AMS 212, Assignment #8

1. Find the first two non-zero terms in the asymptotic expansion of

$$\int_0^5 \exp\left(-\lambda t^2\right)\frac{1}{1+t}\,dt\,, \qquad \lambda \to +\infty$$

2. Use Laplace method to find the leading term in the asymptotic expansion of

$$\int_0^{\frac{\pi}{4}} \exp\left(-\lambda \sin\left(2\theta\right)\right)d\theta$$

3. Use Laplace method to find the leading term in the asymptotic expansion of

$$\int_0^{\infty} \exp\left(-\lambda t - \frac{1}{t}\right)dt\,, \qquad \lambda \to +\infty$$

<u>Hint:</u> Use change of variable $s = \sqrt{\lambda}\, t$

4. Find the first two terms in the asymptotic expansion of

$$\int_0^1 \log(2+t)\cos\left(\lambda t\right)dt\,, \qquad \lambda \to +\infty$$

5. (Optional) First the first two terms in the Stirling's approximation for the Gamma function

$$\Gamma\left(\lambda+1\right) = \int_0^{\infty} e^{-t}\, t^{\lambda} dt = \int_0^{\infty} e^{\lambda \log(t) - t} dt$$

Change of variable: $t = \lambda s$

$$\Gamma\left(\lambda+1\right) = \lambda^{\lambda+1}\int_0^{\infty} e^{\lambda\left(\log(s)-s\right)}ds \equiv \lambda^{\lambda+1}\int_0^{\infty} e^{\lambda h(s)}ds\,, \qquad h\left(s\right) = \log\left(s\right) - s$$

$$\sim \lambda^{\lambda+1}\int_0^{\infty}\exp\left(\lambda\left[h\left(1\right) + \frac{h^{(2)}\left(1\right)}{2!}\left(s-1\right)^2 + \frac{h^{(3)}\left(1\right)}{3!}\left(s-1\right)^3 + \frac{h^{(4)}\left(1\right)}{4!}\left(s-1\right)^4 + \cdots\right]\right)ds$$

<u>Hint:</u> Use change of variable $u^3 = \lambda\left(s-1\right)^3$