

# SYSTEM IDENTIFICATION FOR PRECISION CONTROL OF A GPS-AUTONOMOUS CATAMARAN

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## ABSTRACT

**KEYWORDS:** system identification, autonomous system, distributed control, navigation, GPS.

An autonomous catamaran, based on a modified Prindle-19 day-sailing catamaran was built to test the viability of GPS-based system identification for precision control. The catamaran was fitted with several sensors and actuators to characterize the dynamics. Using an electric trolling motor, and lead ballast to match all-up weight, several system identification passes were performed to excite system modes and model the dynamic response. LQG controllers were designed based on the results of the system identification passes, and tested with the electric trolling motor. Line following performance was excellent, with cross-track error standard deviations of less than 0.15 meters. The wing-sail propulsion system was fitted, and the controllers tested with the wing providing all forward thrust. Line following performance and disturbance rejection were excellent, with the cross-track error standard deviations of approximately 0.30 meters, in spite of wind speed variations of over 50% of nominal value.

## INTRODUCTION

This paper details the progress of the Atlantis project, pictured in Figure 1, which began with the conception of an unmanned, autonomous, GPS-guided, wing-sail-propelled sailboat. From this point of conception back in March of 1997, a full four years have passed bringing this vision to fruition. This has been “systems” approach, with substantial innovations in the areas of wind-propulsion, overall system architecture, sensors, system identification and control.

Functionally, the Atlantis is the marine equivalent of an unmanned aerial vehicle, and would serve similar purposes. The Atlantis project has been able to demonstrate an advance in control precision of a wind-propelled marine vehicle from typical commercial autopilot accuracy of 100 meters to an accuracy of better than one meter. This quantitative improvement enables new applications, including unmanned station-keeping for navigation or communication purposes, autonomous “dock-to-dock” capabilities, emergency “return unmanned” functions, and many others still to be developed. The prototype is based on a modified Prindle-19 light catamaran.

The wind-propulsion system is a rigid wing-sail mounted vertically on bearings to allow free rotation in azimuth about a stub-mast. Aerodynamic torque about the stub-mast is trimmed using a flying tail mounted on booms joined to the wing. This arrangement allows the wing-sail to automatically attain the optimum angle to the wind, and weather vane into gusts without inducing large heeling moments. Modern airfoil design allows for an increased lift-drag (L/D) ratio over a conventional sail, thus providing increased thrust while reducing the overturning moment.



Figure 1: Atlantis with wing-sail, Jan. 2001

The system architecture is based on distributed sensing and actuation, with a high-speed digital serial bus connecting the various modules together. Sensors are sampled at 100 Hz., and a central guidance navigation and control (GNC) computer performs the estimation and control tasks at 5 Hz. This bandwidth has been demonstrated to be capable of precise control of the catamaran. The distributed architecture is both more robust and less expensive than systems that employ a high-speed, and often analog, star-configuration topology with centralized sensor interpretation and actuation.

The sensor system uses differential GPS (DGPS) for position and velocity measurements, augmented by a low-cost attitude system based on accelerometer- and magnetometer-triads. Accurate attitude determination is required to create a synthetic position sensor that is located at the center-of-gravity (CG) of the boat, rather than at the GPS antenna location.

Experimental trials recorded sensor and actuator data intended to excite all system modes. A system model was assembled using Observer/Kalman System Identification (OKID) techniques. An LQG controller was designed using the OKID model, using an estimator based on the observed noise statistics. Experimental tests were run to sail on a precise track through the water, in the presence of currents, wind and waves.

## SYSTEM DESCRIPTION

In order to experimentally validate the concepts presented in this research, a prototype system was built based on a heavily modified Prindle-19, day-sailing catamaran. The catamaran was 7.2 meters. long, 3 meters wide, and was originally equipped with a sloop rig sail with 17 m<sup>2</sup> of sail area. Directional control is based on rudders at the end of each hull, and retractable centerboards approximately ½ meter behind the main crossbeam. Several sensors and actuators were installed within the hulls, and the entire sailing system (mast, boom, main and jib sails) was replaced with a vertical self-trimming wing (wing-sail) suspended on spherical roller bearings.

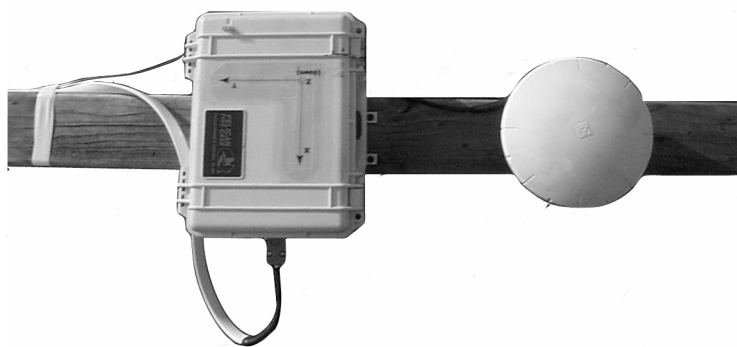


Figure 2: Attitude system and GPS antenna

advantages to this setup, but the ease of troubleshooting and flexibility of physical topology are at the forefront of utility in this design.

The main subsystems are: attitude system, anemometer, hullspeed, rudder angle, rudder actuator, GPS receiver, and wing-sail. These subsystem communicate to the main GNC computer that computes the current estimate of the state, and returns the required commands to the actuator in order to achieve control.

The attitude system, pictured in Figures 2 and 3, consists of a three-axis magnetometer, a two-axis accelerometer, and a Siemens 515 microcontroller. It functions based on a novel gyro-free quaternion based solution to the vector matching problem first proposed by Whaba in 1966[7]. The algorithm is discussed extensively in [6]. The attitude system is

There are several main subsystems on the Atlantis, and all of them are connected to each other via a high-speed serial network. The network utilized is the Controller Area Network (CAN) bus, which was designed by Bosch electronics for robust component communication in an automotive environment[2]. The entire wiring bus on the Atlantis consists of four (4) wires: power (+12V), ground, CAN\_hi, and CAN\_low. There are many



Figure 3: Close-up of magnetometers and accelerometers



Figure 5: GNC computer box

through-hull speed sensor, pictured in Figure 7, in the bottom is the starboard hull, and a LoHet magnetic field effect sensor between two magnets on the rudder hinge line to measure rudder angle. The actuator is a fractional horsepower DC motor, with a lead screw assembly, constrained to rotate only in yaw, and an Infineon H-bridge mosfet drive for electronic control of the motor.

Inside the stub-mast, a Mercotac slip ring allows for a full 360 degree rotation without twisting the four wires that comprise the wiring harness of the Atlantis. The wing itself is built in three sections that are assembled on site. The lower section contains an electronics pod with the batteries, ballast, and battery-charging electronics. A microcontroller and DC motor are used to control the trailing edge flap. It also contains the anemometer microcontroller, with the Standard Communications Electronics marine transducer head, pictured in Figure 4, attached to the top of the electronics pod lid.

The anemometer is used to measure the wind speed and direction relative to the angle of the wing-sail. Essentially, this is a measure of angle of attack of the wing. Experimental data shows this to vary  $\pm 20$  degrees from nominal. This demonstrates the requirement for a self-trimming wing; without the ability to trim quickly to a new angle of attack, the wing would remain stalled most of the time, and the thrust generated would be minimal. The ability to respond quickly to a new angle of attack by rotating into the new trim condition, allows the wing to absorb these transient gusts and continue to provide full thrust with a reduced heeling moment.



Figure 7: Hullspeed transducer

tails must be reversed, which corresponds to tacking or jibing depending on whether the wind crosses the centerline facing aft or forward respectively.

mounted alongside the Global Positioning System (GPS) antenna, inside a waterproof Pelican box on a wooden crossbeam at the forward stay location. The wooden crossbeam was added for increased structural rigidity of the hulls, due to the stresses induced by the wing.



Figure 4: Anemometer

The GNC computer, shown in Figure 5, a Pentium class laptop, is placed in another waterproof case, along with the Trimble Ag122 GPS receiver. The GNC computer is equipped with an ESD parallel port dongle that allows communication over the CAN bus. A DC/DC converter insures that the laptop draws power from the boat power bus rather than its own internal batteries.

Inside the starboard hull, beneath the rear inspection cover, are two Siemens 505 microcontrollers, one for the hullspeed and rudder angle sensor, and the other for the rudder actuator, pictured in Figure 6. There is a Standard Communications Electronics marine



Figure 6: Rudder actuator

transducer head, pictured in Figure 4, attached to the top of the electronics pod lid. Within the wing are four actuators that are identical to the rudder actuator, which actuate the trailing edge flaps and the tail. The moment balance between the wing and the tail keeps the wing-sail at a constant angle of attack relative to the wind. As long as the wind does not cross the centerline of the boat, then the wing continues to provide thrust in the correct direction for forward motion through passive stability of the wing-sail system. Should the wind cross through the centerline of the boat, then the position of the flap and

## SYSTEM IDENTIFICATION METHODOLOGY

In order to control the Atlantis, a system model needed to be assembled. While several good modelling techniques exist to model a powered boat through the water[12], they remain complicated and difficult to calculate. In order to reduce the model order, and obtain a model that would have sufficient fidelity for active control, several different methodologies were attempted. In order to formulate the equations of motion, the Atlantis is assumed to be travelling upon a straight line, conveniently assumed to be coincident with the X-axis, through the water at a constant velocity,  $V_x$ . The distance along that line is referred to as  $X$ , the along track distance. The perpendicular distance to the line is referred to as  $Y$ , the cross track error, and the angle that the centerline of the Atlantis makes with respect to the desired path is defined as  $\Psi$ , the angular error. Figure 8 illustrates the mathematical model of the assumed path of the Atlantis.

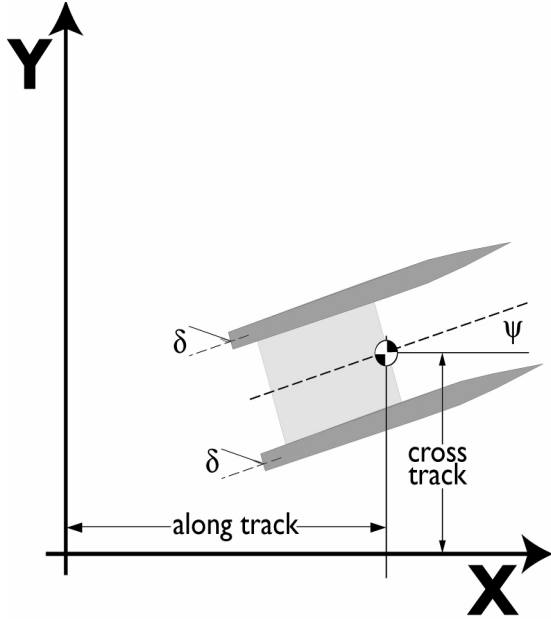


Figure 8: Basic E.O.M. definitions

The first model is a simple kinematic model that assumes that the rudders cannot move sideways through the water. This places a kinematic constraint upon the motion of the entire boat, and the linearized analysis produces the following continuous time state-space equations:

$$\begin{bmatrix} \dot{Y} \\ \dot{\Psi} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & V_x & 0 \\ 0 & 0 & \frac{V_x}{L} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y \\ \Psi \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad (\text{EQ 1})$$

Where,  $Y$ ,  $\Psi$ , and  $\delta$  are defined as the cross-track error, azimuth error, and rudder angle, and  $L$  is the length from the C.G. of the boat to the center of pressure of the rudders. These simplified equations of motion are insufficient to control the boat to great precision, but are excellent for generating intuition for the system identification process. Equation 1, when cast into transfer function form, becomes a triple integrator, and cannot be stabilized by simple proportional control. In addition, the assumption of constant  $V_x$  is poor, since unless the wind can be controlled, the velocity will always be dependent on the speed of the wind. Closer inspection of Equation 1 shows that the errors in azimuth and cross-track integrate not with time, but rather with distance travelled forward. What this means is that if the boat is sitting still in the water, no amount of rudder deflection will cause the azimuth to change, likewise, when moving very quickly through the water, only very small inputs are required to turn the boat through a considerable angle.

Thus, recharacterizing the variables of interest, to make the system velocity invariant, both the azimuth and cross-track error are normalized by velocity. Thus, the new variables of interest become  $\tilde{y}$ , and  $\tilde{\Psi}$ , with  $\delta$  remaining the same as previously defined. The new simplified equations of motion become:

$$\begin{bmatrix} \dot{\tilde{y}} \\ \dot{\tilde{\Psi}} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{1}{L} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{y} \\ \tilde{\Psi} \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad (\text{EQ 2})$$

where:

$$\begin{aligned} \tilde{y} &\equiv \frac{y}{V_x} \\ \tilde{\Psi} &\equiv \frac{\Psi}{V_x} \end{aligned} \quad (\text{EQ 3})$$

Note that this causes the state transition matrix to be constant. This concept of velocity invariance has

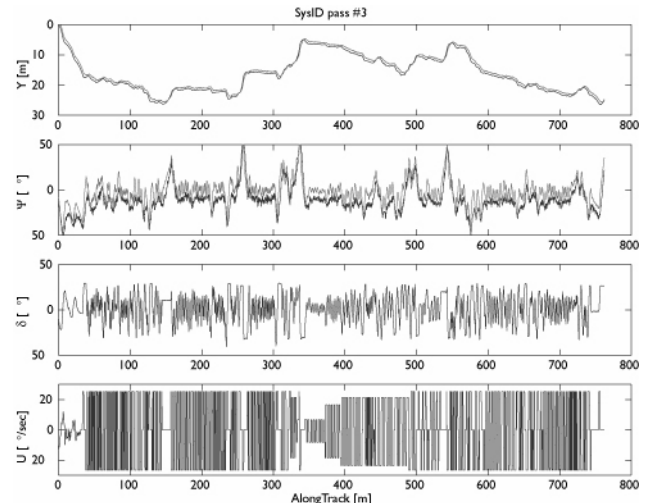


Figure 9: Typical sysID pass

been tested extensively on GPS-controlled farm tractors [9], and shown to work very well. Functionally, this means that controller design is reduced to a single (non-gain-scheduled) controller that automatically adjusts for the changing velocity based on a decreasing input ranges as velocity increases.

In order to gather data to perform a proper system identification of the Atlantis, a series of open-loop line-following tests were conducted in which a human driver, through the GNC computer, caused the rudders to either slew left or right at the maximum slew rate ( $\sim 25$  degrees/sec.). Also, the driver commanded the rudder slew rate to zero through the rudder actuator in order to track a roughly straight line. This “pseudo”-random input was designed to apply the maximum power to the Atlantis through the controls and produce a rich output that would contain information from all modes of interest. A typical pass for system identification is pictured in Figure 9. The controller was designed using a standard LQR methodology. The quadratic cost, as calculated below in Equation 3, minimized the weighted sum of the outputs ( $y_{\max}$  and  $u_{\max}$  are design parameters).

$$J = \sum_{k=0}^{\infty} \left( \hat{x}_k^T C^T \begin{bmatrix} \frac{1}{y_{\max}^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} C \hat{x}_k + \hat{u}_k^T \begin{bmatrix} \frac{1}{u_{\max}^2} \end{bmatrix} \hat{u}_k \right) \quad (\text{EQ } 4)$$

In terms of system identification, this becomes useful as the inputs are scaled by velocity before being assembled into the system identification algorithm. Thus, the identified system is one that is the best model for the velocity invariant control. The system identification methodology used for this work is the Observer Kalman IDentification (OKID)[8]. Among its many advantages is a formulation that presumes a discrete-time, linear system. Since OKID’s development at NASA Langley for the identification of lightly-damped space-structures, many advances on the basic theory have been published[10]. Given a linear discrete-time state-space system, the equations of motion can be written as follows:

$$\begin{aligned} \hat{x}_{k+1} &= A\hat{x}_k + B\hat{u}_k \\ \hat{y}_k &= C\hat{x}_k + D\hat{u}_k \end{aligned} \quad (\text{EQ } 5)$$

It has been shown that the triplet,  $[A,B,C]$  is not unique, but can be transformed through any similarity transform (i.e. the outputs are unique, but the internal states are not). However, the system response from rest when perturbed by a unit pulse input, known as the system Markov parameters, are invariant under similarity transforms. These Markov parameters are:  $[Y_0, Y_1, \dots, Y_k]$

When these Markov parameters are assembled into a specific form—the generalized Hankel matrix of equation 6—this matrix can be decomposed into the Observability matrix, a state transition matrix, and the Controllability matrix; thus the Hankel matrix (in a noise-free case) will always have rank  $n$ , where  $n$  is the system order.

$$H(k-1) = \begin{bmatrix} Y_k & Y_{k+1} & \dots & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & \dots & Y_{k+\beta} \\ \dots & \dots & \dots & \dots \\ Y_{k+\alpha-1} & Y_{k+\alpha} & \dots & Y_{k+\alpha+\beta-1} \end{bmatrix} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \dots \\ CA^{\alpha-1} \end{bmatrix} A^{k-1} \begin{bmatrix} B & AB & A^2B & \dots & A^{\beta-1}B \end{bmatrix} \quad (\text{EQ } 6)$$

Because noise will corrupt this rank deficiency of the Hankel matrix (the Hankel matrix will always be full rank) the Hankel matrix is truncated using a singular value decomposition (SVD) at an order that sufficiently describes the system. This truncated Hankel matrix is then used to reconstruct the triplet  $[A,B,C]$  in a balanced realization that ensures that the controllability and observability Grammians are equal. This is referred to as the Eigensystem Realization Algorithm (ERA); a modified version of this algorithm that includes data correlation is used to identify the Atlantis. A more complete treatment of the subject can be found in [11].

For any real system, however, system pulse response cannot be obtained by simply perturbing the system with a pulse input. A pulse with enough power to excite all modes would likely saturate the actuator or respond in a non-linear fashion. The pulse response of the system can, however, be reconstructed from a continuous stream of rich system input and output behavior. Under normal circumstances, there are not enough equations available to solve

for all of the Markov parameters. Were the system asymptotically stable, such that  $A_k=0$  for some  $k$ , then the number of unknowns could be reduced. The identification process would be of little value if it could only work with asymptotically stable systems.

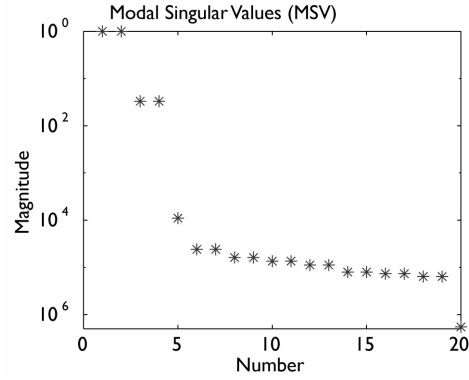
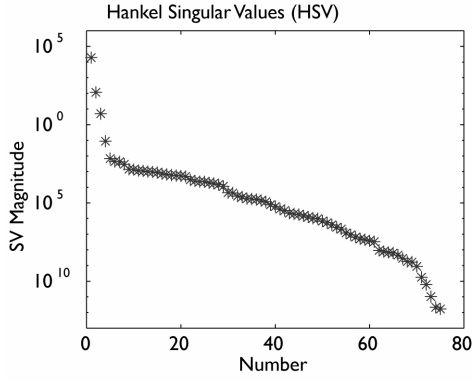


Figure 10: Hankel SVD for catamaran data

By adding an observer to the linear system equations, the following transformation can take place:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Gy_k - Gy_k \\ x_{k+1} &= [A + GC]x_k + [B + GD]u_k - Gy_k \\ x_{k+1} &= \hat{A}x_k + \hat{B}v_k \end{aligned} \quad (\text{EQ } 7)$$

$$\hat{A} \equiv [A + GC], \hat{B} \equiv [B + GD], v_k \equiv \begin{bmatrix} u_k \\ y_k \end{bmatrix}$$

Thus, the system stability can be augmented through an observer and the ideal Markov parameters established through a least-squares solution [8]. It is useful to note that the realization also provides a pseudo-Kalman observer.



Figure 11: Atlantis trolling motor test

both input and output. Utilizing the separation lemma and the provided Kalman filter, only the controller gains

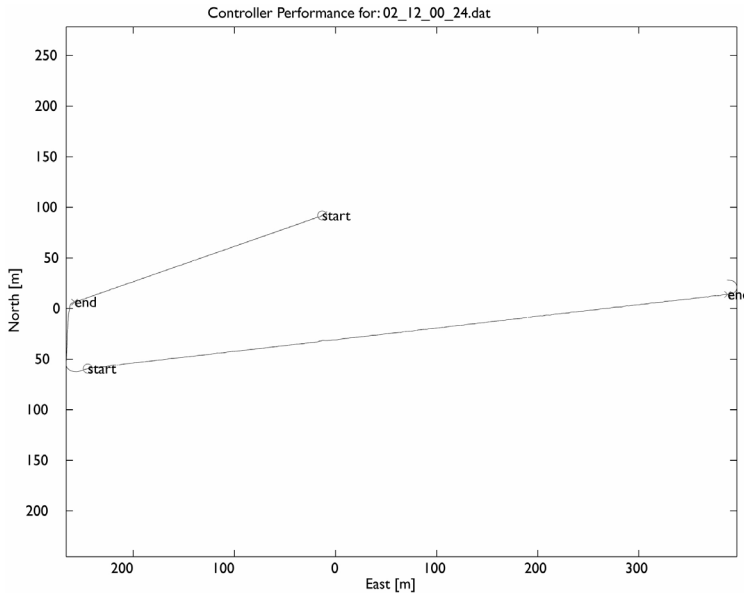


Figure 12: Overhead trace of trolling motor data

carried out to validate the controller performance. Once satisfied with these simulations, experimental trials were performed in order to validate the concept.

The observer orthogonalizes the residuals to time-shifted versions of both input and output. Utilizing the separation lemma and the provided Kalman filter, only the controller gains need be designed to implement a full-state-feedback linear quadratic gaussian (LQG) controller. An improved version of the OKID process, which includes residual whitening [11], was used to identify the sailboat dynamics from the experimental data.

An SVD of the aggregate velocity-normalized data for the Atlantis demonstrated a large drop in the magnitude of the singular values from the fourth to the fifth, indicating a system order,  $n$ , of four (Figure 10, top). In addition, modal singular values (Figure 10, bottom) of all catamaran models of order higher than four exhibited a two order-of-magnitude drop from the fourth modes to modes higher than four. System reconstruction for the identified dynamics also matches well. Thus using these results, the LQG controller was designed, and simulations

## TROLLING MOTOR TESTS

While the wing-sail was still under construction at Cris Hawkins Consulting in Santa Rosa, the system identification and controller tasks had already been completed. At this point, in order to test out the controllers, a MinKota electric trolling motor was used to simulate the presence of the wing-sail and wind. This was done by mounting the trolling motor at the sailboat C.G., and turning the trolling motor such that its direction of thrust was canted off the centerline by more than 40 degrees.

Since the dynamics of the catamaran are greatly affected both by the velocity through the water, as well as the displacement weight of the hulls, the Atlantis was ballasted with an additional 75 kg. of lead ballast to bring the all-up weight of the boat to the same as weight as it would have had with the wing-sail installed. Also, in order to test the controllers at various speeds, the MinKota trolling motor was run with 12, 24, and 36 volts at approximately 65 amps. This changes the speed of the boat through the water, simulating changes in wind velocity. In order to simulate changes in wing direction, the MinKota trolling motor was turned through various angles while the controller was regulating the path to a line.

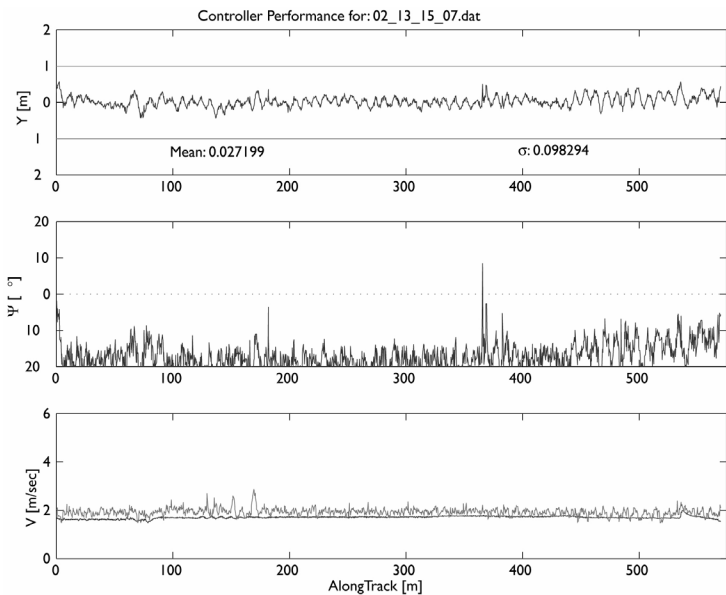


Figure 13: Close up of trolling motor data

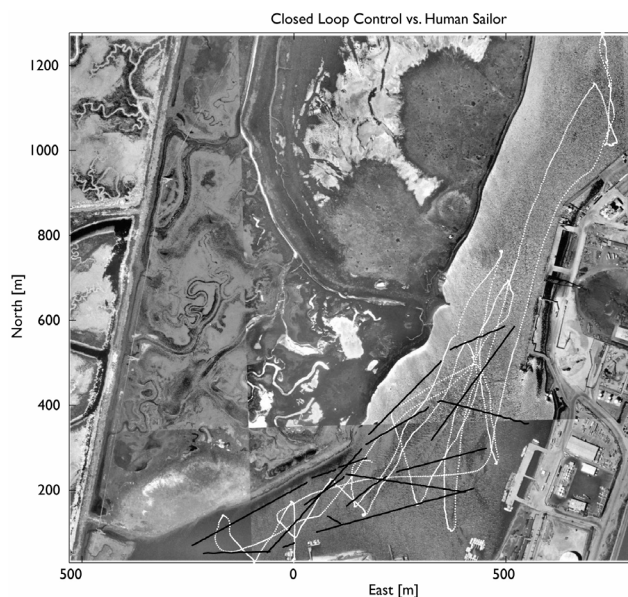


Figure 14: Satellite photograph of harbor

shown in Figure 12 reveals that the mean was less than 10 cm., and the standard deviation was less than 3 cm.

Of interest, the azimuth shows a -20 degree bias for most of the path length of the run pictured in Figure 13, which is due to current. This can be verified by looking at the velocity plot at the bottom of Figure 13, where the top line is the hullspeed sensor, and the smooth lower line is GPS velocity. The difference in these two is current, and it can be seen in spite of the high frequency noise of the hullspeed sensor.

In Figure 11, the Atlantis with the trolling motor can be seen. The trolling motor is at the center of the boat, and the lead batteries provide the ballast. As pictured, the boat was run unmanned, with the GNC computer providing all navigation. Of note is the fact that the anemometer is located at the front wooden crossbeam. This is only a temporary location, and moving the sensor's physical location is very easy due to the bus architecture employed on the Atlantis.

Figure 12 shows a typical autonomous pass while under computer control. Note that the computer regulates the path to the line, but that the turn is performed open loop with a feed-forward command. To the scale pictured in Figure 12, the recorded position data shows very little variation. This is in spite of the fact that the currents were changing, and the wing and waves were all injecting disturbances into the system. In Figure 13, a close look at the errors in the first part of the path

## WINGSAIL TESTS

In order to validate the performance of the controllers and all up system, closed loop control experiments were performed in Redwood city harbor, California, on 27-Jan.-2001. These tests were intended to verify that the closed loop controllers were capable of precise line following with the increased disturbances due to the wing-sail propulsion. No modifications were made to the controller design, and the tests were run on a day with approximately 12 knots (or 6 m/s) of wind, with gusts up to the 20 knot (or 10 m/s) range.

Qualitatively, the wing-sail performed even better than anticipated. With the tail centered, there was no tendency for the Atlantis to heel what-so-ever, and the absence of aeroelastic instability (sail luffing) made the entire event very quiet. Upon turning the trailing edge of the tail in the direction of desired travel, the Atlantis smoothly accelerated to speed and quietly continued on her course. Even large gusts simply caused the Atlantis' wing to quickly stall and, with only a slight shudder, reposition to a new angle of attack (as evidenced by the tell-tails on the wing surface).

More impressive was the ability to sail pointed very high into the wind. Upon analyzing the data, it was demonstrated that the Atlantis was capable of sailing to within 25 degrees of the true wind direction. At one point, a conventional sailboat came about behind the Atlantis and started luffing a full 15 degrees off the wind from where the Atlantis was making headway. This is clearly a result of the improved aerodynamics of the rigid wing, and a vindication of the self-trimming arrangement over a conventional sail. While further experimental studies are required to quantitatively measure the performance increase of the wing-sail, current results indicate very promising discoveries ahead.

Figure 14 shows a satellite picture of the harbor where both the trolling motor and wing-sail tests were performed. The white dots are from a previous year, when the Atlantis was conventionally sailed with a sloop rig, and was sailed by a human pilot. The black dots indicate the various closed loop control passes from the recent tests. Note that the white trace has a curving, "human," look to it, whereas the black trace looks like a ruler was placed upon the photograph and a line drawn. Qualitatively, the computer control simply looks unnatural.

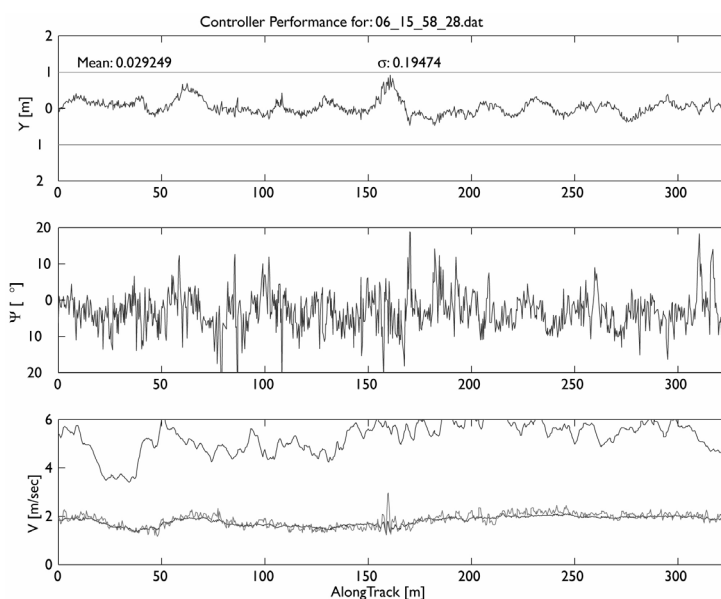


Figure 16: Sailing path errors

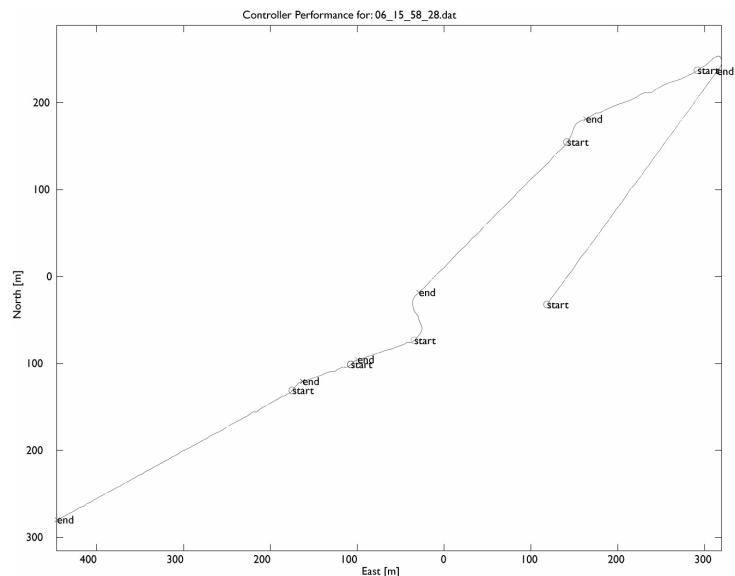


Figure 15: Overhead trace of sailing passes

Figure 15 is, once again, a closer look at a bird's eye view of a set of computer controlled traces. The control system regulated about the lines in between each "start" and "end" pair, and the turns in between were performed open-loop in a feed-forward sense. Figure 16 presents a close-up of the first path of the regulated control, and looks at the cross-track error, azimuth error, and velocities. Note that the dark line in the top of the velocity graph is the wind speed, and can be seen to vary well over 50% of nominal.



The mean of the cross-track error is less than 30 cm., and the standard deviation is less than 3 cm., note that this is the Sailboat Technical Error (STE, the sailing analog of Flight Technical Error). Previous characterization of the coast-guard differential GPS receiver indicated that the Navigation Sensor Error (NSE) is approximately 36 cm., thus the Total System Error (TSE) is less than 1 meter.

Figure 17 presents the aggregate of all controlled sailing runs overlaid one upon the other. Along with bounds indicating +/- 1 meter. The differences in path length have to do with the location of the shore, and the desire not to run aground. Depending on the path chosen, longer or shorter distances were traversed. At no time does the controlled performance of the system exceed the one meter bound.

As a basis for comparison, the specifications for the top-of-the-line AutoHelm autopilot indicate a cross-track accuracy of 0.05 nautical miles, or 92.6 meters.

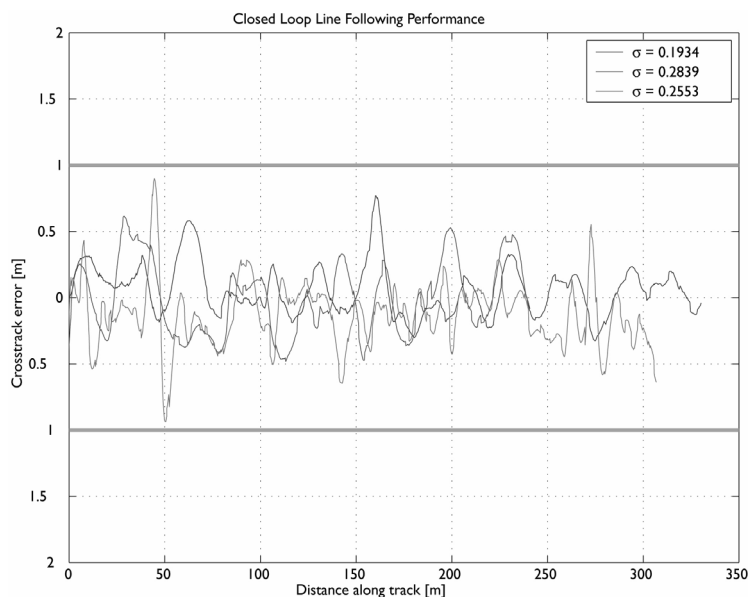


Figure 17: Overlay of all sailing paths

## CONCLUSIONS

It has been demonstrated that with the combined advances in GPS technology, and the advent of low-cost sensors, an unmanned sailboat can be built that can navigate with unprecedented levels of accuracy. By utilizing a novel wing-sail propulsion system, the difficulties of actuating a sail have been overcome, and high authority control can be realized. A demonstrated Sailboat Technical Error (STE) in line following less than 0.3 meters was achieved, in challenging conditions. Combined with a Navigation Sensor Error (NSE) of 0.36 meter, this yields a Total System Error (TSE) of less than 1 meter.

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