AMS 212, Assignment #6

1. Use Frobenius expansion to find ONE solution near $s = 0$ of

$$\frac{d^2y}{ds^2}+\frac{1}{s}\frac{dy}{ds}+y=0$$

We seek a solution of the form

$$y(s)=s^{\alpha}\left[a_0+a_1 s+a_2 s^2+a_3 s^3+\cdots\right]$$

Find coefficients a_0, a_1, a_2, and a_3 (some of them may be zero).

Note: There is only one solution of the given form above.

To find another solution, we need to use the expansion form

$$y_2(x)=b\log(x)y_1(x)+x^{\alpha_2}\sum_{m=0}^{\infty}A_m x^m$$

which is problem 5 below (optional).

2. Consider ODE

$$y''+\frac{2}{\sqrt{x}}y'-\frac{1}{2}\cdot\frac{1}{x^{\frac{3}{2}}}y=0\,,\qquad x\to+\infty$$

Suppose $y(x)$ has the form

$$y(x)=e^{f(x)}$$

Derive the equation for $f(x)$.

Use the method of dominant balance to find ONE solution. Find the first 2 terms.

Note: The method of dominant balance gives us only one solution.

To find two independent solutions, we first need to reduce the ODE from the general form to the special form

$$Y''+\tilde{q}(x)Y=0\,,\qquad x\to+\infty$$

For that purpose, we use $Y(x)=e^{2\sqrt{x}}y(x)$ and derive

$$\frac{d^2Y}{dx^2}-\frac{1}{x}Y=0\qquad \text{as } x\to+\infty$$

which looks familiar …

3. Use the WKB method to find a general solution of

$$y'' + \lambda^2 x^2 y = 0, \qquad \lambda \to +\infty$$

Find the first 2 terms (up to O(1) term).

4. Use the WKB method to find large eigenvalues of

$$\begin{cases} y'' + \lambda^2 x^2 y = 0 \\ y(1) = 0, \quad y(3) = 0 \end{cases}$$

Bonus Problem (optional):

5. In problem 1 above, use the expansion form

$$y_2(x) = b\log(x)\, y_1(x) + x^{\alpha_2} \sum_{m=0}^{\infty} A_m x^m$$

to find the second solution.