$\LaTeX$ math mode exercise, BME 200

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1 Some integrals

First, we derive the density function from the cumulative distribution:

$$\begin{aligned} f(x) &= \frac{d}{dx}(1+e^{-\lambda x})^{-1} \\ &= \lambda(1+e^{-\lambda x})^{-1}(1+e^{\lambda x})^{-1} \end{aligned} \tag{1}$$

and then compute the expected value of the square of the cost:

$$\begin{aligned} E(c^2) &= \int_{-\infty}^{\infty} f(x)x^2 dx \\ &= \lambda \int_{-\infty}^{\infty} \frac{x^2}{(1+e^{-\lambda x})(1+e^{\lambda x})} dx \end{aligned} \tag{2}$$

We can do a change of variable, $y = -\lambda x$ and look up the equation in Gradshteyn and Ryzhik's table of integrals (3.424#2, p. 331 for Equation 6 and 0.234#1, p. 7 for Equation 7):

$$E(c^2) =$$

$$= \lambda^{-2} \int_{-\infty}^{\infty} \frac{y^2}{(1+e^{y})(1+e^{-y})} dy \tag{3}$$

$$= 2\lambda^{-2} \int_{0}^{\infty} \frac{y^2}{(1+e^{y})(1+e^{-y})} dy \tag{4}$$

$$= 2\lambda^{-2} \int_{0}^{\infty} y^2 \frac{e^y}{(1+e^{y})^2} dy \tag{5}$$

$$= 2\lambda^{-2} 2! \sum_{k=1}^{\infty} (-1)^{k+1}/k^2 \tag{6}$$

$$= 2\lambda^{-2} 2! \pi^2/12 \tag{7}$$

$$= (\pi^2/3)\lambda^{-2} \tag{8}$$

2 Dirichlet stuff

2.1 Lemma 2. **Prob** $(\vec{p} \mid \vec{\alpha}) = \frac{\Gamma(|\vec{\alpha}|)}{\prod_{i=1}^{20}\Gamma(\alpha_i)} \prod_{i=1}^{20} p_i^{\alpha_i - 1}$

Proof:

Under the Dirichlet density with parameters $\vec{\alpha}$, the probability of the distribution $\vec{p}$ (where $p_i \geq 0$, and $\sum_i p_i = 1$) is defined as follows:

$$\text{Prob}(\vec{p} \,|\, \vec{\alpha}) = \frac{\prod_{i=1}^{20} p_i^{\alpha_i - 1}}{\int_{\vec{p}\in\mathcal{P}} \prod_i p_i^{\alpha_i - 1} d\vec{p}} . \tag{9}$$

category	alignments		
	total number	dropped number	%dropped
1 V1	23	4	17.4
1 V2	30	0	0
1 V3	28	0	0
2	23	9	39.1
3	11	3	27.3
4	16	4	25
5	12	0	0

Table 1: The number of cases that SAM-T99 failed to align all sequences, rejecting some sequences as too dissimilar. Note that Category 2, which contains "orphan" sequences, has the highest rate of rejection.

We introduce two formulas concerning the Beta function—its definition

$$\begin{aligned} \mathrm{B}(x,y) &= \int_0^1 t^{x-1}(1-t)^{y-1}\,dt \\ &= \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}, \end{aligned}$$

and its combining formula

$$\int_0^b t^{x-1}(b-t)^{y-1}\,dt = b^{x+y-1}\mathrm{B}(x,y)\ .$$

This allows us to write the integral over all $\vec{p}$ vectors as a multiple integral, rearrange some terms, and obtain

$$\int_{\vec{p}\in\mathcal{P}} \prod_i p_i^{\alpha_i-1} d\vec{p} = \mathrm{B}(\alpha_1, \alpha_2+\ldots+\alpha_{20})\mathrm{B}(\alpha_2, \alpha_3+\ldots+\alpha_{20})\ldots\mathrm{B}(\alpha_{19},\alpha_{20}) \tag{10}$$

$$= \frac{\prod_i \Gamma(\alpha_i)}{\Gamma(|\vec{\alpha}|)}\ . \tag{11}$$

We can now give an explicit definition of the probability of the amino acid distribution $\vec{p}$ given the Dirichlet density with parameters $\vec{\alpha}$:

$$\mathrm{Prob}(\vec{p}\,|\,\vec{\alpha}) = \frac{\Gamma(|\vec{\alpha}|)}{\prod_{i=1}^{20}\Gamma(\alpha_i)} \prod_{i=1}^{20} p_i^{\alpha_i-1}\ . \tag{12}$$

3 Typesetting a table

In Section 1, we practiced typesetting some math. Now let's do a table. Table 1 shows a simple table.