AMS 212, Assignment #2

1. Use a perturbation method to solve the IVP (initial value problem)

$$\begin{cases} y' - \varepsilon y + 1 = 0 \\ y(0) = \varepsilon \end{cases}$$

   Find the first two terms in the expansion.

2. Use a perturbation method to solve the IVP (initial value problem)

$$\begin{cases} y'' = 2 - y - \dfrac{1}{(1+\varepsilon y)^2} \\ y(0) = 0, \quad y'(0) = 0 \end{cases}$$

   Find the first two terms in the expansion of y.

   Find the first two terms in the expansion of T, the period of oscillation.

3. Use a perturbation method to solve the BVP (boundary value problem)

$$\begin{cases} y'' - 2y' + \varepsilon y = 0 \\ y(0) = 0, \quad y(1) = 1 \end{cases}$$

   Find the first two terms in the expansion.

4. (Optional) Solve numerically the IVP

$$\begin{cases} y'' = \dfrac{-1}{\varepsilon} \sin(\varepsilon y) \\ y(0) = 1, \quad y'(0) = 0 \end{cases}$$

   Compute $T(\varepsilon)$, the period of oscillation as a function of ε, for ε in [0.01:0.01:1].

   Plot $T(\varepsilon)$ as a function of ε and compare with the asymptotic expansion

$$T(\varepsilon) \sim 2\pi\left(1 + \frac{\varepsilon^2}{16}\right)$$

Plot $\frac{1}{\varepsilon^4}\left(\frac{T(\varepsilon)}{2\pi}-1-\frac{\varepsilon^2}{16}\right)$ as a function of ε to numerically predict the next coefficient.