AMS 212, Assignment #7

1. Use the WKB method to find a general solution for $x > 0$ and a general solution for $x < 0$ of

$$y'' + \lambda^2 x\left(1 + x^2\right)^2 y = 0, \qquad \lambda \to +\infty$$

Find the first 2 terms (up to $O(1)$ term).

2. Use the WKB method to find large eigenvalues of

$$\begin{cases} y'' + \lambda^2 x\left(1 + x^2\right)^2 y = 0 \\ y(-\infty) = 0, \quad y(1) = 0 \end{cases}$$

Hint: You don't need to write out the inner solution near $x = 0$. You only need to use the connection formula to connect the WKB approximation for $x > 0$ to the WKB approximation for $x < 0$.

3. Find the first two non-zero terms in the asymptotic expansion of

$$\int_0^\infty \exp(-\lambda t)\sin\left(\sqrt{t}\right)dt, \qquad \lambda \to +\infty$$

4. (Optional) Use the WKB method to find large eigenvalues of

$$\begin{cases} y'' + \lambda^2 x\left(1 + x^2\right)^2 y = 0 \\ y(0) = 0, \quad y(1) = 0 \end{cases}$$

5. (Optional) Write a code to compute the first 64 eigenvalues of

$$\begin{cases} y'' + \lambda^2 \left(1 - \sin(x)\right)^2 y = 0, \qquad \lambda \to +\infty \\ y(0) = 0, \quad y(1) = 0 \end{cases}$$

Calculate the difference between the numerical eigenvalue and the asymptotic eigenvalue

$$Err(n) = \lambda_n^{(\text{numerical})} - \lambda_n^{(\text{asymptotic})}$$

where $\lambda_n^{(\text{asymptotic})} = \dfrac{n\pi}{\cos(1)}$ (you can derive it!)

Plot $Err(n)$ vs n.