

# Optimal Unicast Capacity of Random Geometric Graphs: Impact of Multipacket Transmission and Reception

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**Abstract**—We establish a tight max-flow min-cut theorem for multi-commodity routing in *random geometric graphs*. We show that, as the number of nodes in the network  $n$  tends to infinity, the maximum concurrent flow (MCF) and the minimum cut-capacity scale as  $\Theta(n^2r^3(n)/k)$  for a random choice of  $k = \Omega(n)$  source-destination pairs, where  $n$  and  $r(n)$  are the number of nodes and the communication range in the network respectively. The MCF equals the interference-free capacity of an ad-hoc network. We exploit this fact to develop novel graph theoretic techniques that can be used to deduce tight order bounds on the capacity of ad-hoc networks. We generalize all existing capacity results reported to date by showing that the per-commodity capacity of the network scales as  $\Theta(1/r(n)/k)$  for the single-packet reception model suggested by Gupta and Kumar, and as  $\Theta(nr(n)/k)$  for the multiple-packet reception model suggested by others. More importantly, we show that, if the nodes in the network are capable of (perfect) multiple-packet transmission (MPT) and reception (MPR), then it is feasible to achieve the optimal scaling of  $\Theta(n^2r^3(n)/k)$ , despite the presence of interference. In comparison to the Gupta-Kumar model, the realization of MPT and MPR may require the deployment of a large number of antennas at each node or bandwidth expansion. Nevertheless, in stark contrast to the existing literature, our analysis presents the possibility of actually increasing the capacity of ad-hoc networks with  $n$  even while the communication range tends to zero!

**Index Terms**—Capacity, Unicast, Random Geometric Graph, Multipacket Transmission, Multipacket Reception.

## I. INTRODUCTION

**G**UPTA and Kumar's [1] seminal work studied the capacity of wireless ad-hoc networks for nodes with single antenna and each node transmits or receives one packet at a time. They cautioned us that the capacity of ad-hoc networks

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does not scale with an increase in network size. However, Gupta and Kumar's analysis [1] applies to the traditional view of ad hoc networking in which protocols are based on a one-to-one communication paradigm aimed at avoiding multiple access interference (MAI). A number of recent advances in cooperative communication and generalizations of routing are challenging the long-held view that avoiding interference is the way to maximize throughput in ad hoc networks. For example, network coding (NC) [2] generalizes routing by permitting processing of packets at intermediate nodes. Many-to-one and many-to-many communication is also feasible under a variety of other cooperative techniques [3]–[5].

Co-operative protocols that provide performance benefits in specific network configurations need not scale well with the network size. In particular, Liu et al. [6] proved another disheartening result: NC cannot increase the throughput order of wireless ad-hoc networks for multi-pair unicast applications under half-duplex communication. However, in a recent challenge, Garcia-Luna-Aceves et al. [4] call for the realization of ad hoc networks that scale by embracing MAI through the use of multipacket reception (MPR) at the receivers. Multipacket reception collectively represents any and all techniques that allow concurrent reception of independent information from multiple transmitters. Under the assumptions of a perfect MPR, it is shown in [4] that the order capacity of a network with  $n$  unicast sessions grows as  $\Theta(r(n))$ , where  $r(n)$  is the communication range. This represents a gain of  $\Theta(nr^2(n))$  over the throughput order of  $\Theta(1/nr(n))$  reported by Gupta and Kumar. The assumptions of MPR in [4] are idealistic and the realization of MPR may require deployment of a large number of antennas for each node or bandwidth expansion. However, [4] demonstrates that there is a significant value in investigating cooperative techniques that lead to increased concurrency at the physical layer.

Interestingly, the prior work on the capacity of wireless networks, which we summarize in Section II, has focused on what is attainable with specific approaches to handle MAI. No prior work has focused on first establishing what is the optimal capacity of a wireless network in the absence of MAI, and then determining whether that capacity is attainable when MAI is present. This is a prime focus of this paper.

Section III presents the first contribution of this paper. We model a *random network* with  $n$  nodes, a homogeneous communication range of  $r(n)$ , and unicast traffic for  $k$  source-

destination (S-D) pairs. In the absence of interference, such a network corresponds to a *random geometric graph* with an edge between any two nodes separated by a distance less than  $r(n)$ . We define a *combinatorial interference model*, and use it to express all the protocol models used in the past, as well as the new multi-packet transmission and reception (MPTR) protocol model introduced in this paper. Under the MPTR protocol model, nodes have the ability to decode correctly multiple packets transmitted concurrently from different nodes, and transmit concurrently multiple packets to different nodes.

Section IV presents the second contribution of this paper, which is the characterization of the optimal interference-free capacity of a wireless network. The task of concurrently maximizing the data rate for  $k$  S-D pairs is an instance of the multi-commodity flow problem. Hence, the *maximum concurrent multi-commodity flow-rate* (MCF) in a random geometric graph equals the interference-free capacity (i.e., the optimal capacity) of the network. To derive upper bounds on the optimal network capacity, we use the fact that the MCF is less than the minimum sparsity vertex cut for any arbitrary graph. The max-flow min-cut theorem by Ford and Fulkerson [7] establishes that this bound is tight for a single commodity. However, in general, the min-cut does not provide a tight bound on the max-flow [8]. The bound is known to be tight only for special cases, such as planar graphs [9], and in general exhibits a gap of at least  $\Theta(\log k)$  [10]. We establish a tight max-flow min-cut theorem for random geometric graphs for the first time, and show that  $\Theta(n^2r^3(n)/k)$  is a tight bound on the optimal capacity of a wireless network.

Section V presents our third contribution, which consists of generalizing prior results by Gupta and Kumar and by Garcia-Luna-Aceves et al., and proving that the optimal capacity of wireless networks is attainable in the presence of MAI. We utilize the max-flow min-cut theorem of Section IV to deduce tight order bounds for the capacity of random networks under various interference models. We show that the per-commodity capacity, under the protocol model suggested by Gupta and Kumar [1], exhibits a tight order bound of  $\Theta(1/r(n)k)$ . This result generalizes Gupta and Kumar's result to any  $k = \Omega(n)$  S-D pairs. Similarly, we generalize Garcia-Luna-Aceves et al.'s analysis for the MPR protocol model. We show that, under the MPR model, the per-commodity capacity of the network scales as  $\Theta(nr(n)/k)$ , which means that it is bounded away from the optimal capacity by a factor of  $\Theta(nr^2(n))$ <sup>1</sup>. We show that MPTR achieves the optimal capacity of  $\Theta(n^2r^3(n)/k)$ . Hence, MPTR provides a gain of  $\Theta(nr^2(n))$  over MPR and any previously reported feasible capacity.

If we choose a transmission range of  $r(n) = \Theta\left(\sqrt{\frac{\log(n)}{n}}\right)$  then, compared to the Gupta-Kumar model, MPR provides an order gain of  $\Theta(\log(n))$ , while MPTR provides a gain of  $\Theta(\log(n)^2)$ . This result may seem counter-intuitive because the maximum number of packets that a node can receive in a single slot remains bounded by  $\Theta(\log(n))$  for both MPR as

<sup>1</sup>Note that from a purely analytical and network perspective, an MPT scheme can be considered to be dual of an MPR scheme. Therefore, the scaling laws of MPT is identical to that of MPR scheme.

well as MPTR. As detailed in the main body of the paper, the difference in performance occurs primarily on account of the increase in concurrency in scheduling the relay of data. The routing in the network is realized by dividing the network area into sub-regions called square-lets and then moving data at a square-let level abstraction. It is observed that there can be only single relay transmission between adjacent square-lets under the Gupta-Kumar model. However, with MPR this value increases to  $\Theta(\log(n))$  and with MPTR to  $\Theta(\log(n)^2)$ . This increase in concurrent transmissions between square-lets is the main cause for the observed order gains.

Our results for MPTR exhibit the possibility that MPTR can achieve the dual objective of increasing capacity and decreasing the transmission range as  $n$  increases. This is in stark contrast to the commonly held view that the capacity of multi-hop wireless networks cannot increase as the number of nodes increases. Indeed, our results demonstrate that the capacity of ad-hoc networks can actually *increase* with  $n$  while the communication range tends to zero! However, we highlight that this positive result requires an idealistic model for MPT and MPR along with an expansion in bandwidth<sup>2</sup> or a large number of antennas<sup>3</sup> on each node. Clearly, the gains attainable in practice due to MPT and MPR are likely to be more modest than what we report because of practical and physical limitations [11]. This paper deals with asymptotic behavior of the networks and it is outside the scope of this paper to analyze more realistic models of MPT and MPR. However, we believe that our analytical techniques are general enough to also prove useful for such future investigations.

Section VI summarizes our key conclusions and addresses the impact of our results on the design of protocols for future wireless ad hoc networks.

## II. RELATED WORK

There have been many contributions on the capacity study of wireless ad hoc networks that span unicast, multicast and broadcast traffic. Due to space limitations, however, we only mention a few of them that focus on unicasting.

A number of papers have extended the results by Gupta and Kumar [1], which showed a gap between the upper and lower bounds on capacity under the physical model. Franceschetti et al. [12] closed this gap using percolation theory.

Several techniques aimed at improving the capacity of wireless ad hoc networks have been analyzed. Grossglauser and Tse [13] demonstrated that a non-vanishing capacity can be attained at the price of long delivery latencies by taking advantage of long-term storage in mobile nodes. It has also been shown that, if bandwidth is allowed to increase proportionally to the number of nodes in the network [14], higher transport capacities can be attained for static wireless networks. Some works demonstrated that changing physical layer assumptions such as using multiple channels [15] or MIMO cooperation [16] can change the capacity of wireless networks.

<sup>2</sup>We refer the reader to [3] for a discussion on bandwidth expansion utilizing MPR.

<sup>3</sup>The number of antennas required to implement MPR or MPT is proportional to  $\Theta(nr^2(n))$ .

Ozgur et al. [16] proposed a hierarchical cooperation technique based on virtual MIMO to achieve linear capacity. They showed that the optimal per-session capacity of an ad-hoc network is bounded as  $O(n \log n)$ , and a constant per-session capacity of  $\Theta(1)$  is achievable. Our work is significantly different from this work, in terms of the model and assumptions used to derive the results. Ozgur et. al. consider the information-theoretic model, and assume that the network employs heterogenous hop-sizes, at times requiring a direct communication between widely separated nodes. In contrast, our work is based on the protocol model and assumes a homogenous transmission range. A comparison of our result with hierarchical MIMO cooperation shows that we can achieve the same capacity with  $r(n) = \Theta(n^{-\frac{1}{3}})$ . If the communication range  $r(n)$  increases beyond  $\Theta(n^{-\frac{1}{3}})$ , then MPTR provides higher capacities.

Cooperation can be extended to the simultaneous transmission and reception at the various nodes in the network, which can result in significant capacity improvement [3]. As we have stated, Garcia-Luna-Aceves et al. [4] showed that using MPR at the receivers can increase the order capacity of wireless networks subject to unicast traffic. We believe that the impact of MPT and MPR on the throughput order has not been studied sufficiently. However, we must highlight that the concepts of MPT and MPR have been around for a long time, and there are various ways of realizing them. In the original introduction of MPR [17], the concept was realized by utilizing spread spectrum techniques, which requires bandwidth expansion and a detailed discussion can be found in [3]. However, more recent advances in the physical layer allows the realization of MPR/MPT by utilizing multiple antennas and the use of beamforming techniques. The work by Peraki and Servetto [18] has identified the upper bound on the performance of beamforming. The scope of this paper is not about different ways of implementing MPR/MPT. In this work we have assumed an implementation of MPR, which does not require any loss in rate. For additional background, we refer the reader to an excellent survey paper [19] and number of contributions on this topic [17]–[24].

A generalization of the max-flow min-cut theorem to multiple commodities is not feasible in arbitrary graphs. However, the seminal paper by Leighton and Rao [8] showed that the gap between the max-flow and min-cut is at most  $\Theta(\log n)$ . In a recent work, Madan et. al. [25] have utilized Leighton and Rao's work to derive bounds for the capacity of ad-hoc networks. They focus on the special case where  $k = \Theta(n^2)$  and the *single packet reception* (SPR) model.

Our work is inspired by the analysis of Leighton and Rao. Given that we are able to deduce a tight max-flow min-cut theorem, our bounds are tighter than those reported by Madan et al. [25]. Furthermore, our work is applicable to a wider variety of protocol models and traffic patterns.

### III. PRELIMINARIES

#### A. Network Model

The area of a continuous region  $R$  is denoted by  $|R|$ . The cardinality of a set  $S$  is denoted by  $|S|$ , and  $\|x - y\|$  represents

the distance between nodes  $x$  and  $y$ . Whenever convenient, we utilize the indicator function  $1_{\{P\}}$ , which is equal to one if  $P$  is true and zero if  $P$  is false.  $Pr(E)$  represents the probability of event  $E$ . We say that an event  $E$  occurs with high probability (w.h.p.) as  $n \rightarrow \infty$  if  $Pr(E) > (1 - (1/n))$ . We employ the standard order notations  $O$ ,  $\Omega$ , and  $\Theta$ .

We assume a random wireless network with  $n$  nodes distributed uniformly in a unit-square. In our model, as  $n$  goes to infinity, the density of the network also goes to infinity. Therefore, our analysis is applicable to dense networks. Furthermore, we assume a fixed transmission range  $r(n)$  for all the nodes in the network. Thus, the network topology can be characterized using a random geometric graph, which we denote by  $G_r$  and define next.

*Definition 3.1: Random Geometric Graph  $G_r$ :* We associate a *directed* graph  $G_r(V_r, E_r)$  with a wireless network formed by distributing  $n$  nodes uniformly in a unit square. We represent the node-set by  $V = \{1, \dots, n\}$ . Let the locations of these nodes be given by  $\{X_1, \dots, X_n\}$ , the edge-set is then  $E = \{(i, j) \mid \|X_i - X_j\| \leq r(n)\}$ .

Note that, as per the above definition, we permit two edges for a pair of connected vertices with possibly different capacity in each direction. Accordingly, the results in this paper can be extended to undirected graphs, but we find it mathematically convenient to use directed graphs. A directed edge allows us to explicitly identify the direction of transmission. Hence, an edge  $e$  can be denoted as  $(e^+, e^-)$ , where  $e^+$  represents the transmitter node and  $e^-$  represents the receiver node.

We assume that the network operates using a slotted channel and, in the absence of interference, the data rate in each time slot for every transmitter-receiver pair is a constant of value  $W$  bits/slot. Given that  $W$  does not change the order capacity, we normalize its value to 1. Therefore, we say that the interference-free capacity of each edge in  $G_r$  is equal to 1. Meanwhile, in presence of interference the edge capacity drops to zero.

The connectivity criteria for  $G_r$  is  $r(n) \geq r_c(n) = \Theta(\sqrt{\log n / n})$  [26]. More specifically, we assume  $r_c(n) = \sqrt{(3 \log n) / n}$ .

In a dense network, interference is the primary constraint on the capacity of the network. Similar to the work by Madan et al. [25], we describe the interference of a network by the following generic model.

*Definition 3.2: Combinatorial Interference Model:* The interference model for the graph<sup>4</sup>  $G(V, E)$  is determined by a function  $I : E \rightarrow P(E)$ , where  $P(E)$  is the power set of  $E$ , i.e., the set of all possible subsets of  $E$ . For every  $e \in E$ ,  $I(e)$  represents an *interference set* such that, a transmission on edge  $e$  is successful iff there are no concurrent transmissions on any  $\hat{e} \in I(e)$ . An interference model can be restricted to a subgraph  $H(V_H, E_H)$  by defining a function  $I_H : E_H \rightarrow P(E_H)$  such that  $I_H(e) = I(e) \cap E_H$ .

The various protocol models that have been proposed in the past can now be expressed as special cases on  $G_r$ .

Gupta and Kumar [1] studied a *single packet reception* (SPR) *protocol model* under which a transmission from node

<sup>4</sup>Note that  $G_r$  denotes random geometric graph while  $G$  represents general graph.

$i$  to receiver  $j$  is successful iff  $\|X_i - X_j\| \leq r(n)$  and if  $\|X_j - X_k\| \geq (1 + \eta)r(n)$  for any other transmitter  $k$ . Here  $\eta$  is a guard-zone that is assumed to be constant for the entire network. Moreover, all the nodes operate in half-duplex mode. The following definition expresses this model in terms of the notation we have introduced.

**Definition 3.3: Single-Packet Reception (SPR) Model:** Let  $e = (e^+, e^-) \in E$ , then the interference set for edge  $e$  is defined as

$$I_{\text{SPR}}(e) = J(e) - e, \quad (1)$$

where,  $J(e) = \{\hat{e} \in E_r \mid \|X_{\hat{e}^+} - X_{e^-}\| \leq (1 + \eta)r(n)\}$ .

Garcia-Luna-Aceves et al. [4] generalized the above model to account for MPR capability at the receivers. According to their *MPR protocol model* a node  $i$  can simultaneously receive all the packets transmitted by nodes within a distance  $r(n)$  iff there are no transmitters at a distance greater than  $r(n)$  but less than  $(1 + \eta)r(n)$ . Furthermore, if a node  $j$  transmits a packet to node  $i$ , then it cannot simultaneously transmit a packet to any other node in the network.

**Definition 3.4: Multi-Packet Reception (MPR) Model:**

$$I_{\text{MPR}}(e) = I_{\text{SPR}}(e) - (A(e) - B(e)) \quad \forall e \in E \quad (2)$$

$$\text{where, } A(e) = \{\hat{e} \in E_r \mid \|X_{\hat{e}^+} - X_{e^-}\| \leq r(n)\},$$

$$B(e) = \{\hat{e} \in E_r \mid \hat{e}^+ = e^+\} \quad (3)$$

We also consider the case in which nodes have MPR and MPT capabilities, i.e., can decode multiple concurrent transmissions and can transmit concurrently multiple packets to different nodes. We capture the MPR and MPT capabilities with a simple yet representative, *multi-packet transmission and reception* (MPTR) protocol model, which is defined below.

**Definition 3.5: Multi-Packet Transmission and Reception (MPTR) Model:**

$$I_{\text{MPTR}}(e) = I_{\text{MPR}}(e) - B(e) \quad \forall e \in E \quad (4)$$

It is important to highlight some of the features of the above model. The MPTR protocol model still restricts the nodes to operate in a half-duplex mode. Moreover, this model is identical to the MPR protocol model in terms of the nodes that are permitted to transmit within the vicinity of a receiver  $i$ , i.e., both models prohibit transmission from a node  $j$  such that  $r(n) < \|X_i - X_j\| \leq (1 + \eta)r(n)$ . Thus, the difference between the MPR and MPTR protocol models stems purely from the fact that, under the MPTR model, a node  $j$  transmitting a packet to node  $i$  can simultaneously transmit packets to other nodes in the network.

Figure 1 provides an illustrative example of a graph and various interference sets defined on it.

We assume that the traffic in the network is generated by unicast communication between  $k$  source-destination (S-D) pairs. The source and the destination are chosen at random from the  $n$  nodes in an independent and uniform fashion. We associate a rate vector  $\lambda = [\lambda_1, \dots, \lambda_k]$  with these  $k$  pairs. We assume the data rate for each S-D pair to be non-zero and to be within a constant factor  $0 \leq \epsilon \leq 1$  of each other, i.e.,  $\min_i(\lambda_i) \geq \max_i(\epsilon\lambda_i)$ . Hence, without loss of generality (w.l.g.), the rate vector can be written as  $\lambda = [fD_1, \dots, fD_k]$ , where  $f \in \mathbb{R}_+$  and  $D_i \in [\epsilon, 1]$  for  $1 \leq i \leq k$ . We

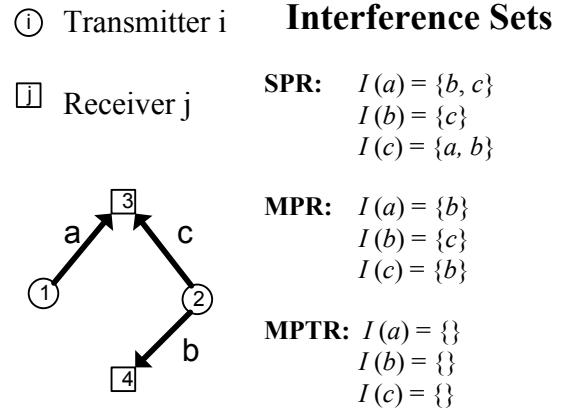


Fig. 1. A network of four nodes is used to illustrate the Interference Sets under different model assumptions. A directed edge is drawn between a transmitter and a receiver if and only if a transmission from the node can be successfully decoded at the receiver. Two edges interfere if they cannot carry independent information in the same slot

refer to the parameter  $f$  as the *concurrent flow rate* and to  $D = [D_1, \dots, D_k]$  as the *demand vector*.

**Definition 3.6: Feasible Flow Rate:** Given  $k$  S-D pairs  $\{(s(1), d(1)), \dots, (s(k), d(k))\}$ , a rate vector  $\lambda = [fD_1, \dots, fD_k]$  is feasible if there exists a spatial and temporal scheme for scheduling transmissions such that, by operating the network in a multi-hop fashion and buffering at intermediate nodes when awaiting transmission, every source  $s(i)$  can send  $\lambda_i$  bits/sec on average to its chosen destination  $d(i)$ . A flow rate  $f$  is feasible for a demand vector  $D = [D_1, \dots, D_k]$  iff  $\lambda = [fD_1, \dots, fD_k]$  is a feasible rate vector.

**Definition 3.7: Capacity of Random Networks:** The capacity per commodity of a network is  $\Theta(f(n))$  if under a random placement of  $n$  nodes, a random choice of  $k$  S-D pairs and for an arbitrary demand vector we have:

$$\lim_{n \rightarrow \infty} \Pr(c f(n) \text{ is feasible flow rate}) = 1 \quad (5)$$

$$\liminf_{n \rightarrow \infty} \Pr(c' f(n) \text{ is infeasible flow rate}) < 1 \quad (6)$$

for some  $c > 0$  and  $c < c' < +\infty$ .

In the following sections we repeatedly utilize the well known Chernoff bounds [27].

**Lemma 3.8: Chernoff Bounds:** Consider  $N$  i.i.d random variables  $Y_i \in \{0, 1\}$  with  $p = \Pr(Y_i = 1)$ . Let  $Y = \sum_{i=1}^N Y_i$ . Then for every  $c > 0$  there exist  $0 < \delta_1 < 1$  and  $\delta_2 > 0$  such that

$$\Pr(Y \leq (1 - \delta_1)Np) < 2e^{-\frac{\delta_1^2}{3}Np} \quad (7)$$

$$\Pr(Y \geq (1 + \delta_2)np) < 2e^{-\frac{\delta_2^2}{3}Np} \quad (8)$$

## B. Graph Theory Results

The task of identifying a feasible flow rate can be posed as a multi-commodity flow problem, specifically the  $k$ -commodity flow problem.

**Definition 3.9:  $k$ -Commodity Flow Problem:**

Consider a directed graph  $G(V, E)$  with a capacity function  $c : E \rightarrow [0, 1]$ . Let  $\{(s(1), d(1)), \dots, (s(k), d(k))\}$  be  $k$  S-D

pairs, with a demand vector  $D \in [\epsilon, 1]^k$ . Let  $f \in \mathbb{R}_+$  be a concurrent flow rate. Find flow functions  $f_i : E \rightarrow \mathbb{R}_+$  for  $1 \leq i \leq k$ , which satisfy the following flow constraints:

$$\text{Capacity Constraint : } \forall e \in E \sum_{1 \leq i \leq k} f_i(e) \leq c(e) \quad (9)$$

$$\text{Flow Conservation : } \forall v \neq s(i), d(i) \sum_{e: e^+ = v} f_i(e) = \sum_{e: e^- = v} f_i(e) \quad (10)$$

$$\text{Demand Satisfaction : } 1 \leq i \leq k$$

$$\sum_{e: e^+ = s(i)} f_i(e) = \sum_{e: e^- = d(i)} f_i(e) = f D_i \quad (11)$$

Flow functions that satisfy the above constraints are called feasible. Other inputs to the problem being fixed, a flow rate  $f$  is said to be feasible iff the above problem has a solution. Furthermore, let  $f^*$  be the MCF, such that the above problem has a feasible solution. A wireless network can be represented by an equivalent graph with capacity functions determined by the interference. Thus, the MCF in an equivalent graph can be perceived as the maximum flow that can be routed in a network. Additionally, if w.h.p.  $f^*$  is the MCF for any graph formed by a random distribution of nodes, sources and destinations, then the capacity of the wireless network is also  $f^*$ . Consider the following additional definitions.

**Definition 3.10: Vertex Cut:** Given a node set  $V$ , a cut is the separation of the vertex set  $V$  into two disjoint and exhaustive sets  $(S, S^C)$ . We shall often reference a cut just by the set  $S$ .

**Definition 3.11: Sparsity Cut:** Given a graph  $G(V, E)$ , a capacity function  $c : E \rightarrow [0, 1]$  and a cut  $(S, S^C)$ . The sparsity of a cut is defined as

$$\Upsilon_{G,S} = \frac{\sum_{e \in E} 1_{[e^+ \in S, e^- \in S^C]} c(e)}{\sum_{i: s(i) \in S, d(i) \in S^C} D_i} \quad (12)$$

**Definition 3.12: Minimum Sparsity Cut:** Given a graph  $G(V, E)$ , a capacity function  $c : E \rightarrow [0, 1]$  and a cut  $(S, S^C)$ . The minimum sparsity cut is defined as

$$\Upsilon_G = \min_{S \subseteq V} \frac{\sum_{e \in E} 1_{[e^+ \in S, e^- \in S^C]} c(e)}{\sum_{i: s(i) \in S, d(i) \in S^C} D_i} \quad (13)$$

It is well-known that the minimum cut-capacity provides an upper bound on the maximum flow rate.

**Lemma 3.13:** For any  $k$ -commodity flow  $f^* \leq \Upsilon_G$

#### IV. OPTIMAL CAPACITY

In this section we show that MCF provides a tight approximation of the minimum sparsity cut (MSC) for random geometric graphs with a random traffic pattern. In general the MCF is not equal to the MSC, we include a well known counter example in Fig. 2. Moreover, it can be shown that there exist graphs where the gap between the MCF and MSC increases with the network size [8]. Hence, the approximate max-flow min-cut theorem established in this section plays a crucial role in providing a tight characterization of the interference-free as well as interference-constrained capacity of wireless ad-hoc networks.

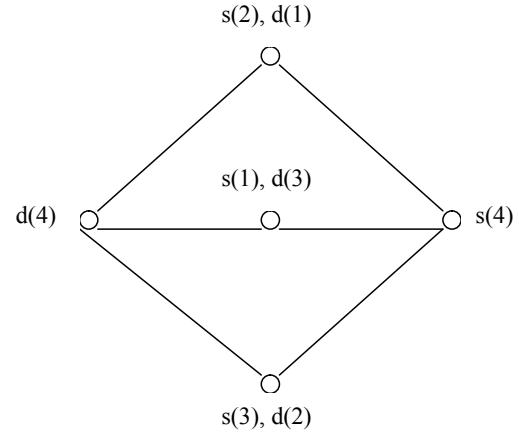


Fig. 2. The Okamuro-Seymour example [28] for which the max-flow is  $3/4$  and the min sparsity cut is  $1$ .

This relationship implies a tight characterization of the interference-free capacity of wireless ad-hoc networks with a homogenous transmission range. Our approach can be summarized as follows: For a particular demand vector, we provide an upper bound by showing that there exists a multi-commodity cut in  $G_r$  of order  $O(g(n))$  and a lower bound by constructing a flow of order  $\Omega(g(n))$  in a sub-graph  $H_r \subseteq G_r$ . These results along with the following lemmas prove that the capacity of  $G_r$  has a tight bound  $\Theta(g(n))$ .

Lemma 4.1 provides some properties for general graphs that include random geometric graphs.

**Lemma 4.1:** A graph  $G(V, E)$  and a sub-graph  $H(V_H, E_H)$  such that  $V = V_H$  satisfy the following two properties: (a) If  $f$  is a feasible flow-rate in  $H$  then  $f$  is feasible in  $G$ ; and (b) the capacity of a cut  $(S, S^C)$  in  $G$  is always greater than or equal to the capacity of the cut in  $H$ .

**Proof:** To prove Part (a) of the lemma, let  $f_{H,i}$  for  $1 \leq i \leq k$  be the flow functions associated with the feasible flow of rate  $f$  in  $H$ . Note that these flow functions satisfy the constraints in Definition 3.9. We construct a flow in  $G$  of rate  $f$  with the following flow functions: For  $1 \leq i \leq k$  let  $f_{G,i} : E \rightarrow \mathbb{R}_+$  such that

$$\begin{aligned} f_{G,i}(e) &= f_{H,i}(e) \text{ if } e \in E_H, \\ &= 0 \text{ if } e \in E - E_H \end{aligned} \quad (14)$$

Now, if we show that the functions  $f_{H,i}$  satisfy the flow constraints, then the flow rate  $f$  is feasible in  $G$ . Definition 3.9 states that  $c(e) \geq 0$  for all the edges. As a result,  $\forall e \in E - E_H$  the capacity constraints are satisfied trivially, given that  $f_{G,i}(e) = 0$  for such edges. Furthermore,  $\forall e \in E_H$  we have

$$\sum_{1 \leq i \leq k} f_{H,i}(e) = \sum_{1 \leq i \leq k} f_{G,i}(e) \leq c(e) \quad (15)$$

Therefore,  $f_{G,i}$  satisfy the capacity constraint. In addition, note that the following equations hold  $\forall v \in V$ :

$$\begin{aligned} \sum_{e \in E: e^- = v} f_{G,i}(e) &= \sum_{e \in E_H: e^- = v} f_{G,i}(e) + \sum_{e \in E - E_H: e^- = v} f_{G,i}(e) \\ &= \sum_{e \in E_H: e^- = v} f_{H,i}(e) + 0 \end{aligned} \quad (16)$$

and

$$\begin{aligned} \sum_{e \in E: e^+ = v} f_{G,i}(e) &= \sum_{e \in E_H: e^+ = v} f_{G,i}(e) + \sum_{e \in E - E_H: e^+ = v} f_{G,i}(e) \\ &= \sum_{e \in E_H: e^+ = v} f_{H,i}(e) + 0 \end{aligned} \quad (17)$$

Eqs. (16) and (17) imply that the net in-flow and the net out-flow, under  $f_{G,i}$  and  $f_{H,i}$ , is identical for all nodes and commodities. Therefore,  $f_{G,i}$  satisfies the flow conservation and demand constraints.

To show Part (b), observe that

$$\begin{aligned} \Upsilon_{G,S} &= \frac{\sum_{e \in E_H} 1_{[e^+ \in S, e^- \in S^C]} c(e)}{\sum_{i: s(i) \in S, d(i) \in S^C} D_i} \\ &\quad + \frac{\sum_{e \in E - E_H} 1_{[e^+ \in S, e^- \in S^C]} c(e)}{\sum_{i: s(i) \in S, d(i) \in S^C} D_i} \\ &= \Upsilon_{H,S} + \text{a non-zero term} \geq \Upsilon_{H,S}. \end{aligned} \quad (18)$$

We have  $\Upsilon_{G,S} \geq \Upsilon_{H,S}$  because  $c(e) \geq 0$  for all edges. ■

#### A. Upper Bound

We utilize the following properties of  $G_r$ .

**Lemma 4.2:** If  $r(n) \geq r_c(n)$  then w.h.p. graph  $G_r$  is such that: (a) The minimum vertex degree  $\nabla = \Omega(nr^2(n))$ , and (b) the maximum vertex degree  $\Delta = O(nr^2(n))$ .

*Proof:* We first show that  $\nabla = \Omega(nr^2(n))$ . A node  $v$  in  $G_r$  is connected to all the nodes in a disk of radius  $r(n)$  centered at  $v$ . The area of this disk is  $\pi r^2(n)$ . Given a uniformly random distribution of nodes, the probability of another node  $u$  lying within this disk is  $\pi r^2(n)$ . Consider a random variable  $Y_{v,u} \in \{0, 1\}$  which is equal to one iff node  $u$  is connected to node  $v$ . The degree of node  $v$  can be written as  $\deg(v) = \sum_{u \in V - \{v\}} Y_{v,u}$ . Therefore, the Chernoff Bound (7) implies that there exists a  $0 \leq \delta \leq 1$  such that

$$\Pr(\deg(v) \leq (1 - \delta)n\pi r^2(n)) < 2e^{-\frac{\delta^2}{3}n\pi r^2(n)}. \quad (19)$$

From the union bound, we obtain

$$\Pr(\nabla \leq (1 - \delta)n\pi r^2(n)) < n\Pr(\deg(v) \leq (1 - \delta)n\pi r^2(n)). \quad (20)$$

Given that  $r(n) \geq r_c(n)$  we have  $r(n) \geq \sqrt{\frac{3\log(n)}{n}}$ . Therefore, Eqs. (19) and (20) imply that

$$\begin{aligned} \Pr(\nabla \leq (1 - \delta)\pi nr^2(n)) &< 2ne^{-\frac{n\delta^2}{3} \times \frac{3\pi\log(n)}{n}} \\ &= 1/n^{\delta^2\pi - 1 - \frac{\log(2)}{\log(n)}}. \end{aligned} \quad (21)$$

Hence, Lemma 3.8 tell us that for a large enough  $n$ , for any  $\sqrt{\frac{2}{\pi}} \leq \delta \leq 1$ , we have

$$\Pr(\nabla \geq (1 - \delta)\pi nr^2(n)) > 1 - (1/n). \quad (22)$$

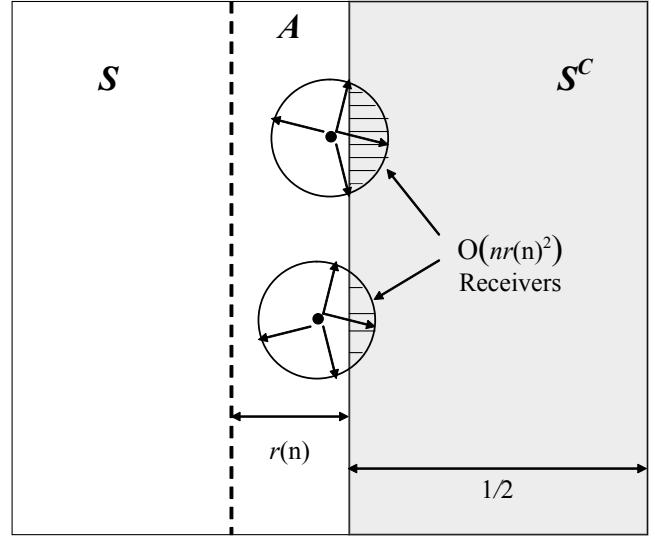


Fig. 3. The network area is divided into two regions: shaded and unshaded. This division defines the vertex cut  $(S, S^C)$ . Any node that successfully transmits across the cut has to fall within a distance  $r(n)$  of the cut boundary, and, hence, it should belong to region  $A$ . Here  $r(n)$  is the transmission range. Furthermore, a transmitter from  $S$  cannot have more than  $\Theta(nr(n)^2)$  receivers in  $S^C$

We use similar arguments to show that  $\Delta = O(nr^2(n))$  w.h.p.. This fact follows from Eq. (8), which implies that  $\forall \delta \geq 0$

$$\Pr(\deg(v) \geq (1 + \delta)n\pi r^2(n)) < 2e^{-\frac{\delta^2}{3}\pi nr^2(n)}. \quad (23)$$

The union bound and the fact that  $r(n) \geq \sqrt{\frac{3\log(n)}{n}}$  implies that

$$\Pr(\Delta \geq (1 + \delta)\pi nr^2(n)) < n2e^{-\frac{\delta^2}{3}\pi nr^2(n)}. \quad (24)$$

Therefore, for a large enough  $n$  and  $\forall \delta \geq \sqrt{\frac{2}{\pi}}$  we have

$$\Pr(\Delta \leq (1 + \delta)\pi nr^2(n)) > 1 - (1/n). \quad (25)$$

Consider the cut  $S$  which, as described by Fig. 3, divides the network area into two equal parts of area  $\frac{1}{2}$  each.

**Lemma 4.3:** If the network consists of  $k = \Omega(n)$  S-D pairs then, w.h.p., a region  $R$  of constant area  $|R| = \frac{1}{2}$  contains  $\Theta(k)$  sources with destinations outside region  $R$ .

*Proof:* Let  $Y_i \in \{0, 1\}$  be a random variable that is equal to one iff the  $i^{th}$  S-D pair is such that  $s(i)$  belongs to region  $R$  and  $d(i)$  does not. Under a uniformly random placement of nodes

$$\Pr(Y_i = 1) = |R|(1 - |R|) = \frac{1}{4}. \quad (26)$$

The total number of S-D pairs satisfying the required condition can be represented by  $Y = \sum_1^k Y_i$ . If  $k = \Omega(n)$ , then for a large enough  $n$ , the Chernoff Bounds implies that there exists a  $c > 0$  such that for all  $0 \leq \delta_1 \leq 1$  and  $\delta_2 > 0$  we have

$$\begin{aligned} \Pr(Y \geq (1 - \delta_1)|R|(1 - |R|)k) &> 1 - 2e^{-\frac{\delta_2^2}{3}k|R|(1 - |R|)} \\ &\geq 1 - 2e^{-cn} \geq 1 - \frac{1}{n}, \end{aligned} \quad (27)$$

$$\begin{aligned} \Pr(Y \leq (1 + \delta_2)|R|(1 - |R|)k) &> 1 - 2e^{-\frac{\delta_2^2}{3}k|R|(1 - |R|)} \\ &\geq 1 - 2e^{-cn} \geq 1 - \frac{1}{n}. \quad (28) \end{aligned}$$

■

Furthermore, consider the subset  $A$  in  $S$  defined by a strip of dimension  $1 \times r(n)$ . The total number of vertices in  $A$  is  $\Theta(nr(n))$  because of the uniform distribution of nodes in the network. More specifically, with arguments similar to those used for the proofs of the above lemmas we can show that, w.h.p., we have  $\forall 0 \leq \delta \leq 1$ ,

$$(1 - \delta)nr(n) \leq |A| \leq (1 + \delta)nr(n). \quad (29)$$

**Theorem 4.4:** If  $r(n) \geq r_c(n)$  and  $k = \Omega(n)$ , then w.h.p. the sparsity of the cut  $S$  is  $\Upsilon_{G_{r,S}} = O(n^2r^3(n)/k)$

*Proof:* According to the definition of  $G_r$ , two nodes are connected iff they are separated by a distance smaller than  $r(n)$ . Consequently, if an edge cuts across  $S$  then it has to be incident upon a node at a distance smaller than  $r(n)$  from the boundary separating  $S$  and  $S^C$ , i.e., the head of the edge should lie in the subset  $A$  of dimension  $1 \times r(n)$ . Furthermore, for each node in  $A$  the maximum number of edges cutting across the cut is bounded by  $\Delta$ . Hence,

$$\sum_{e \in E} 1_{[e^+ \in S, e^- \in S^C]} \leq |A|\Delta. \quad (30)$$

In the absence of interference,  $c(e) = 1$  for all the edges. Hence,

$$\Upsilon_{G_{r,S}} = \frac{\sum_{e \in E_r} 1_{[e^+ \in S, e^- \in S^C]}}{\sum_{i:s(i) \in S, d(i) \in S^C} D_i} \leq \frac{|A|\Delta}{\sum_{i:s(i) \in S, d(i) \in S^C} D_i} \quad (31)$$

According to Lemma 4.3, there exists a constant  $c_1 > 0$  such that the total number S-D pairs across the cut is at least  $c_1 k$ . Furthermore, by Definition 3.9 the demand for each pair is at least  $\epsilon$ . Therefore,

$$\Upsilon_{G_{r,S}} \leq \frac{|A|\Delta}{c_1 \epsilon k}. \quad (32)$$

Lemma 4.2 implies that there exists a constant  $c_2 > 0$  such that  $\Delta < c_2 nr^2(n)$ , while Eq. (29) implies that there exists a constant  $c_3 > 0$  such that  $|A| < c_3 nr(n)$ . Accordingly,

$$\Upsilon_{G_{r,S}} \leq \frac{c_2 nr^2(n) \times c_3 nr(n)}{c_1 \epsilon k} = \frac{c_2 c_3 n^2 r(n)^3}{c_1 \epsilon k} \quad (33)$$

■

Any cut in  $G_r$  has a sparsity greater than the minimum sparsity cut  $\Upsilon_G$ . Consequently, Theorem 4.5 implies the following Corollary.

**Corollary 4.5:** If  $r(n) \geq r_c(n)$  and  $k = \Omega(n)$ , then w.h.p.  $\Upsilon_{G_r}$  is upper bounded by  $O(\frac{n^2 r(n)^3}{k})$ .

### B. Lower Bound

To describe a capacity-achieving flow in a more generic setting, we use an important result from parallel and distributed computing. Consider a mesh of  $l^2$  processing units with  $l$  processors in each row and column. Let each processor be a source and destination of exactly  $h$  packets. The problem of

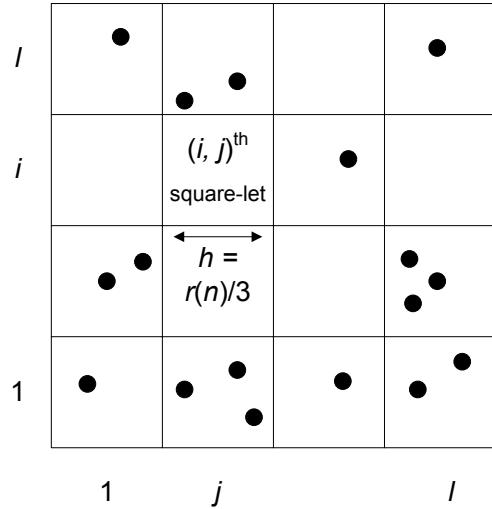


Fig. 4. The network area is decomposed into  $l^2$  logical sub-regions called square-lets. Each square-let has a side-length of  $r(n)/3$ . The square-lets can be indexed by an integer pair  $(i, j)$  such that the  $(i, j)^{th}$  square-let belongs to the  $i^{th}$  row from bottom and the  $j^{th}$  column from left.

routing the  $hl^2$  packets to their destinations is known as  $h \times h$  permutation routing and can be characterized by the following result [29].

**Lemma 4.6:** If in a single slot, each processor can transmit one packet each to its immediate horizontal and vertical neighbors, then an  $h \times h$  permutation routing in a  $l \times l$  mesh can be performed deterministically in  $hl/2 + o(hl)$  steps.

We utilize the following corollary that can be readily deduced from the above Lemma.

**Corollary 4.7:** If a processor is capable of transmitting at least  $\psi$  packets to each of its neighbors in each slot, then an  $h \times h$  permutation routing in a  $l \times l$  mesh can be performed deterministically in  $\frac{hl}{2\psi}$  steps.

Now consider a sub-graph  $H_r \subseteq G_r$  obtained by employing location based constraints on the edge-set. In order to describe these constraints, we first define a location-dependent hash function.

**Definition 4.8: Index Function  $\zeta$ :** Divide the network area into  $l^2$  square-lets [26] of side-length  $a = r(n)/3$ , as shown in Figure 4. Let  $\zeta$  be a function that associates an index  $(i, j)$  with a square-let in the  $i^{th}$  column and  $j^{th}$  row. Furthermore, the index assigned to each square-let is associated with each vertex in the square-let.

We obtain  $H_r$  by removing all edges, except those connecting two nodes in vertically or horizontally adjacent square-lets. We do not necessarily have to consider  $H_r$  in order to obtain a lower bound on the interference-free capacity. However, the performance bounds for  $H_r$  play an important role when we analyze interference-constrained networks in Section V.

**Definition 4.9: Geographically Restricted Sub-Graph  $H_r$ :** The graph  $H_r(V_r, E_{r,H})$  is a sub-graph of  $G_r$  with an identical node-set and an edge-set defined as:

$$E_{r,H} = \{e \in E \mid \zeta(e^-) = (a, b) \Rightarrow \zeta(e^+) = (a \pm 1, b \pm 1)\} \quad (34)$$

Consider some of the properties of the square-lets and  $H_r$ .

**Lemma 4.10:** [26] If  $r(n) \geq r_c(n)$  then w.h.p. the total number of nodes in any square-let are  $\Theta(nr^2(n))$ .

*Proof:* The area of a square-let is equal to  $\Theta(r^2(n))$ . Hence, the proof is identical to that of Lemma 4.2. ■

**Lemma 4.11:** If  $r(n) \geq r_c(n)$  and  $k = \Omega(n)$ , then w.h.p. the total number of sources<sup>5</sup> in any square-let is  $\Theta(kr^2(n))$  and the total destinations in any square-let are  $\Theta(kr^2(n))$ .

*Proof:* For  $1 \leq i \leq k$  let  $Y_{i,m} \in \{0, 1\}$  be a random variable that is equal to one iff source  $s(i)$  belongs to the  $m^{th}$  square-let. Let  $Y_m = \sum_1^k Y_{i,m}$  represent the total number of sources in the square-let. Because  $r(n) \geq r_c(n)$ , Eq. (7) implies

$$\Pr(Y_m \leq (1 - \delta)kr^2(n)) < e^{-(ck \log n)/n} \quad (35)$$

The total number of square-lets in a unit square is equal to  $(3/r(n)) \times (3/r(n)) \leq c_1 n / \log n$ . Therefore, the union bound implies that

$$\begin{aligned} \Pr & (\text{min. no. of nodes in a square-let} < \Theta(kr^2(n))) \\ & \leq (\text{total no. of square-lets}) \times e^{-(ck \log n)/n} \\ & \leq (c_1 n / \log n) \times e^{-(ck \log n)/n} \\ & = c_1 / (n^{(ck/n)-1} \log n) \end{aligned} \quad (36)$$

Thus,  $k \geq \Theta(n)$  guarantees the required convergence and hence we can say that each square-let has at least  $\Theta(kr^2(n))$  sources. The upper bound on the number of sources and the bounds on the number of destinations can be calculated similarly. ■

**Lemma 4.12:** When  $k$  S-D pairs are chosen uniformly randomly, then the maximum number of sources plus destinations placed on any node are at most  $\frac{\lceil(\frac{2k}{n})\rceil 3 \log(n)}{\log(\log(n))}$ .

*Proof:* The lemma follows from a well established result on randomized load balancing. Consider an experiment where  $n$  balls are being randomly distributed among  $n$  bins. It is well known that w.h.p. none of the bins contain more than  $\frac{3 \log(n)}{\log(\log(n))}$  balls. If we repeat the same experiment with  $\lceil(\frac{2k}{n})\rceil$  batches of  $n$  balls, then no bin would contain more than  $\frac{\lceil(\frac{2k}{n})\rceil 3 \log(n)}{\log(\log(n))}$  balls. There is a direct analogy between the balls-bins problem and the random placement of sources and destinations. The sources and destinations can be considered to be the balls, while the nodes can be considered to be the bins. ■

**Theorem 4.13:** If  $k = \Omega(n)$ ,  $r(n) \geq r_c(n)$  and  $r(n) = O\left(\frac{\log(\log(n))}{\log(n)}\right)$ , then w.h.p. the maximum flow rate  $f_H^*$  in  $H_r$  is at least  $\Theta(n^2 r^3(n)/k)$ .

*Proof:* The proof follows from mapping various components of the problem defined above to the  $h \times h$  permutation routing problem. Let us map each square-let to a processor. Consequently, for the chosen size of square-lets, the network equates to a mesh of  $l^2$  processors with  $l = 3/r(n)$ . Assume that each source intends to transmit  $D_i$  as the  $i^{th}$  element of the demand vector. Because  $D_i \leq 1$ , Lemma 4.11 implies that the total number of bits to be transmitted to and from each square-let are at most  $h \leq ckr^2(n)$ . Finally, note that any two nodes in adjacent square-lets are within a distance  $r(n)$ . Fig. 5 provides a geometric proof for this fact; an alternative proof

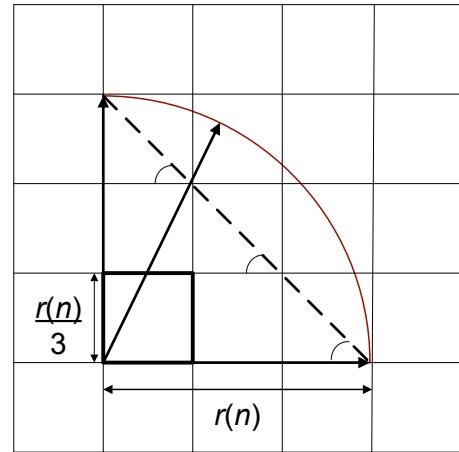


Fig. 5. A geometric proof to show that any two points in adjacent square-lets are within a distance  $r(n)$  of each other. The proof follows from the fact that the chord of a circle lies within it.

can be easily obtained by employing the Pythagoras theorem. In each slot, we can send one packet along each edge between two adjacent square-lets. Therefore, Lemma 4.10 implies that

$$\begin{aligned} \psi &= (\text{min. no. of edges between adjacent square-lets}) \\ &\geq (\text{min. no. of nodes per square-let})^2 \geq c_1 n^2 r^4(n) \end{aligned}$$

Therefore, the total number of slots  $\gamma$  required to complete the desired routing is

$$\begin{aligned} \gamma &\leq (c_2 h l / \psi) \\ &\leq c_2 \times (ckr^2(n)) \times (3/r(n)) \times (1/c_1 n^2 r^4(n)) \\ &= (3c_2 ck / c_1 n^2 r^3(n)) \end{aligned} \quad (37)$$

We can repeat the above routing periodically to guarantee a flow rate of  $f = (1/\gamma) \geq \Theta(n^2 r^3(n)/k)$ . By definition, the max-flow rate is greater than any other feasible flow rate, and the theorem follows. We have over-looked an important subtlety in the above argument. We have assumed that distinct nodes in a square-let can be treated as single processor. This assumption is valid if and only if the flow through a particular node does not become a bottleneck. The maximum rate at which a node can send or receive information is bounded by  $\Theta(nr(n)^2)$ . Lemma 4.12 shows that the maximum number of source and destinations on a single node are bounded by  $\frac{\lceil(\frac{2k}{n})\rceil 3 \log(n)}{\log(\log(n))}$ . Hence, the above argument is sufficient to obtain a lower bound if  $\frac{cnr(n)^2}{\lceil(\frac{2k}{n})\rceil 3 \log(n)} = \Omega\left(\frac{n^2 r(n)^3}{k}\right)$ . This condition is satisfied only when  $r(n) = O\left(\frac{\log(\log(n))}{\log(n)}\right)$ . ■

In the remainder of the paper, unless stated otherwise, it is assumed that  $r(n) = O\left(\frac{\log(\log(n))}{\log(n)}\right)$ . Aggregating the above results, we have the following conclusion.

**Theorem 4.14:** If  $r(n) = \Omega(\sqrt{\log n / n})$  and  $k = \Omega(n)$ , then the max-flow  $f_G^*$  in  $G_r$  is order optimal and can be used to prove a tight approximation of the min-sparsity cut  $\Upsilon_{G_r}^*$  in  $G_r$ . Moreover, the  $f_G^*$  and  $\Upsilon_{G_r}^*$  scale as  $\Theta(n^2 r^3(n)/k)$ .

*Proof:* Lemma 4.1 implies that  $f_G^* \geq f_H^*$ . Hence, the result follows from the lower bound provided by Theorem 4.13 and the upper bound provided by Corollary 4.5. ■

<sup>5</sup>Note that a single node can be selected as source for many destinations and each one is counted separately in this Lemma.

Scaling laws for  $k = \Theta(n)$  have been given special attention in the literature. Accordingly, it is worth stating explicitly the above results for this scenario.

**Corollary 4.15:** Consider an ad-hoc network described by a random placement of  $n$  nodes in a unit square, with  $\Theta(n)$  S-D pairs and a homogenous transmission range of  $r(n) = \Omega(\sqrt{\log n/n})$ . The interference-free capacity of the network scales as  $\Theta(nr^3(n))$ .

## V. INTERFERENCE-LIMITED CAPACITY

### A. General Results on Interference Models

Interference can severely limit the network capacity. In this section, we obtain scaling laws for the SPR, MPR and MPTR interference models. We primarily focus on deducing lower bounds. Moreover, to facilitate a succinct analysis, we develop some additional terminology and establish some important results.

Recall from Section III that, for two different communication schemes  $A$  and  $B$  (e.g., SPR and MPR) defined in the same graph, their corresponding interference functions  $I_A$  and  $I_B$  are different.

**Definition 5.1: Dual-Interference-Set:** Consider an edge set  $E$  and an interference set  $I(e)$  for an edge  $e \in E$ , as defined in Definition 3.2. The dual interference-set for  $e$  is defined by  $F(e) = \{\hat{e} \in E \mid e \in I(\hat{e})\}$ , which is the set of edges that experience a collision on account of a transmission on edge  $e$ .

**Definition 5.2: Dual Conflict Graph:** Given a graph  $G(V, E)$  and an interference function  $I$ , we define the *dual conflict graph* as  $G_D(E, E_D)$ , where  $E_D = \{(e, \hat{e}) \mid \hat{e} \in I(e)\}$ .

**Definition 5.3: Total Degree in Dual Conflict Graph:** The total degree of each node in a dual conflict graph is equal to  $|M(e)|$  where  $M(e) = I(e) \cup F(e)$ .

Similar to the work in [25], we have the following Lemma.

**Lemma 5.4:** Consider a graph  $G(V, E)$  and interference  $I$ . Let  $\kappa = \max_{e \in E} |M(e)|$ . If  $f$  is a feasible flow rate in the absence of interference, then flow rate  $f_I = f/(1 + \kappa)$  is feasible in presence of interference  $I$ .

*Proof:* In the absence of interference the capacity of each edge is assumed to be one. However, because of interference, all edges cannot be activated simultaneously. Let  $\sigma$  be the minimum frequency with which each edge is activated without causing any interference conflicts. Then, for each edge we have  $c(e) \geq 1/\sigma$ .

Observe that  $\kappa$  is the maximum vertex degree of the dual conflict graph  $G_D$ . It is well known that, if  $\kappa$  is the maximum vertex degree, then  $\kappa + 1$  colors are sufficient to provide a proper vertex coloring [30]. Thus, we can partition the edge-set  $E$  into  $1 + \kappa$  subsets such that no two edges in the same subset interfere. Consequently, we can periodically activate these subsets to realize  $c(e) \geq 1/(1 + \kappa)$  for each edge. Thus, a feasible flow rate  $f_I = f/(1 + \kappa)$  can be obtained by scaling the flow functions associated with  $f$  by a factor of  $1/(1 + \kappa)$ . ■

The maximum vertex degree does not provide a tight bound on the minimum number of colors required to provide a proper vertex coloring. Accordingly, to analyze a wider variety of

protocol models, we introduce the concept of *interference clones*.

**Definition 5.5: Interference Clone:** Two edges  $e_1, e_2$  are said to be interference-clones under function  $I$  if they satisfy the conditions that  $M(e_1) = M(e_2)$ .

**Lemma 5.6: Clone Piggy-backing Lemma:** Consider a graph  $G(V, E)$  along with interference functions  $I_A$  and  $I_B$ . Then  $I_A$  and  $I_B$  are such that:

- 1)  $\kappa = \max_{e \in E} |M_A(e)|$ ,
- 2)  $\forall e \in E$ , there exists a set  $M_{A,\bar{B}}(e) \subseteq M_A(e)$  such that every edge belonging  $M_{A,\bar{B}(e)}$  is an interference-clone of  $e$  under  $I_B$ . Further, let  $\mu = \min_{e \in E} |M_{A,\bar{B}}(e)|$ .

If  $f$  is a feasible flow rate in  $G$  without any interference,  $f_{I_A} = f/(1 + \kappa)$  is a feasible flow rate in  $G$  under the  $I_A$  interference function and  $\kappa$  as its corresponding parameter, then  $f_{I_B} = f(1 + \mu)/(1 + \kappa)$  is feasible in presence of interference defined by  $I_B$ .

*Proof:* Consider the interference defined by  $I_A$ . From Lemma 5.4 we know that there exists a conflict free periodic schedule which can activate each edge at least once every  $(1 + \kappa)$  slots. Let us represent this schedule by an indicator function  $\alpha(e, \tau)$ , which equals one iff edge  $e$  is active in slot  $\tau$ . Note that the capacity of each edge under schedule  $\alpha$  is given by

$$c_\alpha(e) = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{\tau=1:T} \alpha(e, \tau) = 1/(1 + \kappa). \quad (38)$$

Now let us use this schedule in the presence of interference  $I_B$ . Observe that  $\alpha$  allocates a distinct slot for each edge in  $M_{A,\bar{B}}(e) \cap \{e\}$  for every  $e$ . Consequently, every edge has  $|M_{A,\bar{B}}(e)|$  interference clones scheduled in slots distinct from each other and the edge itself. In addition, note that if an edge is activated in a time slot meant for one of its interference clones, then it does not lead to any conflict. Therefore, we can define a new conflict-free schedule  $\beta$  such that  $\beta(e, \tau) = 1$  iff there exists an  $e_1 \in M_{A,\bar{B}}(e) \cap \{e\}$  such that  $\alpha(e_1, \tau) = 1$ . Given that  $\mu = \min_{e \in E} |M_{A,\bar{B}}(e)|$ , the capacity of each edge under schedule  $\beta$  for interference  $I_B$ , is given by

$$\begin{aligned} c_\beta(e) &= \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{e_1 \in M_{A,\bar{B}}} \sum_{\tau=1:T} \alpha(e_1, \tau) \\ &= (1 + \mu) \times (1/(1 + \kappa)) \end{aligned} \quad (39)$$

Accordingly, a feasible flow of  $f_{I_B} = f(1 + \mu)/(1 + \kappa)$  can be obtained by scaling all the flow functions associated with the interference-free flow by a factor of  $(1 + \mu)/(1 + \kappa)$ . ■

In the subsequent discussion, we find it convenient to deduce a bound for a particular interference model and then show that it applies to a wider set of models. In order to facilitate such arguments, we define the following partial order.

**Definition 5.7: Partial Order of Interference Models:** An interference function  $I_A$  is said to be more restrictive than  $I_B$ , represented as  $I_A \preceq I_B$ , iff every edge satisfies the condition that  $I_B(e) \subseteq I_A(e)$ .

**Lemma 5.8:** Consider a graph  $G(V, E)$  along with interference  $I_A$  and  $I_B$ . If  $I_A \preceq I_B$ , then a feasible flow rate under  $I_A$  remains feasible under  $I_B$ .

*Proof:* A conflict free schedule under  $I_A$  remains conflict free under  $I_B$ . Hence we can say that  $c_A(e) \leq c_B(e)$ , where  $c_A(e)$  and  $c_B(e)$  represent the edge capacities under each

interference model. Therefore, if a particular flow satisfies the capacity constraints under  $I_A$ , it necessarily satisfies those same constraints under  $I_B$ . ■

### B. Lower Bounds

Direct analysis of the interference models can be tedious. Hence, for mathematical convenience, we consider the following restrictive models. Lemma 5.8 allows us to utilize the performance limits under these restrictive models to indirectly bind the capacity under the interference models for SPR, MPR and MPTR.

We define a restrictive interference model that introduces more restrictions (i.e., collisions) on the interference set for each edge than those strictly dictated by the original interference model. We will show that, under this restrictive model, the order of the lower bound capacity achieves the upper bound under the original (non-restrictive) interference model.

*Definition 5.9: Restricted SPR (RSPR) Model:*

$$I_{\text{RSPR}}(e) = W(e) - \{e\} \quad \forall e \in E_{r,H}$$

$$\text{where } W(e) = \bigcup_{\hat{e}: \zeta(\hat{e}^-) = \zeta(e^-)} M_{\text{SPR}}(\hat{e}) \quad (40)$$

*Definition 5.10: Restricted MPR (RMPR) Model:*

$$I_{\text{RMPR}}(e) = W(e) - U(e) \quad \forall e \in E_{r,H}$$

$$\text{where } U(e) = \{\hat{e} \in E_{r,H} \mid \hat{e}^- = e^-\} \quad (41)$$

*Definition 5.11: Restricted MPTR (RM PTR) Model:*

$$I_{\text{RM PTR}}(e) = W(e) - V(e) \quad \forall e \in E_{r,H}$$

$$\text{where } V(e) = \bigcup_{\hat{e}: \zeta(\hat{e}^-) = \zeta(e^-)} U(\hat{e}) \quad (42)$$

Consider the following properties of the restricted models.

*Lemma 5.12:* For the graph  $H_r$ , we have a partial order defined by (i)  $I_{\text{SPR}} \preceq I_{\text{MPR}} \preceq I_{\text{MPTR}}$ , (ii)  $I_{\text{RSPR}} \preceq I_{\text{RMPR}} \preceq I_{\text{RM PTR}}$ , (iii)  $I_{\text{SPR}} \preceq I_{\text{RSPR}}$  (iv)  $I_{\text{RMPR}} \preceq I_{\text{MPR}}$  (v)  $I_{\text{RM PTR}} \preceq I_{\text{MPTR}}$

*Proof:* (Sketch) The proof for the partial orders (i) to (iii) follows directly from the definitions. Partial order (iv) follows from the fact that  $J(e) \subseteq W(e)$  and  $U(e) \subseteq A(e) - B(e)$ . Similarly, the fact that  $A(e) \subseteq V(e)$  implies partial order (v). ■

*Lemma 5.13:* If  $r(n) \geq r_c(n)$ , then all edges  $e \in E_{r,H}$  have  $|M(e)| = \Theta(n^2r^4(n))$  under interference described by either of the models:  $I_{\text{SPR}}, I_{\text{MPR}}, I_{\text{MPTR}}, I_{\text{RSPR}}, I_{\text{RMPR}}, I_{\text{RM PTR}}$ .

*Proof:* From Lemma 5.12 we can conclude that RSPR is the most restrictive model, while MPTR is the least restrictive model. Further note that  $F_{\text{RSPR}}(e) \subseteq W(e)$  and  $I_{\text{MPTR}}(e) \subseteq M_{\text{MPTR}}(e)$ . Hence, it suffices to prove the following:

$$\gamma_{\max} = \max_{e \in E_{r,H}} |W(e)| = O(n^2r^4(n)) \quad (43)$$

$$\gamma_{\min} = \min_{e \in E_{r,H}} |I_{\text{MPTR}}(e)| = \Omega(n^2r^4(n)) \quad (44)$$

Recall that a node in  $H_r$  is connected to all and only those nodes that are placed in adjacent square-lets. Hence Lemma 4.10 tells us that the degree of each vertex  $v \in H_r$  is bounded as

$$4c_1nr^2(n) \leq \deg(v) \leq 4c_2nr^2(n) \quad (45)$$

Now lets prove the lower bound by considering the MPTR model. According to Definition 3.5, the transmission on edge  $e$  experiences interference from any transmission by a node  $v$  such that  $r(n) < \|X_{e^-} - X_v\| \leq (1 + \eta)r(n)$ . Therefore, there exists an annular ring around  $e^-$  of width  $\eta r(n)$  such that any transmission from a node in this ring interferes with  $e$ . The area of this annular ring is given by

$$\text{area of annular ring} = \eta(2 + \eta)\pi r^2(n) \quad (46)$$

We have already seen ( Lemma 4.2) that an area of  $\Theta(r^2(n))$  contains at least  $\Theta(nr^2(n))$  nodes. Hence, there exists a  $c_3$  such that

$$\gamma_{\min} \geq c_3nr^2(n) \times 4c_1nr^2(n) \quad (47)$$

Eq. (47) proves the required lower bound. The proof for the upper bound is obtained with a similar argument. First, let us inspect the transmission along edge  $e$  under the SPR model. Any transmission from a node in a disk of radius  $(1 + \eta)r(n)$  around  $e^-$ , interferes with  $e$ . Moreover, a transmission from  $e^+$  interferes with any reception in a disk of radius  $(1 + \eta)r(n)$  around  $e^+$ . Given that  $\|X_{e^+} - X_{e^-}\| \leq r(n)$ , there exists a disk of radius  $(2 + \eta)r(n)$  around  $e^-$  containing all the nodes that may have transmission conflicts with  $e$ . Further note that a square-let can be completely inscribed in a circum-circle of radius  $(1/3\sqrt{2})r(n)$ . Hence, there exists a circle of maximum radius  $R = (2 + \eta + (1/3\sqrt{2}))r(n)$  containing all the nodes that may have transmission conflicts with  $e$  under the model RSPR. The area of  $R$  is  $\Theta(r^2(n))$  and hence there exists a constant  $c_4$  such that

$$\begin{aligned} \gamma_{\max} &= \max_{e \in E_{r,H}} |W(e)| \\ &\leq (\text{max. no. of nodes in } R) \times (\text{max. node degree}) \\ &\leq c_4n(2 + \eta + (\sqrt{2}/6))^2r(n)^2 \times 4c_2nr^2(n) \\ &= O(n^2r^4(n)) \end{aligned} \quad (48)$$

■

*Lemma 5.14:* Consider the graph  $H_r$  with  $r(n) \geq r_c(n)$  and  $k = \Omega(n)$ . In such a graph, each edge  $e$  has at least  $\Theta(nr^2(n))$  clones under interference  $I_{\text{RMPR}}$  and  $\Theta(n^2r^4(n))$  clones under interference  $I_{\text{RM PTR}}$ , such that these clones interfere with each other and  $e$ , under the interference  $I_{\text{RSPR}}$ .

*Proof:* According to Definitions 5.10 and 5.11,  $U(e)$  and  $V(e)$  represent the desired set of clones for  $I_{\text{RMPR}}$  and  $I_{\text{RM PTR}}$  respectively. Lemma 4.10 implies that there exist  $c_1$  and  $c_2$  such that

$$\begin{aligned} \mu_{\text{RMPR}} &= \min_{e \in E_{r,H}} |U(e)| \\ &\geq \text{min. vertex degree} \\ &= c_1nr^2(n) \end{aligned} \quad (49)$$

$$\begin{aligned} \mu_{\text{RM PTR}} &= \min_{e \in E_{r,H}} |V(e)| \\ &\geq [\min_{e \in E_{r,H}} |U(e)|] \times \text{min. nodes per square-let} \\ &\geq c_1nr^2(n) \times c_2nr^2(n) \end{aligned} \quad (50)$$

■

*Theorem 5.15:* For  $r(n) \geq r_c(n)$  and  $k = \Omega(n)$ , the capacity of random geometric network is at least: (a)  $\Theta(1/r(n)k)$  under the SPR model, (b)  $\Theta(nr(n)/k)$  under the MPR model, and (c)  $\Theta(n^2r^3(n)/k)$  under the MPTR model.

*Proof:* Recall that the capacity of the random network is greater than the feasible flow rate in  $H_r$ . Theorem 4.13 shows that a rate of  $f = c_1 n^2 r^3(n)/k$  is feasible in  $H_r$ . Additionally, Lemma 5.13 shows that the size of the largest interference set under RSPR is at most  $\kappa = c_2 n^2 r^4(n)$ . Hence, Lemma 5.4 implies that a rate of

$$f_{RSPR} = f \times (1/(1 + \kappa)) \geq (c_3/r(n))k \quad (51)$$

is feasible under the RSPR model. If we take into consideration the interference clones, then Lemma 5.6 further implies that the rate

$$\begin{aligned} f_{RMPR} &= f \times (1/(1 + \kappa)) \times (1 + \mu_{RMPR}) \\ &\geq (c_3/r(n))k \times (c_4 n r^2(n)) \\ &= (c_3 c_4 n r(n)/k) \end{aligned} \quad (52)$$

is feasible under the RMPR model. Similarly, the rate

$$\begin{aligned} f_{RM PTR} &= f \times (1/(1 + \kappa)) \times (1 + \mu_{RM PTR}) \\ &\geq (c_3/r(n))k \times (c_5 n^2 r^4(n)) \\ &= (c_3 c_5 n^2 r^3(n)/k) \end{aligned} \quad (53)$$

is feasible under the RM PTR model. Finally, note that a feasible rate under a restricted model is necessarily feasible under the original model. Hence, the result proven in Lemma 5.13 completes the proof. ■

It can be easily shown that the upper bound for SPR and MPR is the same as the lower bound but the details are omitted here and can be found in [31].

The interference-free capacity provides an upper bound on the capacity under any model, and Theorem 5.15 already shows that the MPTR model achieves this capacity. However, we need to provide additional arguments to obtain a tight bound on the capacity under SPR and MPR. Our arguments are similar to those used in [6] and [4], and we briefly sketch the proof of the following result for the sake of completeness.

**Theorem 5.16:** For  $r(n) \geq r_c(n)$  and  $k = \Omega(n)$ , the capacity of random geometric network is at most: (a)  $\Theta(1/r(n))k$  under the SPR model, (b)  $\Theta(nr(n)/k)$  under the MPR model, and (c)  $\Theta(n^2 r^3(n)/k)$  under the MPTR model.

*Proof:* (Sketch) Consider the Cut  $S$  in Figure 3. The capacity of this cut and hence the network is less than

$$\frac{(\text{no. of transmitters in } A) \times (\text{max. transmission per node})}{(1/2)k} \quad (54)$$

The total number of transmitters in  $A$  is smaller than the total number of nodes in  $A$ . We have already shown that the total number of nodes is  $\Theta(nr(n))$ . Under the MPTR model, each node can transmit a maximum of  $\Theta(nr^2(n))$  packets, and under the MPR model each packet transmits just a single packet. This provides the bounds for MPR and MPTR. To obtain the bound for SPR, we note that each transmitter silences all the nodes within an area of  $\Theta(r^2(n))$ . Hence, only  $\Theta(1/r(n))$  nodes are capable of transmitting simultaneously across a cut. Each of these nodes transmits a single packet, and therefore, the above equation provides the bound for SPR. ■

## VI. DISCUSSION

The results we have presented demonstrate that future ad hoc networks can scale well beyond the capacity predicted by Gupta and Kumar [1]. First, we showed that the optimal capacity that *any* protocol architecture can attain in a wireless network is  $\Theta(n^2 r^3(n)/k)$ . Second, we demonstrated that this capacity can indeed be attained under the protocol model when nodes embrace MAI with transmitters and receivers utilizing a combination of MPR and MPT.

In the important case when  $k = \Theta(n)$ , we have the interesting observation that any choice of  $r(n) = \Omega(n^{-1/3})$  allows us to increase the per-session capacity of the network with  $n$ , even as the hop-size decreases to zero. This result can be understood intuitively by observing that, even though the transmission range goes to zero, the concurrent transmission between two square-lets increase as  $\Theta(nr(n)^2) = \Theta(n^{1/3})$ . On the contrary if we choose the transmission range equal to the minimum value required to guarantee connectivity, then the throughput order due to MPTR is  $\Theta\left(\frac{\log(n)^{\frac{3}{2}}}{\sqrt{n}}\right)$ . This represents a comparatively modest gain of  $\Theta(\log(n)^2)$  over the Gupta and Kumar's results.

The above discussion clearly highlights that even though MPTR may have the potential to significantly increase the capacity of ad hoc networks, this increase may require substantial increase in the network resources as well as processing accuracy at the physical layer.

In closing, we should point out that, while our results provide a new outlook on the design of wireless ad hoc networks, much work remains to be done to fully understand their fundamental limits! For example, the results we have presented address only unicast traffic; our model can be used to study the cases of multicast and broadcast information dissemination too. We hope that this paper motivates research on protocol architectures that combine multi-packet reception and transmission to attain scalable ad hoc networks.

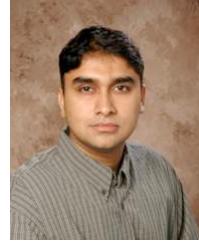
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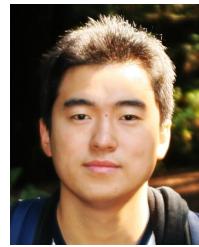
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