AMS 212, Assignment #4

1. Use a perturbation method to solve the BVP (boundary value problem)

$$\begin{cases} \varepsilon y'' - \left(x - \frac{1}{3} \right) y' - y = 0 \\ y(0) = 1, \quad y(1) = -1 \end{cases}, \quad \varepsilon \to 0_+$$

   Find the leading term in the composite expansion.

2. Use a perturbation method to solve the BVP (boundary value problem)

$$\begin{cases} \varepsilon y'' + \sin\left(x - \frac{1}{3} \right) y' = 0 \\ y(0) = 1, \quad y(1) = -1 \end{cases}, \quad \varepsilon \to 0_+$$

   Find the leading term in the composite expansion.

3. Consider the BVP

$$\begin{cases} \varepsilon y'' + x y' + y^2 = 0 \\ y(0) = \alpha, \quad y(1) = \beta \end{cases}, \quad \varepsilon \to 0_+$$

   Use the method of characteristics to determine the location of boundary layer and use the principle of least degeneracy to determine the width of boundary layer.

   (You don't need to calculate the asymptotic expansion).

4. (Optional) In Problem 1 above, the exact solution of the BVP is unknown.

   Write a numerical code to compute a very accurate numerical solution of the BVP.

   Let $y_n(x,\varepsilon)$ denote the numerical solution.

   Let $y_a(x,\varepsilon)$ denote the composite expansion (with the leading term).

   For $\varepsilon = 0.05$, plot $y_n(x,\varepsilon)$ and $y_a(x,\varepsilon)$ in one figure.

   For $\varepsilon = 0.01$, plot $y_n(x,\varepsilon)$ and $y_a(x,\varepsilon)$ in one figure.