AMS 212, Assignment #3

1. Use a perturbation method to solve the BVP (boundary value problem)

$$\begin{cases} \varepsilon y'' + y' + y = 0 \\ y(0) = e, \quad y(1) = 0 \end{cases}, \qquad \varepsilon \to 0_-$$

   Find <u>the first two terms</u> (i.e., O(1) term and O(ε) term) in the composite expansion.

2. Use a perturbation method to solve the BVP (boundary value problem)

$$\begin{cases} \varepsilon y'' + y' + e^{y} = 0 \\ y(0) = 1, \quad y(1) = -\ln 2 \end{cases}, \qquad \varepsilon \to 0+$$

   Find <u>the leading term</u> in the composite expansion.

   <u>Hint:</u> First find a general solution of $y' + e^{y} = 0$.

3. Recall the example we studied in lecture

$$\begin{cases} \varepsilon y'' + y' + y = 0 \\ y(0) = 0, \quad y(1) = 1 \end{cases}, \qquad \varepsilon \to 0+$$

   The leading term composite expansion is

$$y_a(x) = e^{1-x} - e^{1-\frac{x}{\varepsilon}}$$

   The exact solution is

$$y_e(x) = \frac{e^{\lambda_1 x} - e^{\lambda_2 x}}{e^{\lambda_1} - e^{\lambda_2}}$$

   where

$$\lambda_1 = \frac{-1+\sqrt{1-4\varepsilon}}{2\varepsilon}, \qquad \lambda_2 = \frac{-1-\sqrt{1-4\varepsilon}}{2\varepsilon}$$

   Consider the difference between the exact solution and the asymptotic solution

$$\left| y_e(x) - y_a(x) \right|$$

   For $\epsilon = 2^{-3}, 2^{-4}, 2^{-5}, \ldots, 2^{-25}$, calculate numerically

$$x(\varepsilon) = \arg\max_x \left| y_e(x) - y_a(x) \right|$$

$$err(\varepsilon) = \max_x \left| y_e(x) - y_a(x) \right|$$

Use loglog to plot $x(\varepsilon)$ as a function of ε.

Use loglog to plot $err(\varepsilon)$ as a function of ε.

Use semilogx to plot $(err(\varepsilon)/\varepsilon)$ as a function of ε. Is $err(\varepsilon)$ proportional to ε?

4. (Optional) In problem 2 above, find <u>the first two terms</u> (i.e., $O(1)$ term and $O(\varepsilon)$ term) in the composite expansion.

   <u>Note:</u> The $O(\varepsilon)$ term in the expansion will involve the integral $\int_0^{e^{-u}} \frac{\exp(v)-1}{v} dv$.