properties that are well-conserved through evolution, easily predicted, and useful for finding and aligning templates.
\& One popular choice is a 3-valued helix/strand/other alphabet-we have investigated many others. Typically, predictors get about $80 \%$ accuracy on 3-state prediction.
Many machine-learning methods have been applied to this problem, but the most successful is neural networks.

## CASP Competition Experiment

Everything published in literature "works"
CASP set up as true blind test of prediction methods.
$\leftrightarrow$ Sequences of proteins about to be solved released to prediction community.

Eredictions registered with organizers.
E Experimental structures compared with solution by assessors.

* "Winners" get papers in Proteins: Structure, Function, and Bioinformatics.


## Predicting Local Structure

\& Want to predict some local property at each residue.
\& Local property can be emergent property of chain (such as being buried or being in a beta sheet).
\& Property should be conserved through evolution (at least as well as amino acid identity).
\& Property should be somewhat predictable (we gain information by predicting it).
\& Predicted property should aid in fold-recognition and alignment.

E For ease of prediction and comparison, we look only at discrete properties (alphabets of properties).

## Using Neural Net

We use neural nets to predict local properties.
\& Input is profile with probabilities of amino acids at each position of target chain, plus insertion and deletion probabilities.
O Output is probability vector for local structure alphabet at each position.
Each layer takes as input windows of the chain in the previous layer and provides a probability vector in each position for its output.
\& We train neural net to maximize $\sum \log (P($ correct output $)$.

## Neural Net

Typical net has 4 layers and 6471 weight parameters:

| input/pos | window | output/pos | weights |
| ---: | ---: | ---: | ---: |
| 22 | 5 | 15 | 1665 |
| 15 | 7 | 15 | 1590 |
| 15 | 9 | 15 | 2040 |
| 15 | 13 | 6 | 1176 |



Inputs

Hidden Layer 1

Hidden Layer 2

Hidden Layer 3

Output Layer


## DSSP

\& DSSP is a popular program to define secondary structure.
\& 7-letter alphabet: EBGHSTL

- $\mathbf{E}=\beta$ strand
- $\mathbf{B}=\beta$ bridge
- $\mathbf{G}=3_{10}$ helix
- $\mathrm{H}=\alpha$ helix
- $\mathrm{I}=\pi$ helix (very rare, so we lump in with H)
- $\mathrm{S}=$ bend
- T = turn
- $L=$ everything else (DSSP uses space for L )


## STR: Extension to DSSP

\& Yael Mandel-Gutfreund noticed that parallel and anti-parallel strands had different hydrophobicity patterns, implying that parallel/antiparallel can be predicted from sequence.
E We created a new alphabet, splitting DSSP's E into 6 letters:

$$
\begin{array}{cc}
\uparrow \uparrow \uparrow \mathrm{P} & \uparrow \uparrow \mathrm{Q} \\
v \uparrow \vee \mathrm{~A} & \uparrow \vee \mathrm{Z} \\
v \uparrow \uparrow \mathrm{M} & \uparrow=\mathrm{E}
\end{array}
$$

## HMMSTR $\phi-\psi$ alphabet

E For HMMSTER, Bystroff did k-means classification of $\phi-\psi$ angle pairs into 10 classes (plus one class for cis peptides).
\& We used just the 10 classes, ignoring the $\omega$ angle.


## ALPHA11: $\alpha$ angle

\& Backbone geometry can be mostly summarized with one angle per residue:

\& We discretize into 11 classes:


## de Brevern's Protein Blocks

Clustered on 5 -residue window of $\phi$ - $\psi$ angles:


## Burial alphabets

Our second set of investigations was for a sampling of the many burial alphabets, which are discretizations of various accessibility or burial measures:
\& solvent accessible surface area
\& relative solvent accessible surface area
\& neighborhood-count burial measures

## Solvent Accessibility

\& Absolute SA: area in square Ångstroms accessible to a water molecule, computed by DSSP.
\& Relative SA: Absolute SA/ max SA for residue type (using Rost's table for max SA).


## Burial

\& Define a sphere for each residue.
Count the number of atoms or of residues within that sphere.
\& Example: center $=\mathrm{C}_{\beta}$, radius $=14 \AA$, count $=\mathrm{C}_{\beta}$, quantize in 7 equi-probable bins.


## Mutual Information

\& Mutual information between two random variables (letters of alphabet):

$$
M I(X, Y)=\sum_{i, j} P(i, j) \log \frac{P(i, j)}{P(i) P(j)},
$$

\& We look at mutual information between different alphabets at same position in protein. (redundancy)
\& We look at mutual information with one alphabet between corresponding positions on alignments of sequences.

## Information Gain

\& Information gain is how much more we know about a variable after making a prediction.

$$
I(X)=\text { average } \log \frac{\hat{P}_{i}\left(X_{i}\right)}{P_{0}\left(X_{i}\right)}
$$

\& $\hat{P}_{i}$ is predicted probability vector for position $i$
\& $X_{i}$ is actual observation at position $i$
\& $P_{0}$ is background probability vector

## Conservation and Predictability

| Name | alphabet <br> size | entropy | $\begin{gathered} \mathrm{MI} \\ \text { with } \mathrm{AA} \end{gathered}$ | conservation <br> mutual info | predictability info gain per residue |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| str | 13 | 2.842 | 0.103 | 1.107 | 1.009 | 0.561 |
| protein blocks | 16 | 3.233 | 0.162 | 0.980 | 1.259 | 0.579 |
| stride | 6 | 2.182 | 0.088 | 0.904 | 0.863 | 0.663 |
| DSSP | 7 | 2.397 | 0.092 | 0.893 | 0.913 | 0.633 |
| stride-EHL | 3 | 1.546 | 0.075 | 0.861 | 0.736 | 0.769 |
| DSSP-EHL | 3 | 1.545 | 0.079 | 0.831 | 0.717 | 0.763 |
| CB-16 | 7 | 2.783 | 0.089 | 0.682 | 0.502 |  |
| CB-14 | 7 | 2.786 | 0.106 | 0.667 | 0.525 |  |
| CB-12 | 7 | 2.769 | 0.124 | 0.640 | 0.519 |  |
| rel SA | 7 | 2.806 | 0.183 | 0.402 | 0.461 |  |
| abs SA | 7 | 2.804 | 0.250 | 0.382 | 0.447 |  |

## Hidden Markov Models

Hidden Markov Models (hmмs) are a very successful way to capture the variability possible in a family of proteins.
\& An нмм is a stochastic model-that is, it assigns a probability to every possible sequence.
\& An нмм is a finite-state machine with a probability for emitting each letter in each state, and with probabilities for making each transition between states.

* Probabilities of letters sum to one for each state.
\& Probabilities of transitions out of each state sum to one for that state.
\& We also include null states that emit no letters, but have transition probabilities on their out-edges.


## Profile Hidden Markov Model


\& Circles are null states.
\& Squares are match states, each of which is paired with a null delete state. We call the match-delete pair a fat state.
$\Leftrightarrow$ Each fat state is visited exactly once on every path from Start to End.
\& Diamonds are insert states, and are used to represent possible extra amino acids that are not found in most of the sequences in the family being modeled.

## What is single-track HMM looking for?

nostruct-align/3chy.t2k w0.5


## What is second track looking for?

nostruct-align/3chy.t2k EBGHTL


## Multi-track Нммs

We can also use alignments to build a two- or three-track target нмм:
Amino-acid track (created from the multiple alignment).
Local-structure track(s) with probabilities from neural net.
\& Can align template (AA+local) to target model.


## Target-model Fold Recognition

\& Find probable homologs of target sequence and make multiple alignment.
\& Make secondary structure probability predictions based on multiple alignment.
\& Build an нмм based on the multiple alignment and predicted 2ry structure (or just on multiple alignment).
\& Score sequences and secondary structure sequences for proteins that have known structure (all sequences for AA-only, 8,000-11,000 representatives for multi-track).
\& Select the best-scoring sequence(s) to use as templates.

## Template-library Fold Recognition

\& Build an нмм for each protein in the template library, based on the template sequence (and any homologs you can find).
\& The T2K library has over 11,000 templates from PDB.
\& For the fold-recognition problem, structure information can be used in building these models (though we currently don't).
\& Score target sequence with all models in the library.
S Select the best-scoring model(s) to use as templates.

## Combined SAM-T02 method



Combine the costs from the template library search and the target library searches using different local structure alphabets.
\& Choose one of the many alignments of the target and template (whatever method gets best results in testing).
http://www.soe.ucsc.edu/research/compbio/HMM-apps/T02-query.html

## Fold recognition results



## Scoring нымs and Bayes Rule

\& The model $M$ is a computable function that assigns a probability $\operatorname{Prob}(A \mid M)$ to each string $A$.
When given a string $A$, we want to know how likely the model is. That is, we want to compute something like Prob ( $M \mid A$ ).
\& Bayes Rule:

$$
\operatorname{Prob}(M \mid A)=\operatorname{Prob}(A \mid M) \frac{\operatorname{Prob}(M)}{\operatorname{Prob}(A)} .
$$

\& Problem: $\operatorname{Prob}(A)$ and $\operatorname{Prob}(M)$ are inherently unknowable.

## Null models

\& Standard solution: ask how much more likely $M$ is than some null hypothesis (represented by a null model).

$$
\frac{\operatorname{Prob}(M \mid A)}{\operatorname{Prob}(N \mid A)}=\frac{\operatorname{Prob}(A \mid M)}{\operatorname{Prob}(A \mid N)} \frac{\operatorname{Prob}(M)}{\operatorname{Prob}(N)} .
$$

$\leqslant \frac{\operatorname{Prob}_{(M)}}{\operatorname{Prob}(N)}$ is the prior odds ratio, and represents our belief in the likelihood of the model before seeing any data.
$\in \frac{\operatorname{Prob}(M \mid A)}{\operatorname{Prob}(N \mid A)}$ is the posterior odds ratio, and represents our belief in the likelihood of the model after seeing the data.

## Standard Null Model

Null model is an i.i.d (independent, identically distributed) model.

$$
\begin{gathered}
\operatorname{Prob}(A \mid N, \operatorname{len}(A))=\prod_{i=1}^{\operatorname{len}(A)} \operatorname{Prob}\left(A_{i}\right) \\
\operatorname{Prob}(A \mid N)= \\
\operatorname{Prob}(\text { string of length len }(A)) \\
\prod_{i=1}^{\operatorname{len}(A)} \operatorname{Prob}\left(A_{i}\right)
\end{gathered}
$$

\& The length modeling is often omitted, but one must be careful then to normalize the probabilities correctly.

## Problems with standard null

\& When using the standard null model, certain sequences and нммs have anomalous behavior. Many of the problems are due to unusual composition-a large number of some usually rare amino acid.
\& For example, metallothionein, with 24 cysteines in only 61 total amino acids, scores well on any model with multiple highly conserved cysteines.

## Reversed model for null

\& We avoid composition bias (and several other problems) by using a reversed model $M^{r}$ as the null model.
\& The probability of a sequence in $M^{r}$ is exactly the same as the probability of the reversal of the sequence given $M$.
\& If we assume that $M$ and $M^{r}$ have equal prior likelihood, then

$$
\frac{\operatorname{Prob}(M \mid S)}{\operatorname{Prob}\left(M^{r} \mid S\right)}=\frac{\operatorname{Prob}(S \mid M)}{\operatorname{Prob}\left(S \mid M^{r}\right)} .
$$

\& This method corrects for composition biases, length biases, and several subtler biases.

## Composition as source of error

A cysteine-rich protein, such as metallothionein, can match any HMM that has several highly-conserved cysteines, even if they have quite different structures:

|  |  | cost in nats |  |
| :--- | :--- | ---: | ---: |
| HMM | sequence | model - <br> standard null | model - <br> reversed-model |
| 1kst | $4 \mathrm{mt2}$ | -21.15 | 0.01 |
| 1kst | 1tabl | -15.04 | -0.93 |
| $4 \mathrm{mt2}$ | 1kst | -15.14 | -0.10 |
| 4 mt 2 | 1tabl | -21.44 | -1.44 |
| 1 tabl | 1 kst | -17.79 | -7.72 |
| 1tabl | $4 \mathrm{mt2}$ | -19.63 | -1.79 |

## Composition examples

## Metallothionein Isoform II (4mt2)



Kistrin (1kst)


## Composition examples

## Kistrin (1kst)



Trypsin-binding domain of Bowman-Birk Inhibitor (1tabl)


## Helix examples

## Tropomyosin (2tmaA)



Colicin la (1cii)


Flavodoxin mutant (1vsgA)


## Helix examples

Apolipophorin III (1aep)


Apolipoprotein A-I (1av1A)

## What is Statistical Significance?

E The statistical significance of a hit, $P_{1}$, is the probability of getting a score as good as the hit "by chance," when scoring a single "random" sequence.
When searching a database of $N$ sequences, the significance is best reported as an E-value-the expected number of sequences that would score that well by chance: $E=P_{1} N$.
\& Some people prefer the p-value: $P_{N}=1-\left(1-P_{1}\right)^{N}$, For large $N$ and small $E, P_{N} \approx 1-e^{-E} \approx E$.
\& I prefer E-values, because our best scores are often not significant, and it is easier to distinguish between E-values of 10,100 , and 1000 than between p-values of $0.999955,1.0-4 \mathrm{E}-44$, and $1.0-5 \mathrm{E}-435$

## Approaches to Statistical Significance

(Markov's inequality) For any scoring scheme that uses

$$
\ln \frac{\operatorname{Prob}\left(\text { seq } \mid M_{1}\right)}{\operatorname{Prob}\left(\text { seq } \mid M_{2}\right)}
$$

the probability of a score better than $T$ is less than $e^{-T}$ for sequences distributed according to $M_{2}$. This method is independent of the actual probability distributions.
\& (Classical parameter fitting) If the "random" sequences are not drawn from the distribution $M_{2}$, but from some other distribution, then we can try to fit some parameterized family of distributions to scores from a random sample, and use the parameters to compute $P_{1}$ and $E$ values for scores of real sequences.

## Our Assumptions

Bad assumption 1: The sequence and reversed sequence come from the same underlying distribution.
Bad assumption 2: The scores with a standard null model are distributed according to an extreme-value distribution:
$P(\ln \operatorname{Prob}(\operatorname{seq} \mid M)>T) \approx G_{k, \lambda}(T)=1-\exp \left(-k e^{\lambda T}\right)$.
Bad assumption 3: The scores with the model and the reverse-model are independent of each other.
Result: The scores using a reverse-sequence null model are distributed according to a sigmoidal function:

$$
P(\text { score }>T)=\left(1-e^{\lambda T}\right)^{-1} .
$$

## Derivation of sigmoidal distribution

(Derivation for costs, not scores, so more negative is better.)

$$
\begin{aligned}
P(\text { cost }<T) & =\int_{-\infty}^{\infty} P\left(c_{M}=x\right) \int_{x-T}^{\infty} P\left(c_{M^{\prime}}=y\right) d y d x \\
& =\int_{-\infty}^{\infty} P\left(c_{M}=x\right) P\left(c_{M^{\prime}}>x-T\right) d x \\
& =\int_{-\infty}^{\infty} k \lambda \exp \left(-k e^{\lambda x}\right) e^{\lambda x} \exp \left(-k e^{\lambda(x-T)}\right) d x \\
& =\int_{-\infty}^{\infty} k \lambda e^{\lambda x} \exp \left(-k\left(1+e^{-\lambda T}\right) e^{\lambda x}\right) d x
\end{aligned}
$$

## Derivation of sigmoid (cont.)

If we introduce a temporary variable to simplify the formulas: $K_{T}=k(1+\exp (-\lambda T))$, then

$$
\begin{aligned}
P(\operatorname{cost}<T) & =\int_{-\infty}^{\infty}\left(1+e^{-\lambda T}\right)^{-1} K_{T} \lambda e^{\lambda x} \exp \left(-K_{T} e^{\lambda x}\right) d x \\
& =\left(1+e^{-\lambda T}\right)^{-1} \int_{-\infty}^{\infty} K_{T} \lambda e^{\lambda x} \exp \left(-K_{T} e^{\lambda x}\right) d x \\
& =\left(1+e^{-\lambda T}\right)^{-1} \int_{-\infty}^{\infty} g_{K_{T}, \lambda}(x) d x \\
& =\left(1+e^{-\lambda T}\right)^{-1}
\end{aligned}
$$

## Fitting $\lambda$

\& The $\lambda$ parameter simply scales the scores (or costs) before the sigmoidal distribution, so $\lambda$ can be set by matching the observed variance to the theoretically expected variance.
\& The mean is theoretically (and experimentally) zero.
$\&$ The variance is easily computed, though derivation is messy:

$$
E\left(c^{2}\right)=\left(\pi^{2} / 3\right) \lambda^{-2} .
$$

$\Leftrightarrow \lambda$ is easily fit by matching the variance:

$$
\lambda \approx \pi \sqrt{N /\left(3 \sum_{i=0}^{N-1} c_{i}^{2}\right)} .
$$

## Two-parameter family

\& We made three dangerous assumptions: reversibility, extreme-value, and independence.
\& To give ourselves some room to compensate for deviations from the extreme-value assumption, we can add another parameter to the family.
\& We can replace $-\lambda T$ with any strictly decreasing odd function.
\& Somewhat arbitrarily, we chose

$$
-\operatorname{sign}(T)|\lambda T|^{\tau}
$$

so that we could match a "stretched exponential" tail.

## Fitting a two-parameter family

For two-parameter symmetric distribution, we can fit using 2nd and 4th moments:

$$
\begin{aligned}
& E\left(c^{2}\right)=\lambda^{-2 / \tau} K_{2 / \tau} \\
& E\left(c^{4}\right)=\lambda^{-4 / \tau} K_{4 / \tau}
\end{aligned}
$$

where $K_{x}$ is a constant:

$$
\begin{aligned}
K_{x} & =\int_{-\infty}^{\infty} y^{x}\left(1+e^{y}\right)^{-1}\left(1+e^{-y}\right)^{-1} d y \\
& =-\Gamma(x+1) \sum_{k=1}^{\infty}(-1)^{k} / k^{x} .
\end{aligned}
$$

## Fitting a two-parameter family (cont.)

* The ratio $E\left(c^{4}\right) /\left(E\left(c^{2}\right)\right)^{2}=K_{4 / \tau} / K_{2 / t a u}^{2}$ is independent of $\lambda$ and monotonic in $\tau$, so we can fit $\tau$ by binary search.
$\leqslant$ Once $\tau$ is chosen we can fit $\lambda$ using $E\left(c^{2}\right)=\lambda^{-2 / \tau} K_{2 / \tau}$.


## Student's t-distribution

\& On the advice of statistician David Draper, we tried maximum-likelihood fits of Student's t-distribution to our heavy-tailed symmetric data.
\& We couldn't do moment matching, because the degrees of freedom parameter for the best fits turned out to be less than 4, where the 4th moment of Student's $t$ is infinite.
\& The maximum-likelihood fit of Student's $t$ seemed to produce too heavy a tail for our data.
需 We plan to investigate other heavy-tailed distributions.

## Use database, not random sequences

C Calibration with random sequences works ok for 1-track, but not 2-track HMMs.
\& "Random" secondary structure sequences (i.i.d. model) are not representative of real sequences.
\& Fixes:

- Better secondary structure decoy generator
- Use real database, but avoid problems with contamination by true positives by taking only costs $>0$ to get estimate of $E\left(\operatorname{cost}^{2}\right)$ and $E\left(\operatorname{cost}^{4}\right)$.


## What went wrong with Protein Blocks?

\& Brevern's protein blocks provided one of our most predictable local structure alphabets.
\& The 2-track нмms using de Brevern's protein blocks did much worse than AA-only нммs. Why?
\& The protein blocks alphabet strongly violates reversibility assumption.
Encoding cost in bits for secondary structure strings using Markov chains:

|  | alphabet | 0-order | 1st-order | reverse-forward |
| :--- | :--- | ---: | ---: | ---: |
|  | amino acid | 4.1896 | 4.1759 | 0.0153 |
| stride | 2.3330 | 1.0455 | 0.0042 |  |
| dssp | 2.5494 | 1.3387 | 0.0590 |  |
| pb | 3.3935 | 1.4876 | 3.0551 |  |

## Undertaker

\& Undertaker is UCSC's attempt at a fragment-packing program.
\& Named because it optimizes burial.
\& Representation is 3D coordinates of all heavy atoms (not hydrogens).
\& Can replace backbone fragments (a la Rosetta) or full alignments-chain need not remain contiguous.
\& Conformations can borrow heavily from fold-recognition alignments, without having to lock in a particular alignment.
\& Use genetic algorithm with many conformation-change operators to do stochastic search.

## Fragfinder

Fragments are provided to undertaker from 3 sources:
$\Leftrightarrow$ Generic fragments ( $2-4$ residues, exact sequence match) are obtained by reading in 500-1000 PDB files, and indexing all fragments.
\& Long specific fragments (and full alignments) are obtained from the various target and template alignments generated during fold recognition.
\& Medium-length fragments (9-12 residues long) for every position are generated from the нммs with fragfinder, a new tool in the SAM suite.

## Cost function

\& Cost function is modularly designed—easy to add or remove terms.
\& Cost function can include predictions of local properties by neural nets.
\& Clashes and hydrogen bonds are important components.
\& There are over 40 cost function components available: burial functions, disulfides, contact order, rotamer preference, radius of gyration, constraints, ...

## Target T0201 (NF)

\& We tried forcing various sheet topologies and selected 4 by hand.
\& Model 1 has right topology (5.912Å all-atom, 5.219Å $C_{\alpha}$ ).
\& Unconstrained cost function not good at choosing topology (two strands curled into helices).
\& Helices were too short.

## Target T0201 (NF)



## Contact prediction

\& Use mutual information between columns.
\& Thin alignments aggressively (30\%, 35\%, 40\%, 50\%, 62\%).
\& Compute e-value for mutual info (correcting for small-sample effects).
\& Compute rank of log(e-value) within protein.
Feed log(e-values), log rank, contact potential, joint entropy, and separation along chain for pair, and amino-acid profile, predicted burial, and predicted secondary structure for each residue of pair into a neural net.

## Open problem

Given a contingency table for a small sample of pairs of independent discrete random variables, what is the distribution of the mutual information statistic:

$$
M I(X, Y)=\sum_{i, j} P(i, j) \log \frac{P(i, j)}{P(i) P(j)},
$$

where the probabilities are the maximum-likelihood estimates from the observed sample.
Asymptotic results ( $\chi^{2}$ distribution) are known, but neither the shape of the distribution nor how to fit its parameters have been established theoretically (we have good empirical fits).

## Evaluating contact prediction

Two measures of contact prediction:
\& Accuracy:

$$
\frac{\sum \chi(i, j)}{\sum 1}
$$

(favors short-range predictions, where contact probability is higher)
\& Weighted accuracy:

$$
\frac{\sum \frac{\chi(i, j)}{\operatorname{Prob}(\text { contact } \mid \text { separation }=|i-j|)}}{\sum 1}
$$

(1 if predictions no better than chance based on separation).

## Contact prediction results



Weighted-accuracy of contact prediction, by protein


## Target T0230 (FR/A)

Good except for C-terminal loop and helix flopped wrong way.
\& We have secondary structure right, including phase of beta strands.
\& Contact prediction helped, but we put too much weight on it-decoys fit predictions better than real structure does.

## Target T0230 (FR/A)



## Target T0230 (FR/A)

Real structure with contact predictions:


## Web sites

These slides:

```
http://www.soe.ucsc.edu/~karplus/papers/not-just-opt-may-2006.pdf
```

SAM-T06 prediction server:
http://www.soe.ucsc.edu/research/compbio/SAM_T06/T06-query.html
CASP6 all our results and working notes:
http://www.soe.ucsc.edu/~karplus/casp6/
Predictions for all yeast proteins:
http://www.soe.ucsc.edu/~karplus/yeast/
UCSC bioinformatics (research and degree programs) info:
http://www.soe.ucsc.edu/research/compbio/
SAM tool suite info:

