

AMS 212, Assignment #5

1. Use the method of multi-scales with $T_0 = t$ and $T_1 = \epsilon t$ to solve the IVP (initial value problem)

$$\begin{cases} y'' + \epsilon y^2 y' + y = 0 \\ y(0) = 1, \quad y'(0) = 0 \end{cases}, \quad \epsilon \rightarrow 0_+$$

Find the leading term in the expansion.

2. Using the method of strained variable to solve the IVP (initial value problem).

$$\begin{cases} y'' + \left(2 - \frac{1}{\sqrt{1 + \epsilon^2 y^2}} \right) y = 0 \\ y(0) = 1, \quad y'(0) = 0 \end{cases}, \quad \epsilon \rightarrow 0_+$$

Find the first two terms (up to ϵ^2 term) in the expansion.

Find the period of oscillation (find the first two terms, up to ϵ^2 term).

3. Consider the IVP (initial value problem)

$$\begin{cases} y'' + \frac{1}{\epsilon} \sin(\epsilon y) = 0 \\ y(0) = 1, \quad y'(0) = 0 \end{cases}, \quad \epsilon \rightarrow 0_+$$

Its regular expansion is

$$y(t) \sim \cos(t) + \epsilon^2 \left[\frac{t}{16} \sin t + \frac{1}{192} (\cos t - \cos 3t) \right]$$

Use the renormalization technique to find the strained variable expansion.

4. Using the method of strained variable to solve the IVP (initial value problem).

$$\begin{cases} y'' + y + (e^{\epsilon y} - 1) = 0 \\ y(0) = 1, \quad y'(0) = 0 \end{cases}, \quad \epsilon \rightarrow 0_+$$

Find the first two terms (up to ϵ term) in the expansion.

Bonus Problem (optional):

5. In Problem 4 above, find the first three terms (up to ε^2 term) in the expansion.