AMS 147 Computational Methods and Applications

Assignment #1

1. Plot the two functions below

$$f_1(x) = \sqrt{\frac{1+2x}{1+x}}$$

$$f_2(x) = 1 + \frac{1}{2}x - \frac{5}{8}x^2 + \frac{13}{15}x^3$$

for x in [0, 0.6]. Plot the two curves in one figure. Use a legend to show which curve is which function. Use about 200 points to represent each curve. Use linear scales for both the X- and Y- axes.

In which region, $f_2(x)$ is a good approximation to $f_1(x)$?

2. Plot the function of two variables below as a surface

$$f(x,y) = x \cdot \sin\left(\frac{x}{2}\right) - y \cdot \sin\left(\frac{y}{2}\right)$$

for x in [-3, 3] and y in [-4, 4]. Use about 50 points in each dimension to represent the surface. Be careful with the component-wise operations!

3. Write a Matlab code to read in data from file "**data2.txt**" in

http://www.cse.ucsc.edu/~hongwang/Codes/Read_data

Plot the data and plot the fitting function

$$f(x) = \cos(2\exp(cx))$$

in the same figure and in the same horizontal range.

Use a legend to show which is the fitting curve and which is the data.

Use linear scales for both the X- and Y- axes.

Try to manually adjust the value of c to fit the data. Hint: start near c = 1.

4. Consider the non-linear equation

$$f(x) = \exp(x) - 1.5 - \cos(x+c)$$

Here c is a parameter in the equation. The root of the equation varies with c and thus the root is a function of c. Use Newton's method to solve the equation. Plot the root as a function of c for c in [–1.5, 1.5]. Use about 200 points to represent the curve.

5. File "data2.txt" in http://www.cse.ucsc.edu/~hongwang/Codes/Read_data contains a set of data points $\left(x(j), y(j)\right), \quad j = 1, 2, \ldots, N$.

   The distance between the data and the fitting function $f(x) = \cos\left(2\exp(cx)\right)$ is defined as

   $$dd(c) = \sqrt{\sum_{j=1}^{N} \left(f\left(x(j)\right) - y(j)\right)^2}$$

   Calculate *dd*(*c*) for c in [0.5, 1.5]. Plot *dd*(*c*) as a function of c. Use about 200 points to represent the curve.

6. In problem 5 above, use the golden search method to find the optimal value of c at which the distance function dd(c) attains the minimum. Plot both the data and the optimal fitting function in one figure. Report the optimal value of c with at least 6 decimal digits.