UNIVERSITY OF CALIFORNIA

SANTA CRUZ

PERFORMANCE ANALYSIS OF A HORIZONTAL SEPARATION ASSURANCE ALGORITHM FOR SHORT-RANGE CONFLICT DETECTION AND RESOLUTION

A thesis submitted in partial satisfaction
of the requirements for the degree of

MASTER OF SCIENCE

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by

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Chapter One: Introduction

The concept of distributed air-to-air separation assurance envisions aircraft detecting and resolving conflicts without the need for a ground-based air traffic service provider. But without guidance from a ground-based system, a reliable method for distributed separation assurance needs to be developed. The Advanced Airspace Concept (AAC) [1] describes a more automated air traffic control system in which automated separation assurance plays a central role. With the idea of an automated and distributed method, an algorithm for horizontal separation assurance has been developed. This thesis presents the results from measuring this algorithm's performance in current and higher air traffic densities.

The separation assurance algorithm will detect and resolve conflicts for a time to first Loss of Separation (LOS) at three minutes or less. Current automated resolution algorithms, including the AAC auto-resolution algorithm [2] implemented in ACES and CTAS, assume instantaneous heading changes. This assumption can result in an unintentional LOS and so a trajectory engine checks the simplified resolution maneuver using realistic turn dynamics. If separation is lost, a new heading change is requested. For a close-range conflict greatly affected by turn dynamics, this will result in several iterations between the algorithm and the trajectory engine. Erzberger [3] has derived a new algorithm to create a resolution plan by analyzing the turn dynamics for both aircraft.

A simulation is used to measure the performance of the horizontal separation assurance algorithm developed in [1]. While the simulation is running, certain events are recorded, counted, and classified into various types. Plotting measured data, drawing individual conflicts, and re-creating the simulation as an animation are used in this study to visualize events and measure the performance of the algorithm.

In the simulation, pair-wise conflicts are either detected at the time to first LOS parameter, or they are detected at an earlier time as the result of a secondary conflict. A pair-wise conflict resolution may result in a secondary conflict, specifically, when a maneuver is complete, either aircraft may become involved in a conflict with a third aircraft. Some secondary conflicts occur as an immediate result of the maneuver, possibly causing it to be detected at a time less than the time to first LOS, and potentially resulting in an unavoidable LOS.

At present, the algorithm is designed to resolve pair wise conflicts without considering any possible secondary conflicts. However, the simulation will attempt to resolve any secondary conflicts once the initial conflict has been resolved. However, the data from the simulation shows a conclusive need for coordination between aircraft to avoid secondary conflicts. Communication is a necessary step; first it ensures both aircraft involved in a conflict choose the same maneuver and second, it informs other aircraft of their intentions. By including other aircraft's intentions and turn dynamics, future work will extend this algorithm to resolve secondary conflicts.

Chapter Two: Analytical Background

The following is a summary of the analytical formulation presented by Erzberger [1] for horizontal separation assurance between two aircraft. Understanding the derivation for this pair-wise separation assurance algorithm is a key step in the implementation and analysis process. Throughout the thesis, winds are assumed negligible, thus air speed and heading is equivalent to ground speed and heading. A table of nomenclature for use in the rest of this document is now presented:

V_A, V_B	The ground speed of both aircraft, written here as velocity.
X_{B0}, Y_{B0}	Relative initial position of aircraft B after transformation into relative coordinate system.
ψ_A, ψ_B	Relative heading of both aircraft after transformation. ψ_A is initially at zero degrees heading north.
D_{req}	The required minimum separation between any two aircraft.
R_A, R_B	Turn Radius for both aircraft.
$\Delta\psi_A$, $\Delta\psi_B$	Turn Angle for both aircraft.
ϕ	Bank Angle is constant for both aircraft.
P_{AT}, P_{BT}	Position of the aircraft when the turn is complete.
$t(\Delta\psi_A)$, $t(\Delta\psi_B)$	Time for each aircraft to complete a turn.
D_{AB}	Distance between A and B at the end of the turn.
$t_{s\min}$	Time to minimum separation in the straight-line segment beginning at the end of the turn.
$d_{s\min}$	Minimum Distance achieved in the straight-line segment beginning at the end of the turn.
D_m	Minimum distance achieved during the entire maneuver. This may occur during the turn, at the end of the turn, or in the straight-line segment beginning at the end of the turn.

Table 2.1: Nomenclature table [1].

Aircraft Dynamics:

$$X_A = X_A' + V_A \sin(\psi_A) \qquad (2.1)$$

$Y_A = Y_A' + V_A \cos(\psi_A)$	(2.2)

Transformation of Coordinate System:

$\begin{bmatrix} X_{B0} \\ Y_{B0} \end{bmatrix} = \begin{bmatrix} \cos\phi_{A0}' & -\sin\phi_{A0}' \\ \sin\phi_{A0}' & \cos\phi_{A0}' \end{bmatrix} \begin{bmatrix} x_{B0}' - x_{A0}' \\ y_{B0}' - y_{A0}' \end{bmatrix}$ $\psi_B = \psi_{B0}' - \psi_{A0}'$	(2.3)

Turn Dynamics:

$R_A = \dfrac{V_A^2}{g \tan\phi}, \quad R_B = \dfrac{V_B^2}{g \tan\phi}$	(2.4)
$\Delta\psi_A = \dfrac{t \cdot g\tan\phi}{V_A}, \quad \Delta\psi_B = \dfrac{t \cdot g\tan\phi}{V_B}$	(2.5)
$\lvert\Delta\psi_B\rvert = \dfrac{V_A}{V_B}\lvert\Delta\psi_A\rvert$	(2.6)
$t(\Delta\psi_A) = t(\Delta\psi_B) = \dfrac{\lvert\Delta\psi_A\rvert * V_A}{g * \tan\phi}$	(2.7)
$P_{BT} = [x_A, y_A] = [x_{B0} + R_B \cdot sgn(\Delta\psi_B) \cdot (\cos(\psi_B) - \cos(\psi_B + \Delta\psi_B)),$ $y_{B0} + R_B \cdot sgn(\Delta\psi_B) \cdot (-\sin(\psi_B) + \sin(\psi_B + \Delta\psi_B))]$	(2.8)
$P_{AT} = [x_A, y_A] = R_A \cdot sgn(\Delta\psi_A) * [1 - \cos\Delta\psi_A, \sin\Delta\psi_A]$	(2.9)
$P_{BT} = [x_B, y_B] = [x_{B0} + t(\Delta\psi_A) \cdot V_B \cdot \sin(\Delta\psi_B),$ $y_{B0} + t(\Delta\psi_A) \cdot V_B \cdot \cos(\Delta\psi_B)]$	(2.10)
$P_{AT} = [x_A, y_A] = [0, \quad y_{A0} + t(\Delta\psi_B) \cdot V_A]$	(2.11)

$D_{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}.$	(2.12)
$t_{s\min} = \frac{-(\Delta x V_{Rx} + \Delta y V_{Ry})}{\lvert V_R \rvert^2} \geq 0$	(2.13)
$d_{smin} = \sqrt{[\Delta_x + V_{Rx} * t_{smin}]^2 + [\Delta_y + V_{Ry} * t_{smin}]^2}$	(2.14)

Table 2.2: Equations used while calculating separation assurance maneuvers [1].

These equations will be used to find the minimum distance between the two aircraft (A/C) during a maneuver. A single aircraft maneuver has one A/C turn for a specified angle while the other aircraft flies straight. A cooperative maneuver is when both aircraft will turn for the same specified amount of time followed by straight-line flight. There are eight possible pair-wise maneuvers to check, four are cooperative and four are single aircraft. The goal is to maintain separation between the two aircraft. Separation is maintained if the minimum distance is greater than D_{req}.

Figure 2.1 illustrates the scenario to be resolved. The two aircraft involved are known as A and B, where the host aircraft assumes itself to be A, and the intruder aircraft is known as B. A coordinate and heading transformation is used to show the conflict scenario from the host's perspective by placing A at the origin with a heading of zero degrees. Aircraft B's position and heading, but not speed, are transformed into the relative coordinate system.

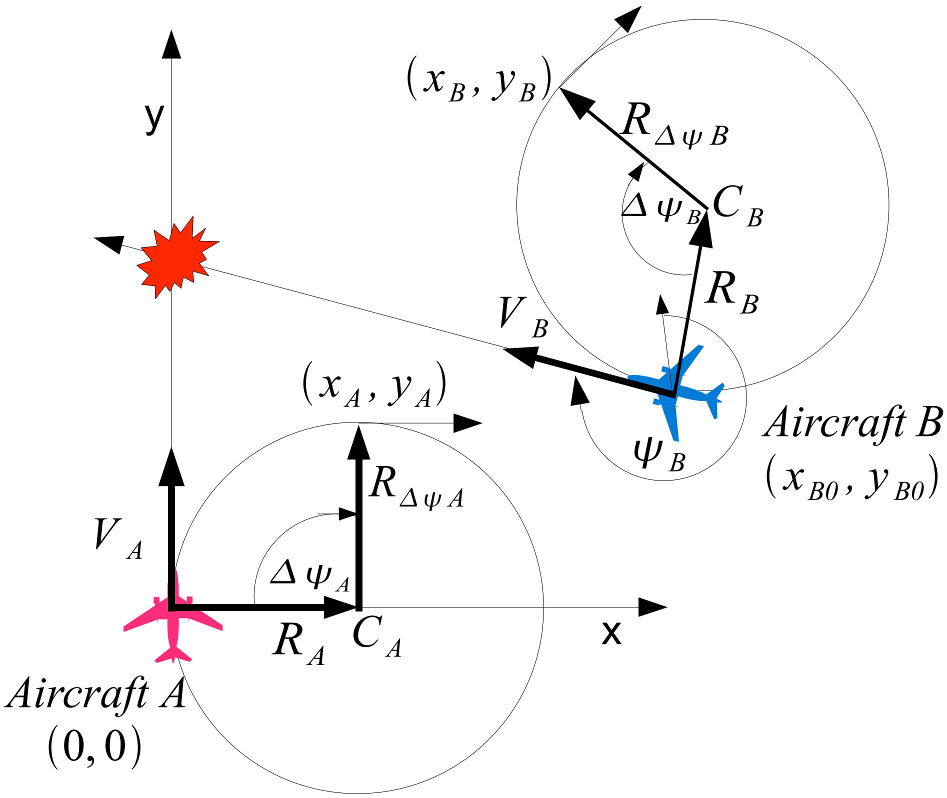

Figure 2.1: Figure 1, from [1], shows a possible conflict scenario translated into A/C A's relative coordinate system.

Generating a Separation Graph

The equations above calculate the minimum distance for a specified turn angle. An algorithm will iterate over a finite set of turn angles within a range and record the minimum distances for each one. This algorithm produces a separation graph showing the minimum separation achieved during and after all such turn angles. The following example shows two single aircraft maneuvers, where aircraft A turns left or right and aircraft B flies straight. Figure 2.2 is the detected conflict scenario. Figure 2.3 is distance at the end of aircraft A's left or right turn of 55°.

Figure 2.4 shows a plot of the distance for every turn angle from -180° to 180° by using Equation 2.12. By using Equation 2.7, the time to complete the maneuver for each turn angle can be calculated. Figure 2.5 is the example of the minimum distance once aircraft A has finished turning and continues to fly in a straight line. After this point the distance between the two aircraft diverges. Figure 2.6 shows the minimum distance during the straight-line flight after each turn angle by using Equation 2.14. The time to reach this minimum distance is calculated in Equation 2.13 and it may vary from zero seconds to several minutes depending on each aircraft's position at the end of the turn. Figure 2.7 combines Figures 2.4 and 2.6 onto one plot to show the relationship between the distance at the end of each turn and the distance during the straight-line flight after each turn. Finally, the total time to minimum separation is shown in Figure 2.8. The total time to minimum separation is the sum of Equations 2.7 and 2.13.

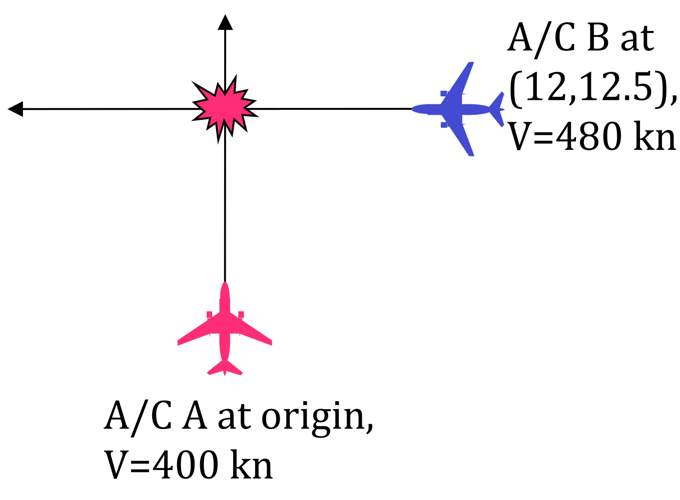

Figure 2.2: A Pair-wise conflict scenario. A/C B's relative heading is -90°.

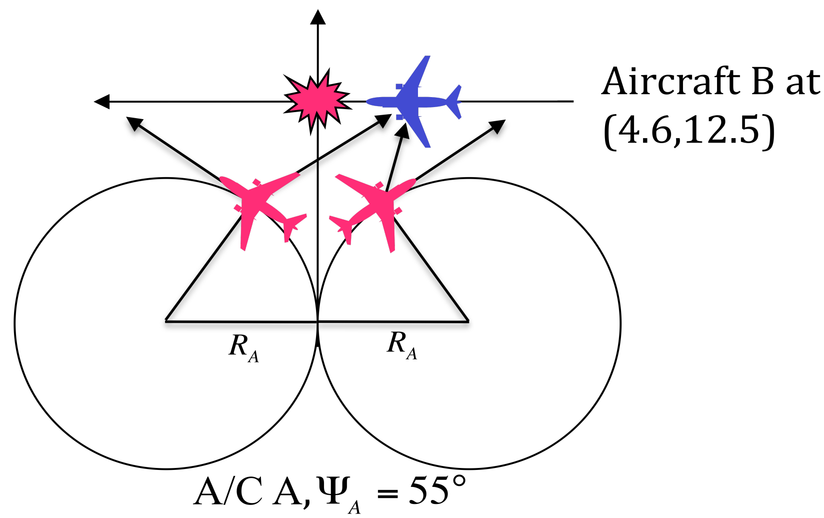

Figure 2.3: Distance between A/C's A and B after A turns left or right for 55° at a bank angle of 20°. B is at the same position because the time for both turns is the same. Note that A is no longer at (0,0), this is only used as the initial position and heading.

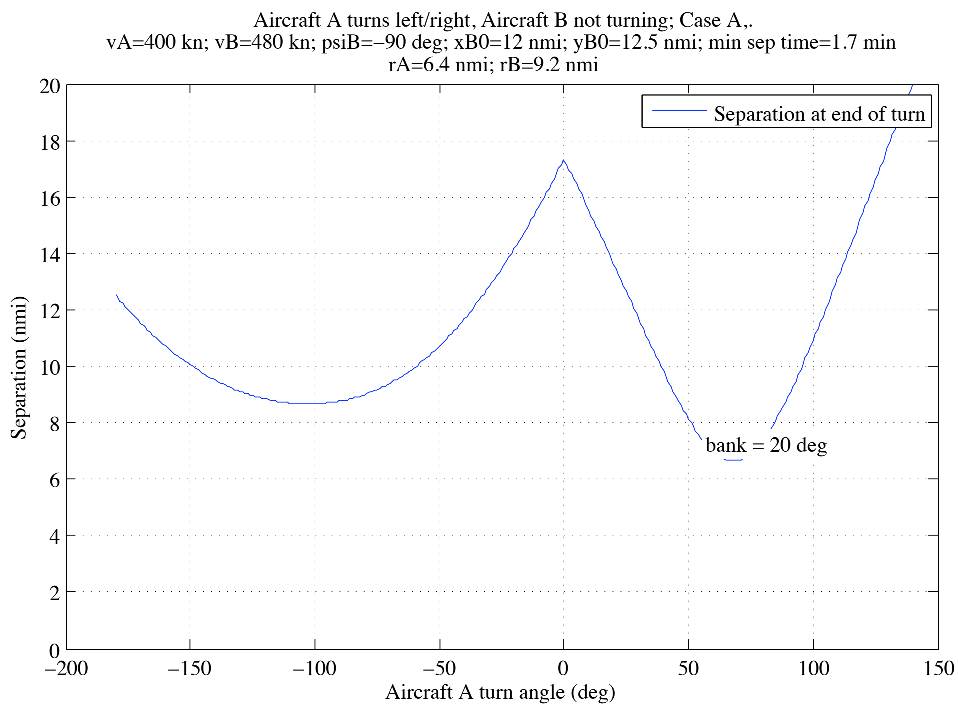

Figure 2.4: Distance at the end of every turn angle using Eq. 2.12. Right turns are positive angles and left turns are negative angles. Generated by [4]. As the turn angle increases, so does the time from the initial conflict position.

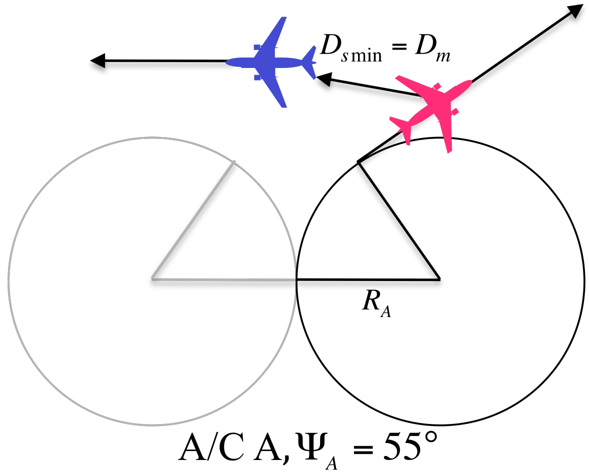

Figure 2.5: Minimum Distance on the straight-line after A/C A's turn is complete. This occurs shortly after the end of the turn.

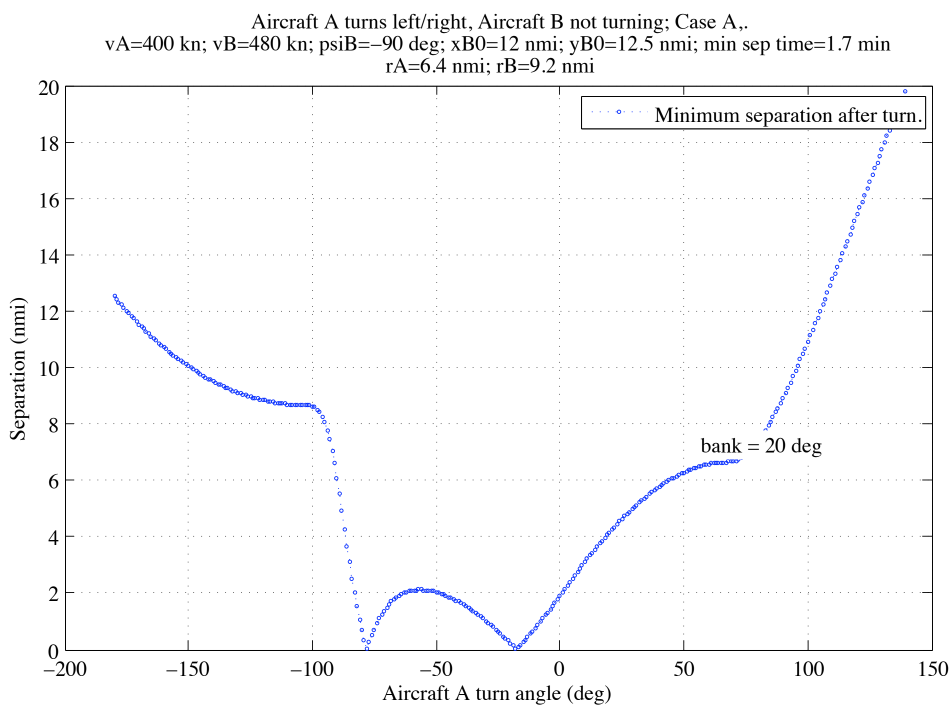

Figure 2.6: Minimum distance during the straight-line flight at each turn angle using Eq. 2.14. Right turns are positive angles and left turns are negative angles. Generated by [4]. The time to the minimum distance after the turn may vary anywhere zero seconds up to several minutes.

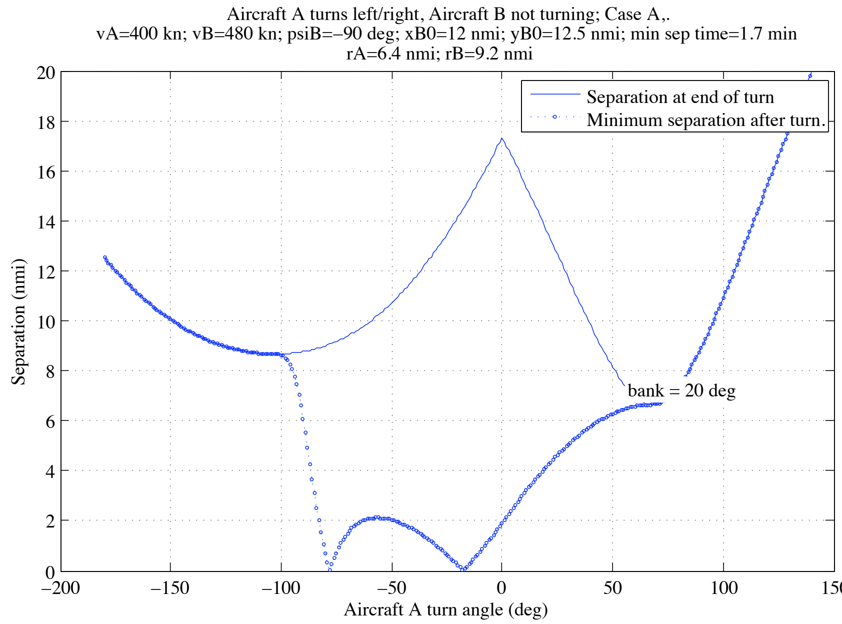

Figure 2.7: Plotting Figs. 2.4 and 2.6 on the same graph. Generated by [4]. At a right turn of 55°, the distance at the end of the turn is slightly larger than the distance in the straight line after the turn.

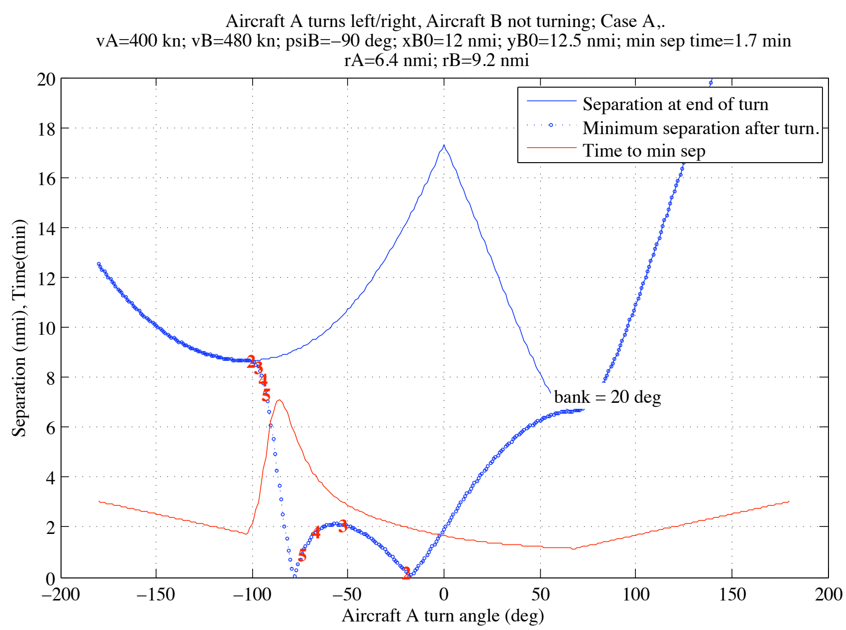

Figure 2.8: Plotting time as well. Y-axis is for minutes and nautical miles. Generated by [4]. The total time to complete a 55° right turn maneuver is just above one minute.

Extracting a Maneuver from the Separation Graph

The algorithm to extract a maneuver from a separation graph will achieve separation in the minimum amount of time. If loss of separation (LOS) cannot be avoided, then it will maximize the minimum distance. Horizontal separation is achieved when the minimum distance between the two aircraft is greater than D_{req} at all points during the maneuver. The algorithm will iterate through the range of turn angles for each maneuver recording the minimum separation at each one. This iteration results in plotting the entire path of the maneuver for both aircraft. It will search for the turn angle where the minimum distance in the straight-line segment is greater than D_{req}. Once this angle is found, it searches the remaining turn angles for the minimum total time to maintain separation. Figure 2.9 shows the turn angle where separation is achieved in the minimum total time. Figure 2.10 shows this maneuver as generated by the algorithm. Aircraft A turns right for 68° at a bank angle of 20°, while aircraft B flies straight. It includes the minimum separation of both aircraft along their original trajectories. It also shows the turn radius of aircraft A, Equation 2.4, the position of aircraft A after the turn, Equation 2.9, the position of aircraft B after the turn, Equation 2.10, and the minimum separation, equation 2.14. The maneuver is complete at the point of minimum separation.

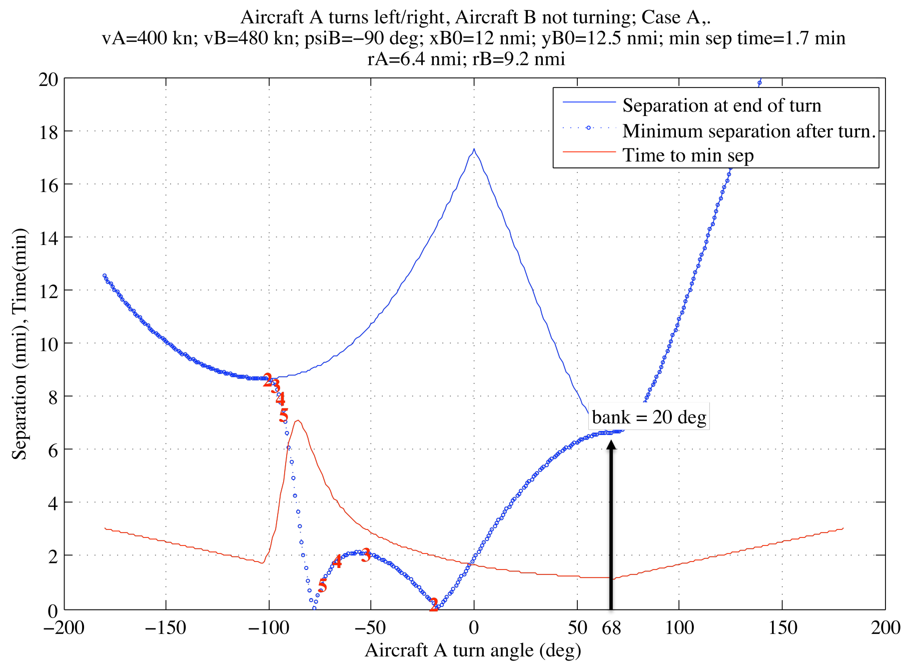

Figure 2.9: Separation is assured in the minimum amount of time at a turn angle of 55° for aircraft A turning right and aircraft B flying straight. Generated by [4].

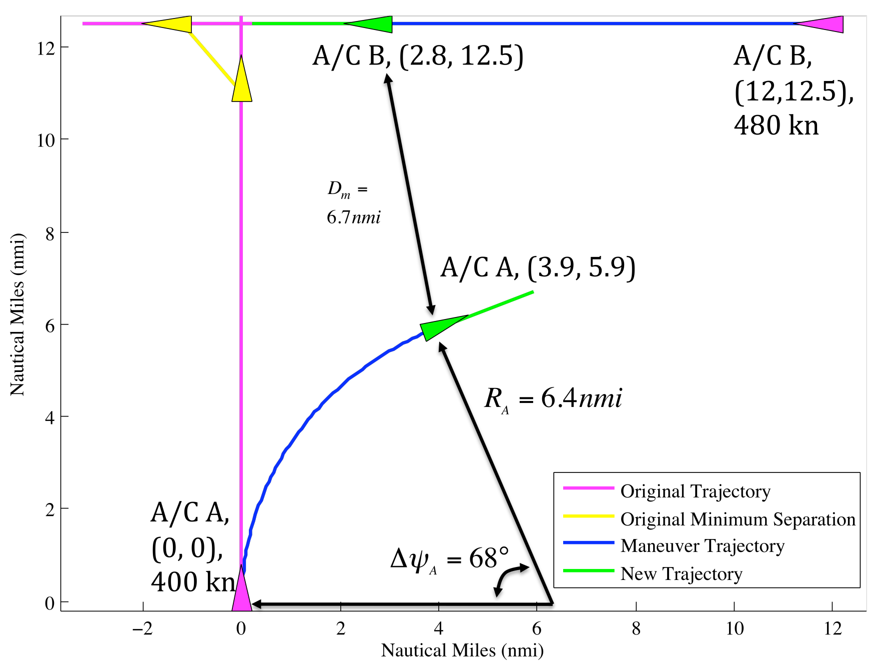

Figure 2.10: A/C A performs a right turn to maintain separation.

Classifying all Eight Possible Maneuvers

Once all eight maneuvers are extracted from the separation graphs, the maneuvers can be classified, sorted, and listed in a table. For the purpose of this thesis, there are two classifications, one where separation is maintained and one where separation is lost. Table 2.3 lists the eight maneuvers for the conflict scenario from Figure 2.2. The maneuver from Figure 2.10 takes 68.4 seconds to turn 68° at a bank angle of 20° with a minimum separation of 6.7 nmi.

Table 2.3 contains one case that requires additional explanation. The ‘R/L’ case suggests a turn angle of zero for both aircraft with a separation of 1.9 nmi. Figure 2.11 shows that the minimum separation drops below 1.9 nmi at a turn angle of 75° for aircraft A and 63° for aircraft B. Figure 2.12 shows the trajectories for both aircraft as generated by the algorithm. For all turn angles up to 75°, the minimum separation in the straight-line segment after the turn is below 1.9 nmi. Since a LOS is unavoidable, the algorithm chooses to maximize the minimum separation, resulting in both aircraft flying straight. Since there are several other maneuvers that maintain separation, this one will not be chosen.

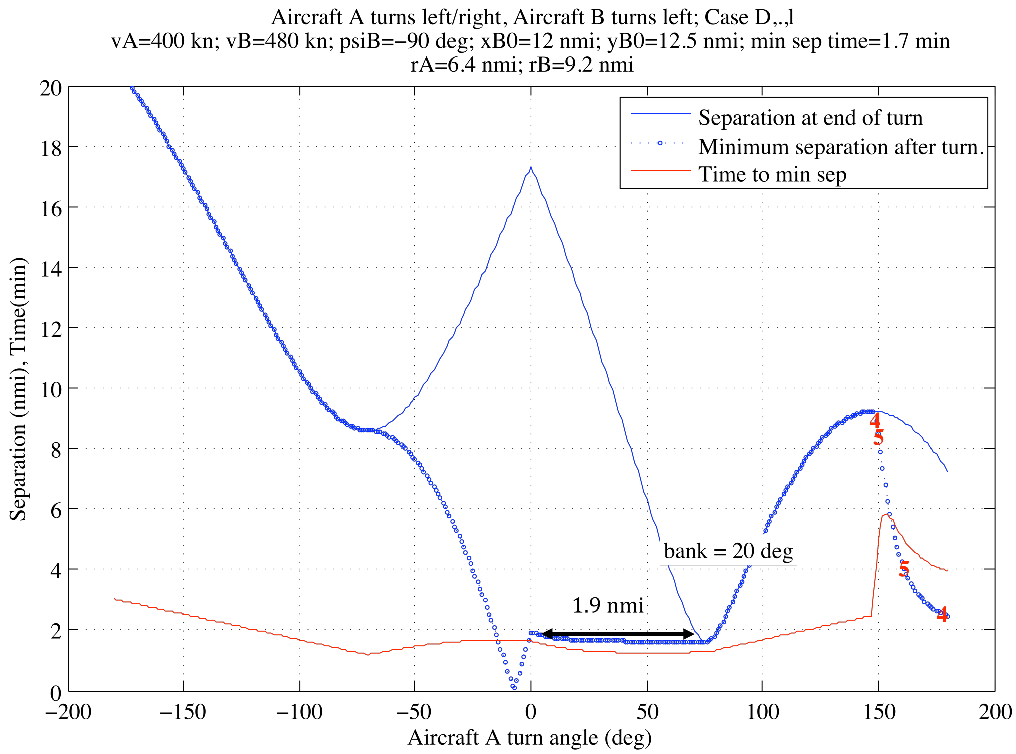

Figure 2.11: LOS is unavoidable and the separation during a turn drops below the original conflict separation of 1.9 nmi. Generated by [4].

Type	$T(\psi_A)$	$T_{s\min}$	$D_{s\min}$	D_{Turn}	ψ_A	ψ_B
(A/B)	(seconds)	(seconds)	(nmi)	(nmi)	(deg)	(deg)
L/L	71.4	0.0	8.6	8.6	-71	-59
L/R	49.3	0.0	12.8	12.8	-49	41
R/R	58.3	0.0	10.4	10.4	58	48
R/L	0.0	99.2	1.92	17.3	0	0
S/L	94.1	0.0	6.0	3.9	0	-78
S/R	86.9	0.0	9.8	9.8	0	72
L/S	103.6	0.0	8.7	8.7	-103	0
R/S	68.4	0.2	6.7	6.7	68	0

Table 2.3: Summary of all eight maneuvers.

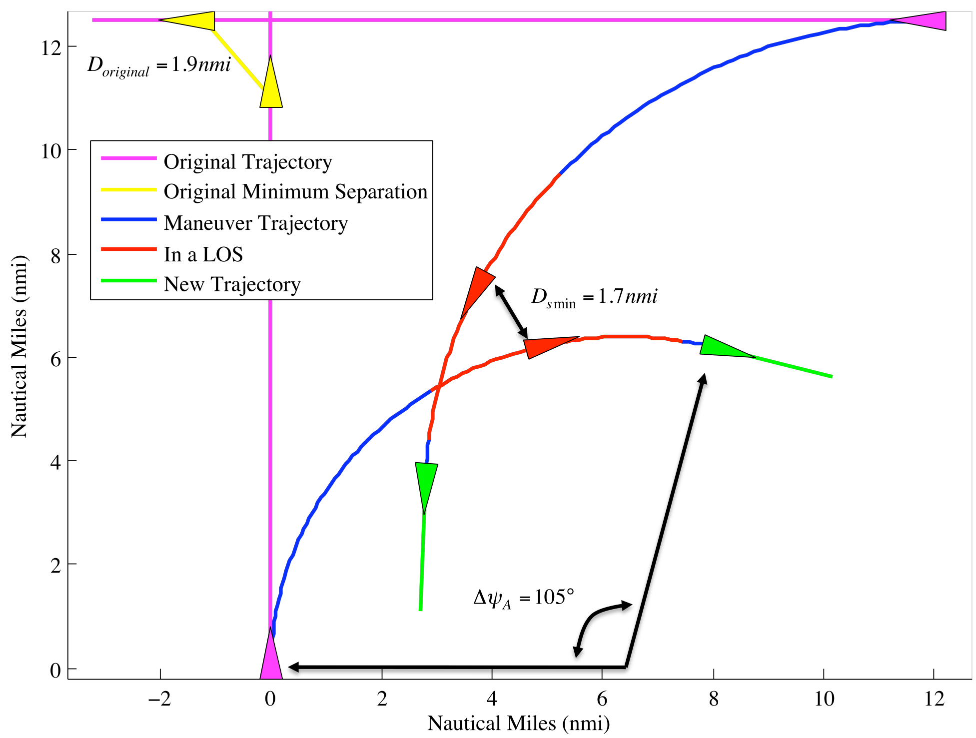

Figure 2.12: Illustrating the 'R/L' maneuver by drawing a large turn angle for both aircraft. A/C A turns 105° and A/C B turns -88°.

Significance of Turn Dynamics

Conflicts detected at two minutes or less time to first LOS are greatly affected by the turn radius and bank angle of each aircraft. At this range, changing speeds will generally have little affect for resolving a conflict [1]. However, the turn radius is greatly affected by the speed of an aircraft. Equation 2.4 shows the turn radius increases by the square of the speed. Any algorithm assuming instantaneous heading change for aircraft at this range will produce an in-adequate resolution maneuver. The Automated Airspace Concept (AAC) contains a horizontal resolution algorithm that generates a maneuver assuming instantaneous heading change. The maneuver is passed to a trajectory engine to check the maneuver. If the resolution fails to maintain separation, an iterative process is used to revise the trajectory and check it again. However at such a close range, this iterative process is inappropriate. Figure 2.13 illustrates why this instantaneous heading change algorithm will not work.

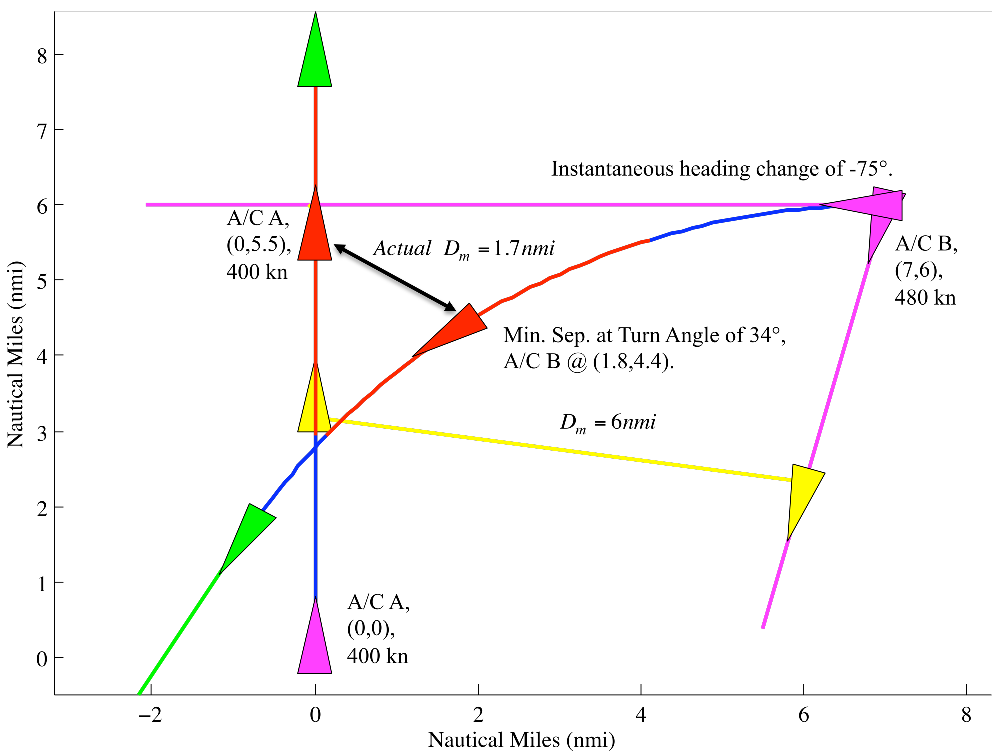

Figure 2.13: An instantaneous heading change algorithm claims separation is maintained, but when the maneuver is generated using turn dynamics, a LOS occurs. A larger turn angle will not make any difference in the minimum separation.

Maneuver Selection Process

The algorithm uses the given equations to generate each aircraft's trajectory for all eight possible maneuvers. It creates the separation graph for each maneuver. It extracts a maneuver that achieves separation in the minimum amount of time. If LOS is unavoidable, it maximizes the minimum separation. Next it organizes the eight maneuvers for selection. Cooperative maneuvers are understood to be faster, however single aircraft maneuvers can achieve separation, so it checks the single aircraft maneuvers first. If none of the single aircraft maneuvers maintain separation, it checks the cooperative maneuvers. If none of the cooperative maneuvers maintain separation, it chooses the maneuver that maximizes the minimum separation. This process is in the following flow chart Figure 2.14.

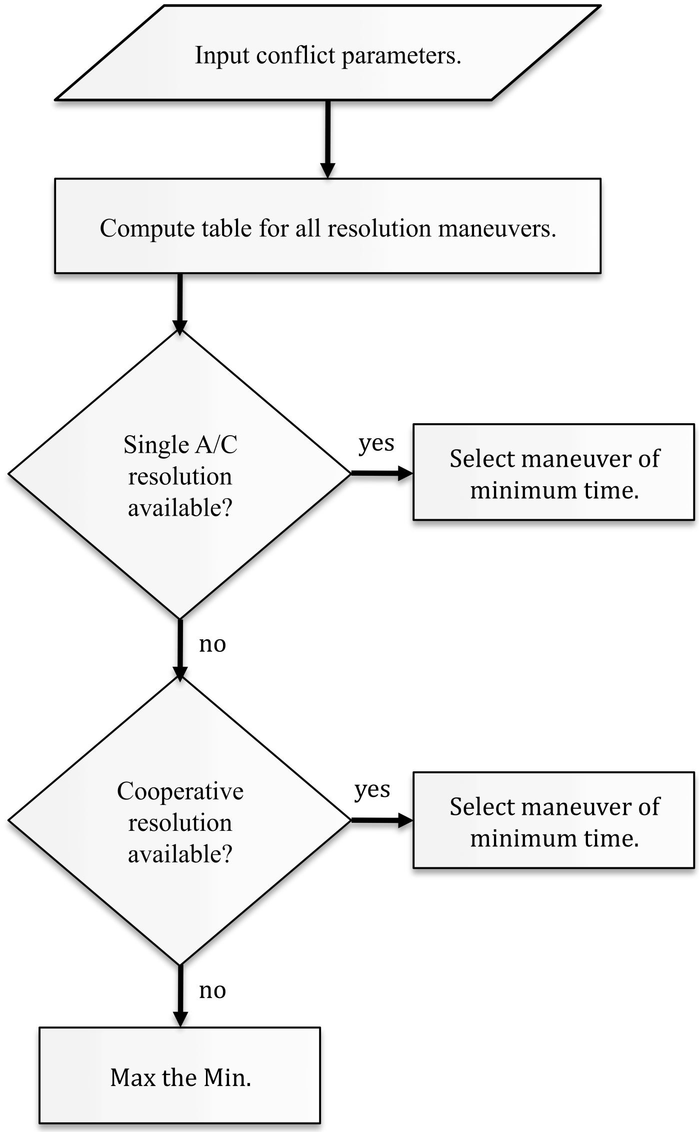

Figure 2.14: Selection algorithm flow chart. If a single A/C maneuver is available it will use it, else if a cooperative maneuver maintains separation it will select it, otherwise it will maximize the minimum separation.

Chapter Three: Simulation of a Pair-Wise Horizontal Conflict Detection and Resolution Algorithm

The goal of the simulation is to assess the performance of a pair-wise conflict resolution process in the presence of more than two aircraft. Having a variable amount of aircraft interact in one area will help measure the effects of this pair-wise separation assurance algorithm. The same dynamics used by the resolution algorithm are used to model the dynamics of aircraft. Aircraft are introduced one at a time into a square and single altitude airspace until a specified density is achieved. A single altitude is sufficient to investigate the performance of a horizontal resolution algorithm. Each aircraft flies on a straight path only altered by the separation assurance algorithm, with no attempt to return to original heading once a maneuver is performed. When an aircraft reaches the edge of the airspace, a new aircraft replaces it. Any new aircraft is introduced at a random position along the edge with new random values for speed and heading. There are many parameters available to this simulation, some are randomly generated during run time, some are set at the beginning, and some are constant throughout all runs. The following table lists the key parameters.

Detection Range	Aircraft can detect each other if within this range. The default value is 50 nautical miles.
Time to First LOS	Conflicts are detected at this look-ahead time usually specified in minutes at the beginning of every simulation.
Delay Before Maneuver	By default this is zero. The simulation may assume all maneuvers will be performed at some time delay after detection unless LOS will occur as a direct result of this

	delay. In this case it will perform an immediate maneuver.
Required Separation	The required separation between any two aircraft is five nautical miles. If two aircraft are closer than this distance, it is recorded as a loss of separation (LOS). The pair-wise separation assurance algorithm requires six nautical miles to create a buffer handling discrete calculation errors in aircraft position.
Aircraft Density	The aircraft density is specified at the beginning of a simulation and it is maintained throughout the entire run. The default density is set to 25 aircraft.
Simulation Run Time	The run time is set at the beginning of each simulation. This is 50 hours by default.
Bank Angle ϕ	Set at a constant of 20° for all simulations.
Range of Speed	Each aircraft is randomly assigned a constant speed when it is initialized. This speed range is 300 knots to 500 knots.
Range of Heading	Each aircraft is assigned a heading appropriate for the border it will enter from. For instance, if it is entering along the west border, then it should be heading east, leaving a heading range from greater than zero degrees to less than 180 degrees.

Table 3.4: Simulation parameters. Some parameters are constant and some can be changed at the beginning of each simulation.

The following figures are snapshots from an animation of the simulation. The initial state of the simulation is shown in Figure 3.15. Figure 3.16 shows each aircraft is represented as a circle of radius 2.5 nmi, showing the required separation and the aircraft's current position. Once the simulation begins, one aircraft enters the airspace every thirty seconds until the required density is reached. Figure 3.17 shows an aircraft is removed once it reaches the border. Figure 3.18 shows a conflict detection with a time to first LOS of one minute. Figure 3.19 shows that the separation assurance algorithm generates a resolution maneuver, in this case both aircraft are maneuvering. Once a maneuver is complete, both aircraft continue to fly straight in Figure 3.20.

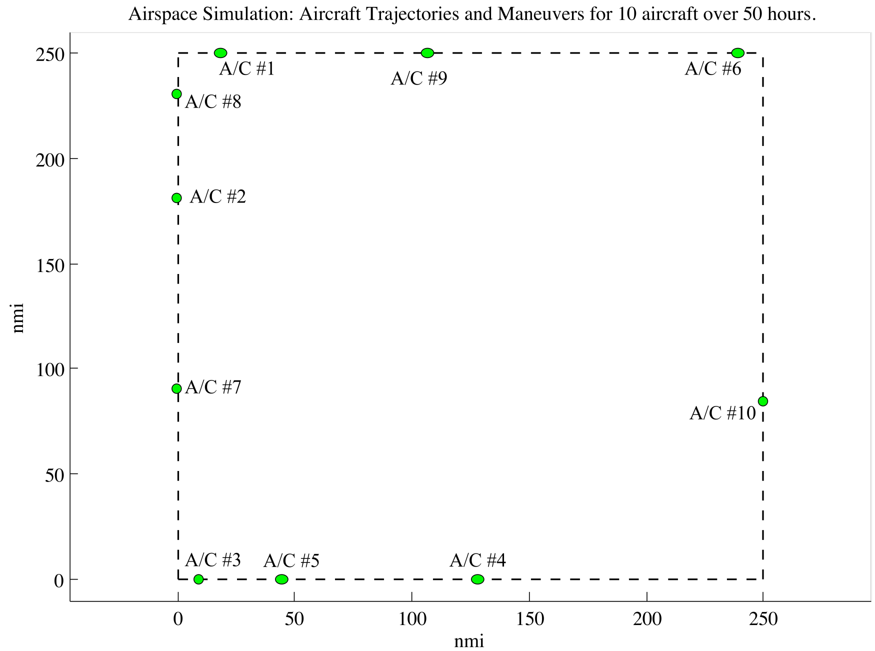

Figure 3.15: Initial configuration for a simulation at a density of 10 aircraft. All aircraft are waiting along the edge of the airspace.

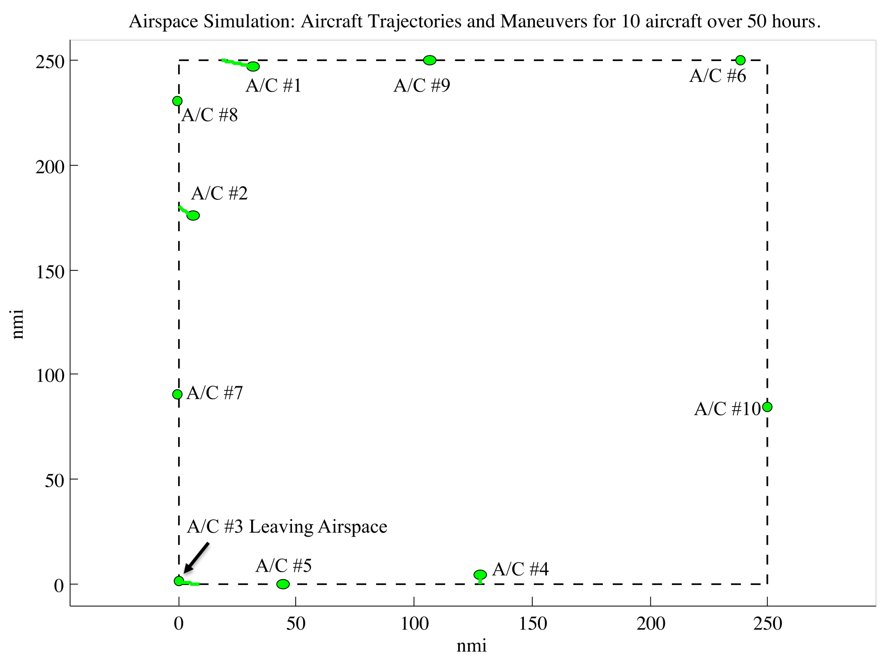

Figure 3.16: One aircraft enters the airspace every 30 seconds. Here A/C's 1 through 4 have entered the airspace. Also, A/C 3 is about to reach a border.

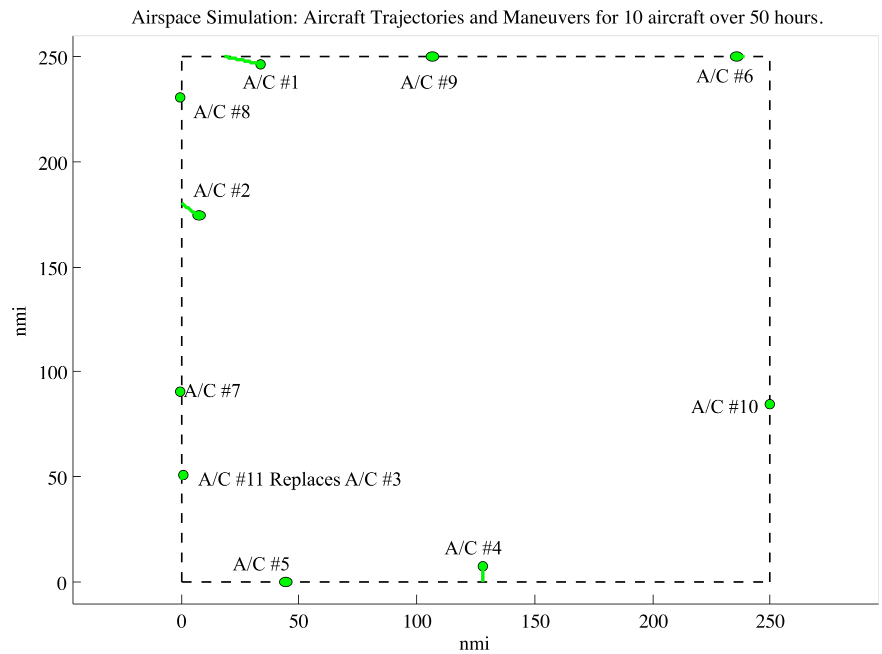

Figure 3.17: A/C 3 has reached the border and is replaced by a new aircraft 11.

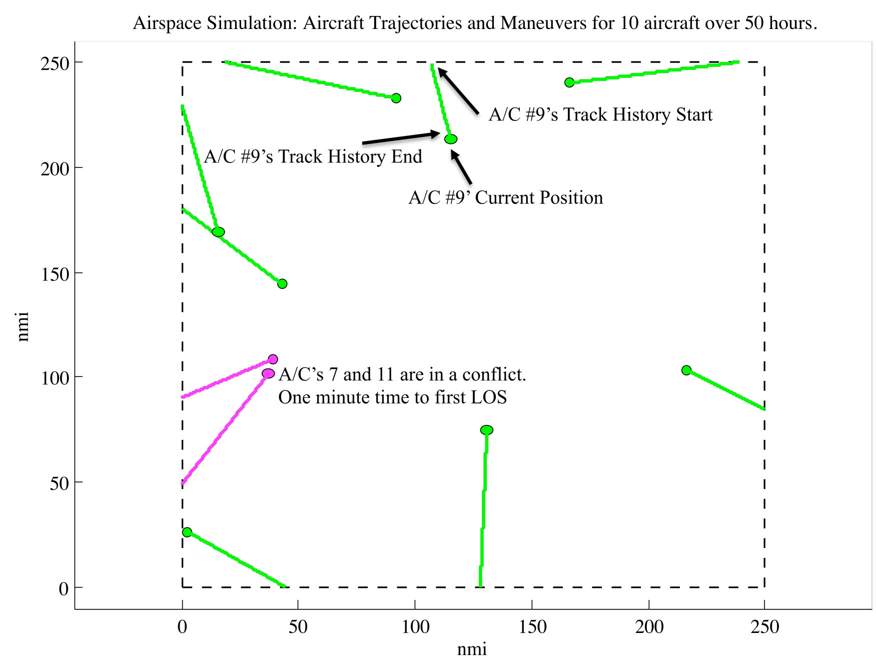

Figure 3.18: The first conflict is detected at one minute Time to First LOS. The line behind each aircraft is its track history since it entered the airspace.

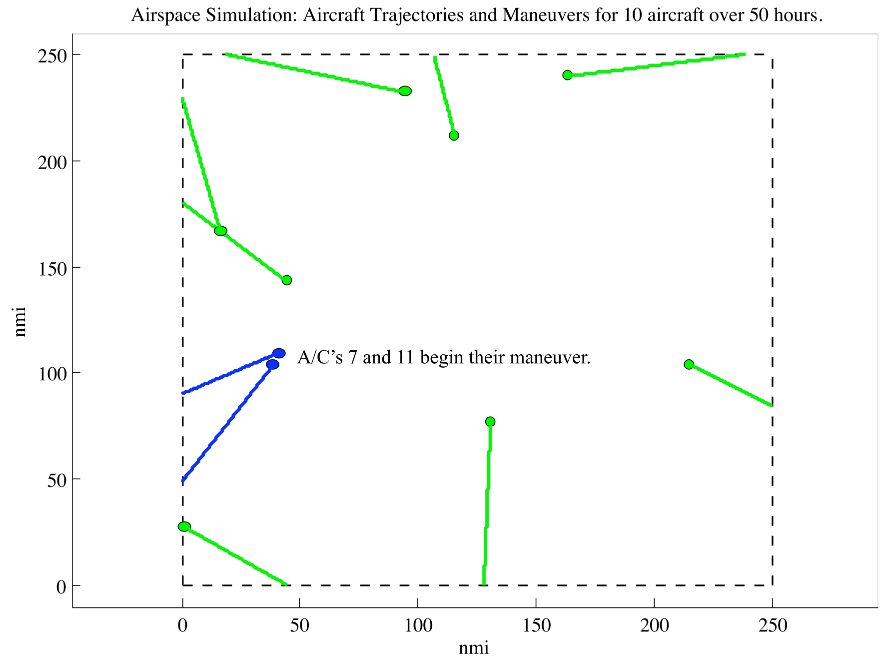

Figure 3.19: Both aircraft have calculated their maneuver and will now begin turning. In this case it is a cooperative maneuver.

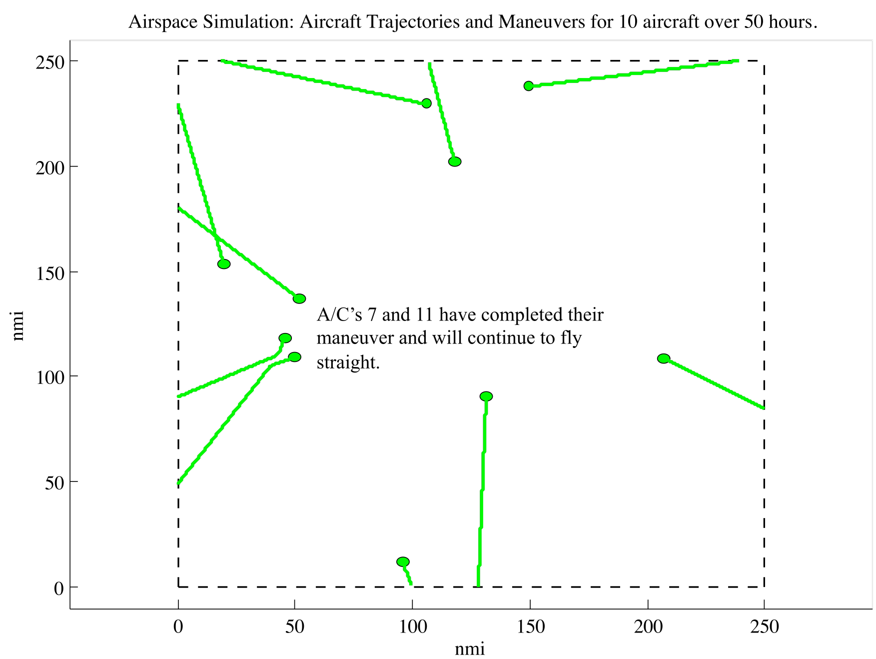

Figure 3.20: Both A/C have finished their maneuvers and will continue to fly straight.

Simulation Program Flow

The main simulation loop operates at one second increments, giving an adequate approximation of aircraft position. There are several processes running every second of simulation time, each one loops either once for each aircraft or it compares each aircraft to every other aircraft in a quadratic loop requiring $O(n^2)$ iterations.

The first loop updates the aircraft position according to Equations 2.1 and 2.2 or if the aircraft is turning, then according to Equation 2.8. Once an aircraft reaches the edge of the airspace, the second loop will remove it. It then generates a new aircraft with random position, heading, and speed. The aircraft is then immediately introduced into the airspace unless it is found to be in conflict with another aircraft. In this case it will be held at the border until it can enter the airspace conflict free. This is similar to a hand-off procedure performed by air traffic controllers. The third loop updates each aircraft's detection table for every other aircraft within the Detection Range.

Every aircraft checks for conflicts against every other aircraft in the fourth loop. Each aircraft will keep track of only one conflict with the lowest time to first LOS. The fifth loop will check for any LOS and records the type. If a conflict has been detected, then the sixth loop will run the separation assurance algorithm.

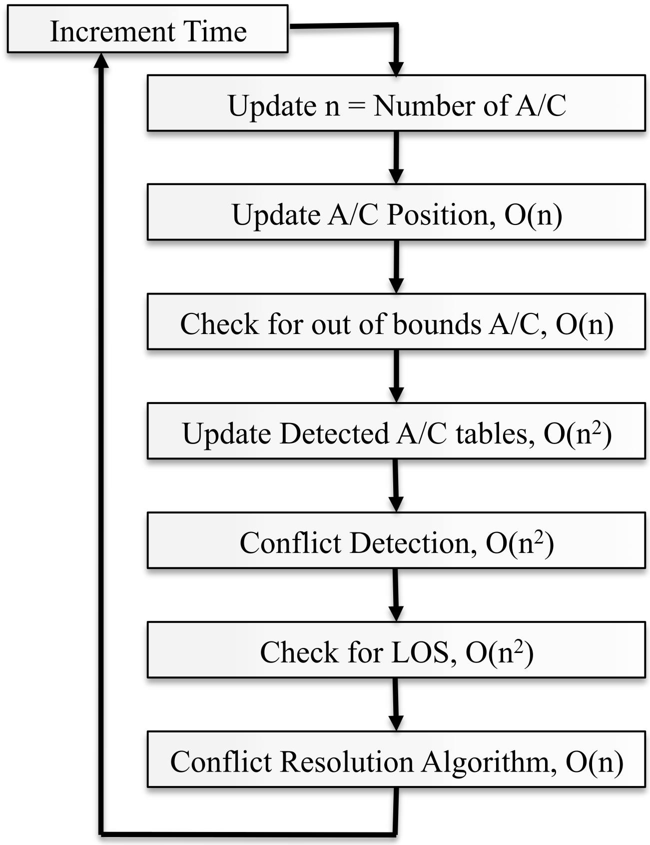

Figure 3.21: Flow Chart for the main simulation loop.

Plotting the number of aircraft, detections, conflicts, and LOS as the density of aircraft increases, shows each algorithm's complexity. The total number of unique aircraft grows linearly as the density of aircraft increases as shown in Figure 3.22. The total number of detections, conflicts, and LOS all grow quadratically as the density of aircraft increases, as shown in Figures 3.23, 3.24, and 3.25. In each figure, the density of aircraft increases in steps of ten in the range of 5 to 75. Each point is the standard deviation and mean over ten simulations at each aircraft density.

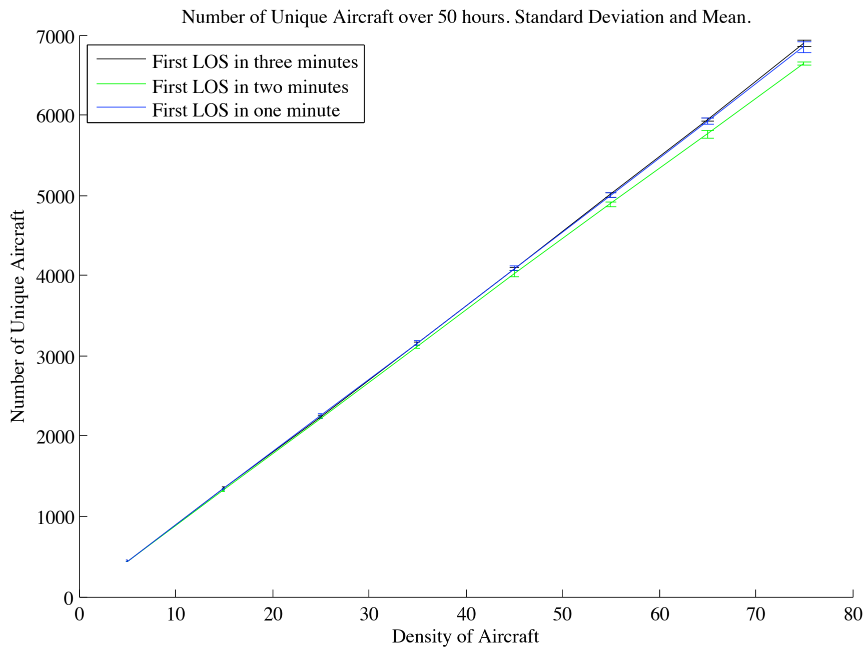

Figure 3.22: Number of unique aircraft grows linearly as the density increases.

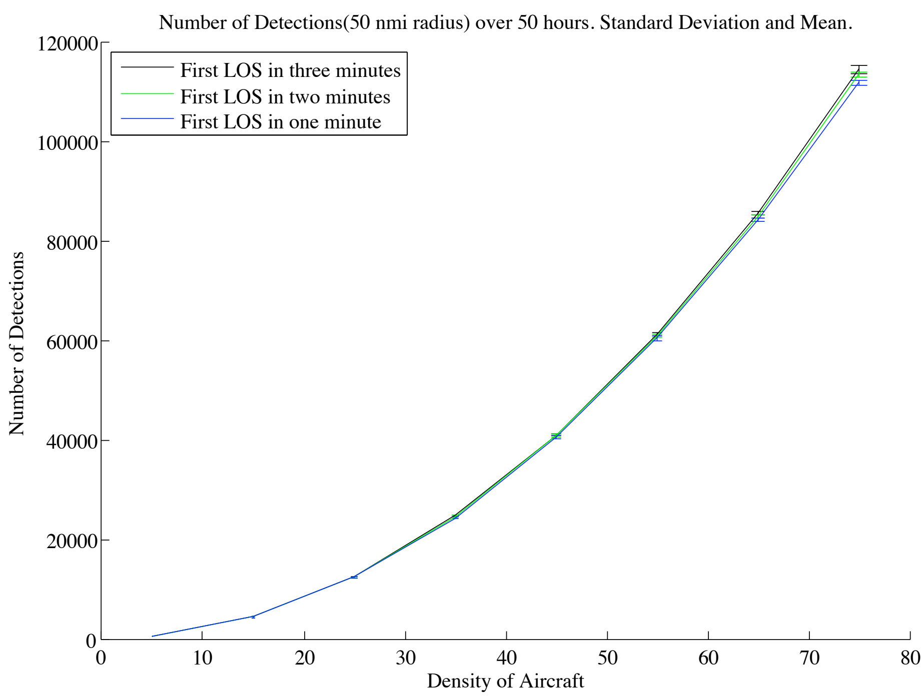

Figure 3.23: The number of detections grows quadratically as the density increases.

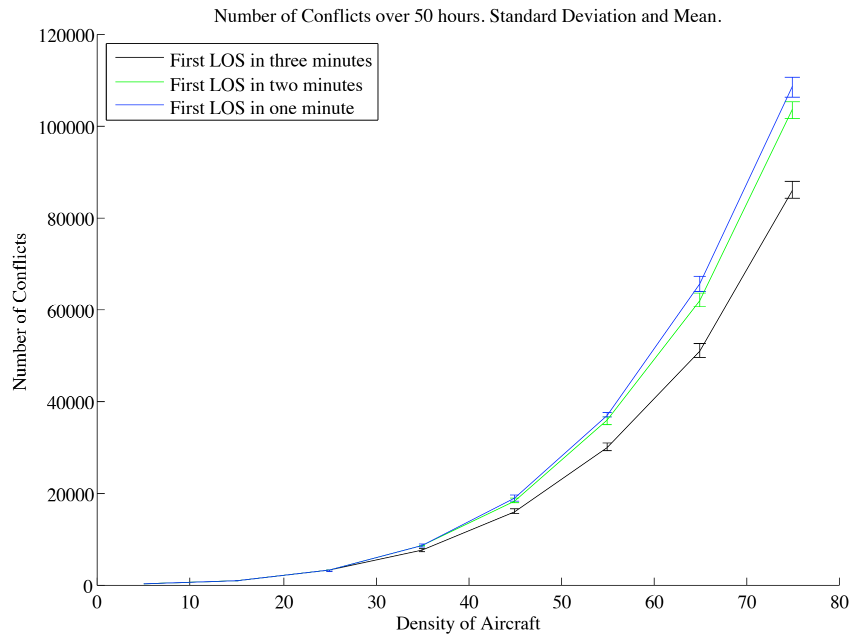

Figure 3.24: Number of conflict detections and resolutions grows quadratically as the density increases.

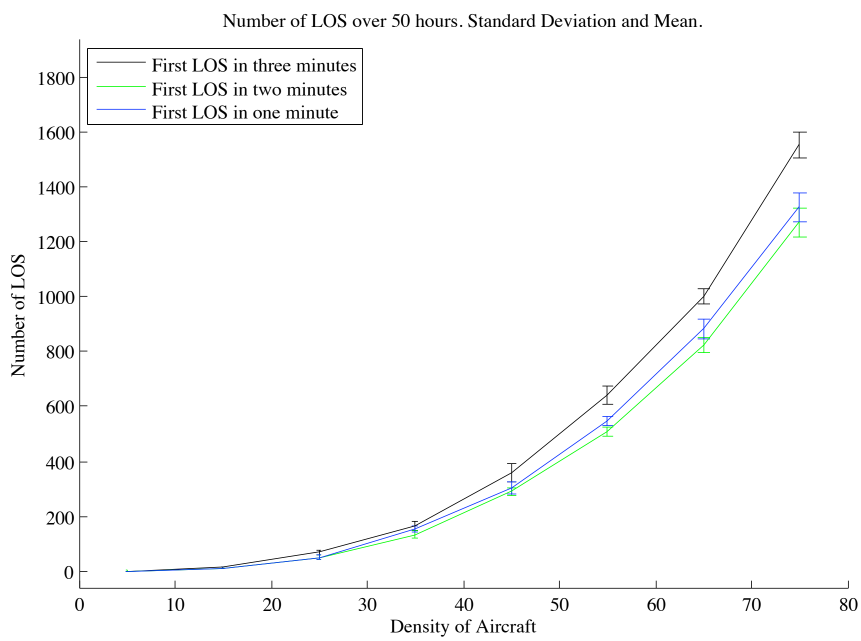

Figure 3.25: Number of LOS grows quadratically as the density increases. One may note the number of LOS at two minutes is less than for both one and three minutes. By varying the time to first LOS, Fig. 4.6, it can be seen the difference is not significant.

Visualization Tools

An important step in developing this simulation is to create effective ways to visualize the collected data. Creating graphs for data, still pictures of one or more conflict scenarios, and to display an animation of the simulation are key ways to visualize the simulation. Figures 3.22 through 3.25 are examples of plotting collected data to a graph. Figure 3.26 is an example still picture capturing a conflict resolution maneuver. Each triangle shows the position of both aircraft at a specific time step. It shows the original position and trajectory of both aircraft as well as the original minimum conflict separation. Then the maneuver path is shown along with the position of both aircraft when the maneuver is complete.

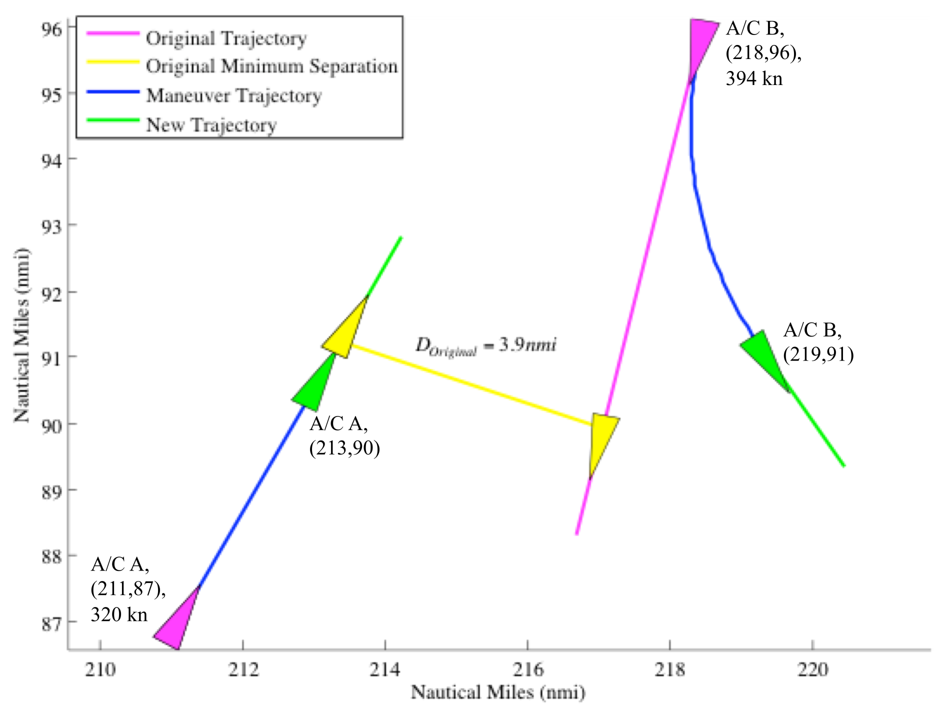

Figure 3.26: A/C B turns left. In the simulation this conflict is being resolved at 40 seconds time to first LOS. The minimum separation of 3.9 nmi occurs in 56 seconds.

Type (A/B)	$T(\psi_A)$ (seconds)	$T_{s\min}$ (seconds)	$D_{s\min}$ (nmi)	$D_{TurnMin}$ (nmi)	ψ_A (deg)	ψ_B (deg)
L/L	33.8	0.8	7.7	7.7	-42	-34
L/R	0.0	56.1	3.9	11.7	0	0
R/R	0.0	56.1	3.9	11.7	0	0
R/L	98.1	0.0	6.0	4.8	122	-99
S/L	45.5	0.4	6.4	6.4	0	-46
S/R	0.0	56.1	3.9	11.7	0	0
L/S	44.2	0.2	6.3	6.3	-55	0
R/S	0.0	56.1	3.9	11.7	0	0

Table 3.5: Of all eight maneuvers for the conflict in Figure 3.10, only three can maintain separation. This table is printed along with the picture.

An animation of the simulation is vital for not only showing what the simulation does, it is also a valuable debugging tool. The simulation saves events containing information about an aircraft. The animation tool interprets and visually recreates each event. All aircraft fly straight except for a resolution maneuver. This fact allows the event history to be a succinct representation of a potentially very long simulation. When each C simulation is complete, it will output the events to a Matlab formatted file. Then in Matlab, an animation function can be called for any event history file. Viewing the animation can quickly reveal the underlying logic, allowing for very quick verification or debug checks.

Figures 3.15 through 3.20 are snapshots of the beginning of an animation and do not show every event that can occur. There are nine events recorded in the simulation. The events are: an aircraft enters the airspace, leaves the airspace, is on a holding pattern, is being initialized at the beginning of the simulation, is no longer being initialized, has detected a conflict, is starting a maneuver, is finishing a maneuver, is in a LOS, and is no longer in a LOS. While there may or may not be any visual changes for a particular event, the exact information for every event is displayed on the command prompt and can be compared to the event history file. These nine events have been sufficient for the purpose of debugging and displaying the simulation. An example shows a LOS due to a secondary conflict in Figures 3.30 through 3.36.

Secondary Conflicts and the Classifications of a LOS

A primary pair-wise conflict is always detected at the parameter time to first LOS. A secondary conflict is detected when one or both aircraft have recently finished a resolution maneuver. An immediate secondary conflict occurs when an aircraft has finished a maneuver and is immediately in conflict with a third aircraft. This may result in a conflict at a time less than the parameter time to first LOS. The conflict detection and resolution algorithm does not look ahead and resolve for secondary conflicts, it will only detect, count, and resolve them once both aircraft are not currently maneuvering. While a secondary conflict is counted up to two minutes after a maneuver is finished, it will many times occur immediately at the end of a maneuver.

If an aircraft is maneuvering to resolve a conflict, it may lose separation with a third aircraft not involved in the conflict during its maneuver. This third aircraft may be flying straight or performing its own maneuver to resolve a separate conflict. These are not counted as conflicts as long as both aircraft are maneuvering while in LOS.

Every LOS is recorded as one of two types. The first type occurs when a pair-wise resolution results in an immediate secondary conflict detection. This secondary conflict causes an unavoidable LOS, as shown in Figure 3.27, and is detected at less than the time to first LOS parameter. The other type, as previously explained, is shown in Figures 3.28 and 3.29. In both cases, while one aircraft is turning as part of a resolution maneuver, it loses separation with a third, uninvolved aircraft. This third aircraft may be flying straight or it may be turning as part of its own conflict resolution.

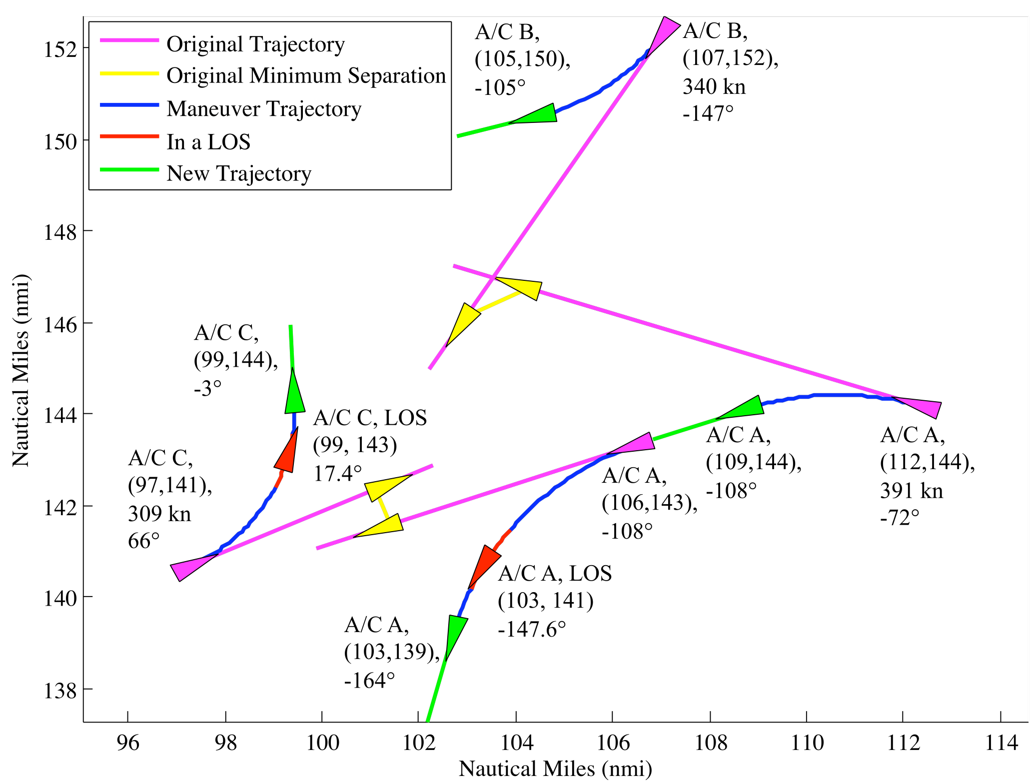

Figure 3.27: A/C A maneuvers into an immediate secondary conflict with A/C C, resulting in a LOS.

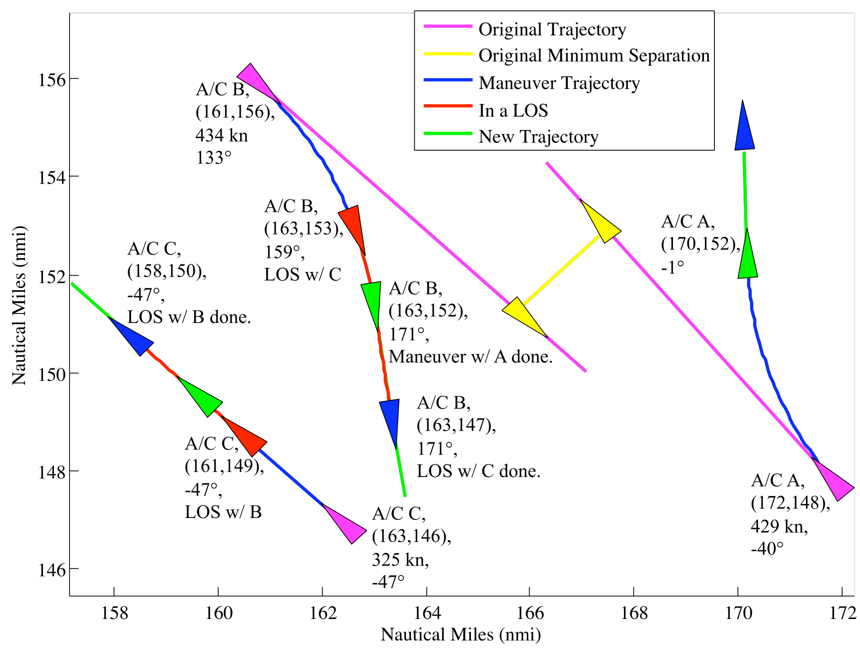

Figure 3.28: As A/C A and B maneuver to avoid each other, B loses separation with A/C C which is flying straight. B finishes its maneuver with A while still in a LOS with C.

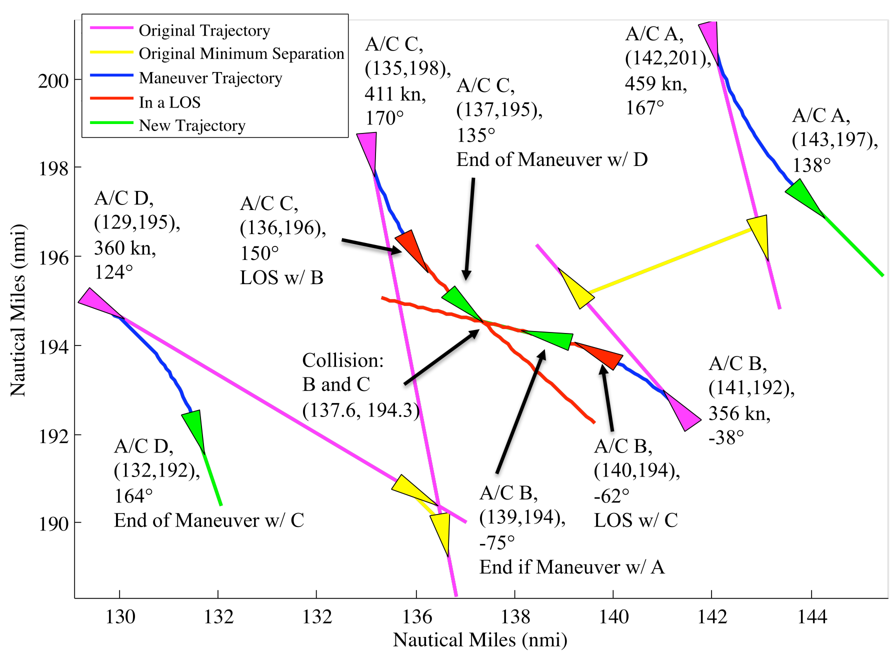

Figure 3.29: Two pair-wise conflicts resolve near each other resulting in a collision between A/C's B and C. When both B and C finish their maneuvers, they are 10 seconds from a collision.

A Secondary Conflict Example

Now that a secondary conflict has been defined, an example from the animation is presented showing a primary conflict resolving into a secondary conflict, resulting in an unavoidable LOS. Figures 3.30 through 3.36 show two aircraft involved in a primary conflict detection and resolution, a secondary conflict and resolution with a third aircraft, and then finally an immediate secondary conflict resulting in an unavoidable LOS.

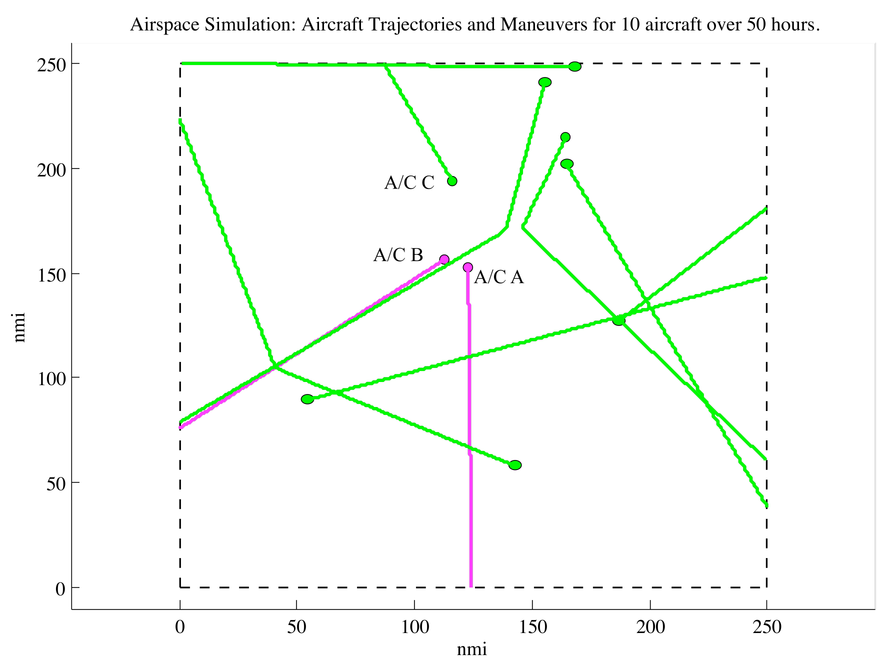

Figure 3.30: Three aircraft A, B, and C are flying towards one area. A and B detect the primary conflict.

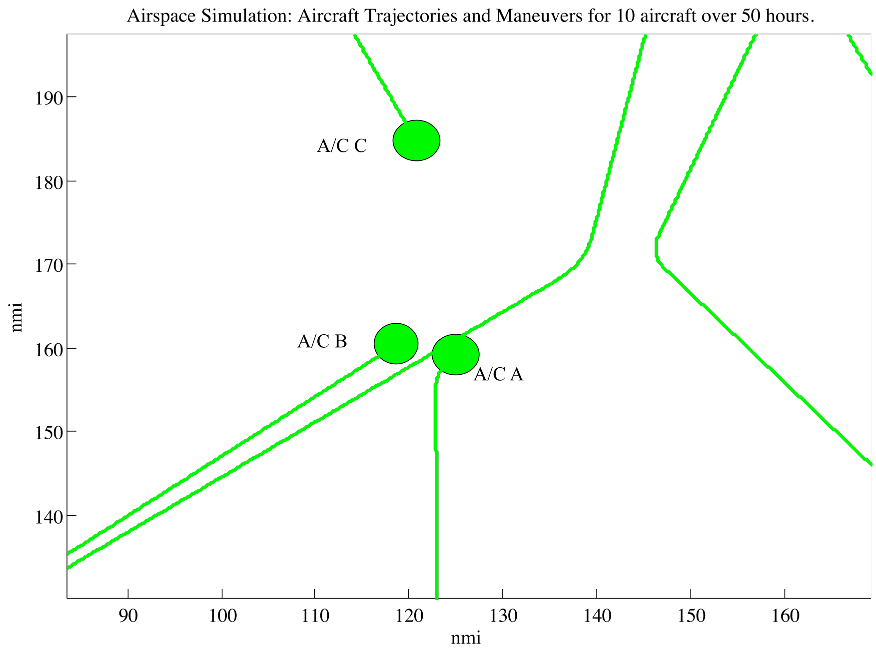

Figure 3.31: Zooming in, A/C's A and B have finished maneuvering. The track history of A/C A and B shows A/C A performed a single A/C maneuver.

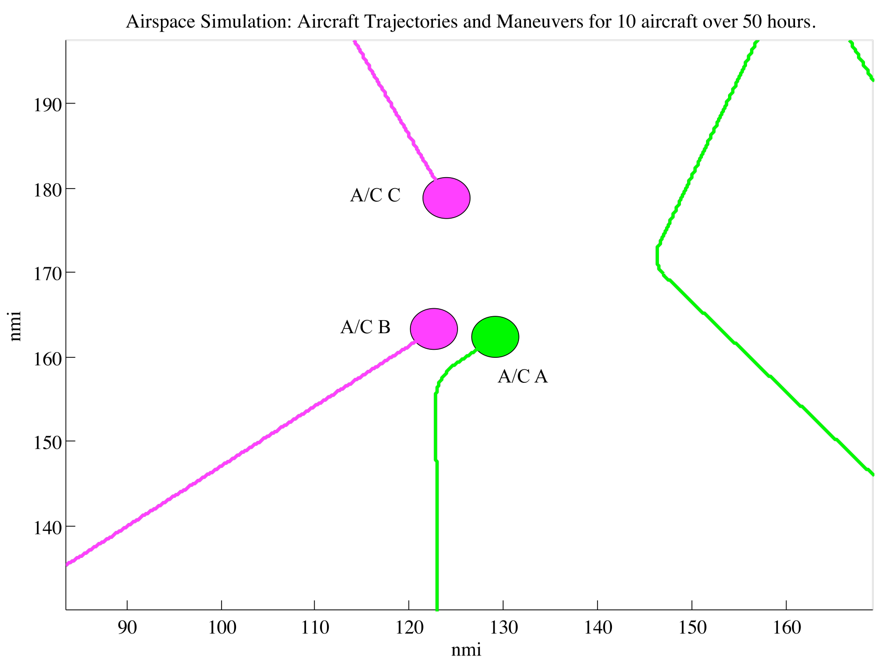

Figure 3.32: A/C's B and C have detected a secondary conflict. This conflict is detected at the time to first LOS parameter.

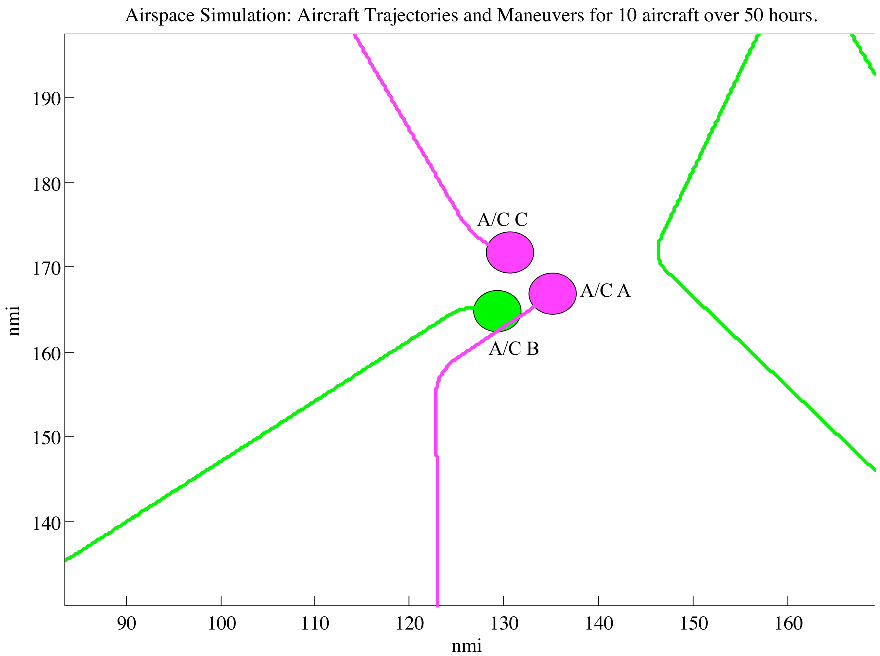

Figure 3.33: After A/C B and C have finished maneuvering, C becomes in an immediate secondary conflict with A. This conflict is detected at less than the time to first LOS parameter.

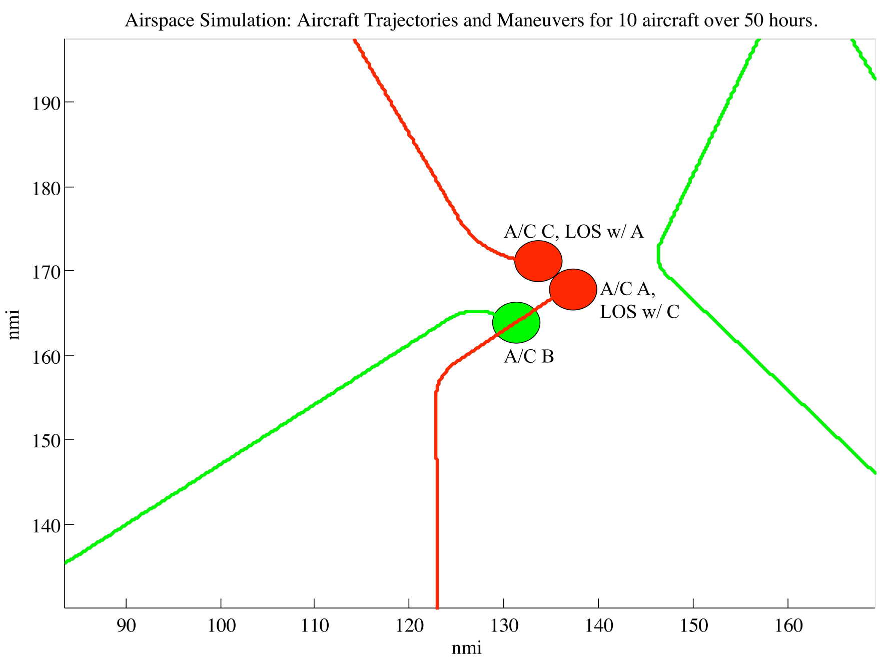

Figure 3.34: A/C's A and C are in an unavoidable LOS.

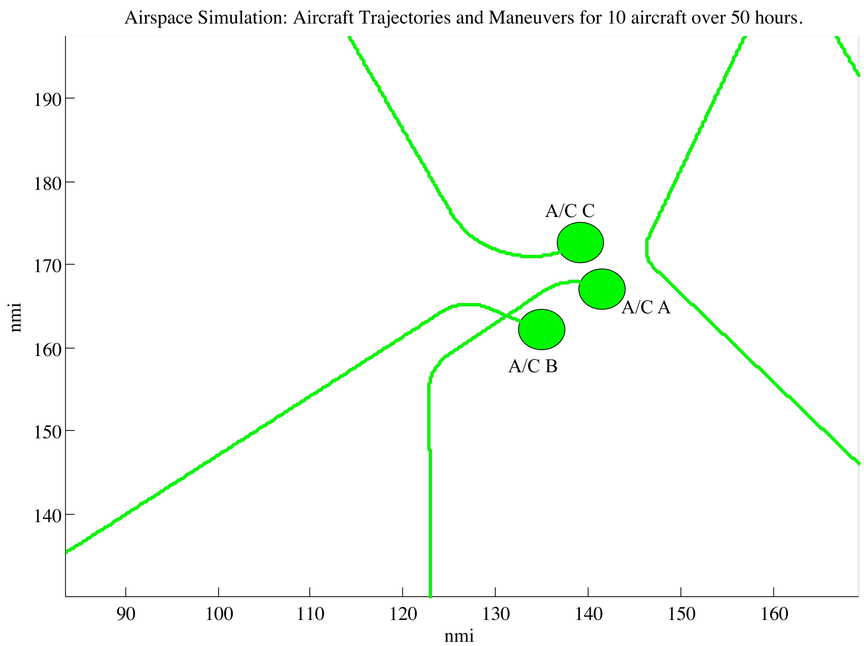

Figure 3.35: A/C A and C have finished maneuvering and are no longer in a LOS. A/C A turned right twice, A/C B turned right once, and A/C C turned left twice. C's two maneuvers look like one continuous large turn angle.

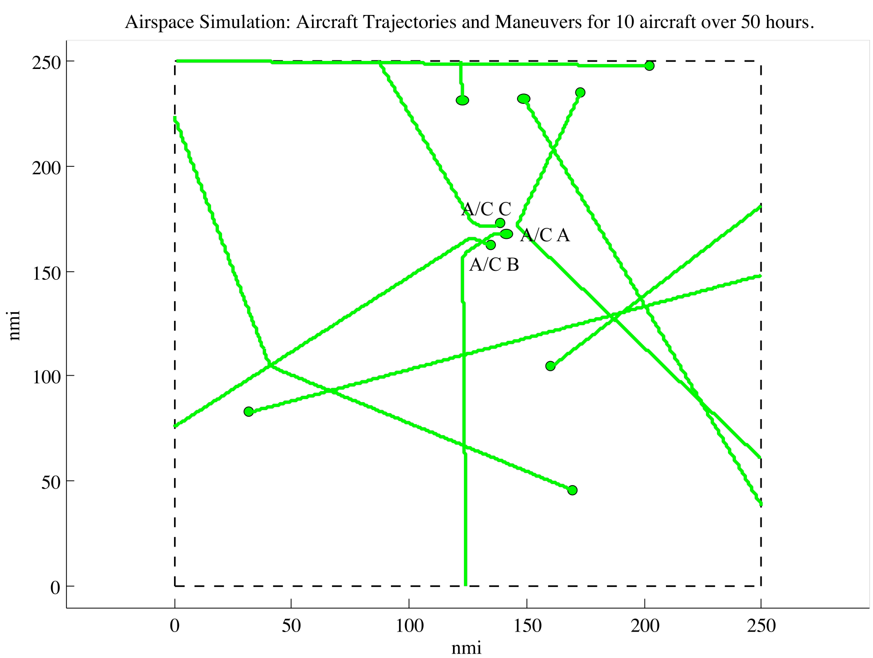

Figure 3.36: Zooming back out, all three aircraft will now fly straight.

Simulation Output and Recorded Data

The data output is organized into two files. Every time a simulation is run, it will produce output for the event history file and for the statistics file. The event history is unique to each run of a simulation, however the statistics file can span many sequential simulations. Each simulation is given a random seed and so multiple simulations can run simultaneously on a multiple core processor, each producing unique results. A script organizes the statistics into a Matlab readable format. Each statistic is a counted result of every simulation. The following table is a list of data collected and output at the end of each simulation to the statistics file.

Density of Aircraft	The density of aircraft set for this specific simulation experiment.
Number of Unique Aircraft	The total number of unique aircraft that entered the airspace during the simulation run-time.
Simulation Length	The run-time of the simulation in seconds.
Time to First LOS	Conflicts are detected at this range or less. Specified in seconds.
Detection Range	Aircraft are allowed to detect each other at this range. Specified in nmi.
Number of Detections	The number of times aircraft are detected.
Number of Conflicts	The number of conflicts detected. The separation assurance algorithm runs once for every conflict detection. If either aircraft is maneuvering, a conflict is not detected.
Number of Close Range Conflict	The number of conflicts detected at less than the Time to First LOS parameter.
Number of Secondary Conflicts for One A/C	The number of secondary conflicts where only one aircraft has finished a maneuver within the last two minutes.
Number of Secondary Conflicts for Both A/C.	The number of secondary conflicts where both aircraft have finished maneuvering within the last two minutes.
Number of LOS	The number of unique LOS.

Number of Immediate Secondary LOS	The number of unavoidable LOS when one or both aircraft are resolving a secondary conflict.
Number of LOS with a Third Aircraft while Maneuvering	The number of LOS that occur when one or both aircraft are currently maneuvering with a different aircraft.
Other LOS	For debugging purposes, any LOS that does not match the above two is recorded as a different type. Under normal experimental setup, any LOS detected as Other is a border case in the logic and a bug. However as the time to first LOS or detection range approaches zero, primary conflicts will result in LOS.
Number of Single Aircraft Maneuvers	Counts the number of single A/C maneuvers.
Number of Cooperative Maneuvers	Counts the number of maneuvers where both aircraft are maneuvering.
Number of coordination failures of algorithm	Both aircraft will run the separation assurance algorithm separately. Sometimes discrete errors will cause the two aircraft to choose different resolution maneuvers. A communication step is used to resolve this.
Number of coordination failures resulting in a LOS	If both aircraft choose different resolution maneuvers, it is possible this will have resulted in a LOS. However the communication step resolves this.

Table 3.6: List of counted events during each simulation.

Separation Assurance Algorithm, Discrete Errors, and Communication

Discrete errors may cause the separation assurance algorithm to select different maneuvers for both aircraft involved in the pair-wise conflict. Each aircraft runs the algorithm in a distributed manner and without communication a discrete error may cause a failure to select the same maneuver given the same information. It is possible this failure to synchronize will cause an un-intended and completely avoidable LOS, thus a communication step is necessary to ensure both aircraft select the same maneuver. The communication step assumes both aircraft send each other its maneuver information and will synchronize on one maneuver.

Communication is assumed to be instantaneous and is not part of this simulation's testing of the separation assurance algorithm. The synchronization process follows the same steps as the algorithm, it will either choose the maneuver of minimum time if separation is assured, or it will maximize the minimum separation if LOS is unavoidable.

Performance of the Simulation

As the aircraft density increases, the run time of many algorithms in the main simulation loop increase quadratically. The following table lists the run time on a compute server with four 2.6 GHz processor cores, each running one copy of the simulation. The memory usage of each process is relatively small. For even 75 aircraft density, the memory usage is less than 20 MB total. These run times are the average over 30 simulations at each aircraft density, 10 for each time to first LOS, rounded up to the nearest multiple of five seconds. The application is compiled using 'g++ -g'; no other optimizations are specified. The times are collected using the UNIX command time.

Aircraft Density	Run Time
15	20 s
25	55 s
35	1 min 45 s
45	2 min 40 s
55	4 min 20 s
65	6 min 5 s
75	7 min 50 s

Table 3.7: Average run time for the simulation given increasing aircraft density.

Chapter Four: Analysis of Simulation Results

The performance of the separation assurance algorithm is being measured by varying key parameters of the simulation, analyzing the recorded statistics, visually confirming the results, and finally plotting collected data. Key parameters include time to first LOS, detection range, and aircraft density. Important results include the counted number of each type of LOS and conflict, the number of cooperative and single aircraft maneuvers, and the number of times the two aircraft fail two agree on the same maneuver.

Varying Time to First LOS

As the time to first LOS approaches zero, certain borders of the separation assurance algorithm are found. These borders include the time when single aircraft maneuvers can no longer maintain separation, causing the algorithm to switch to cooperative maneuvers, and the time when primary conflicts will result in unavoidable LOS.

When a single A/C maneuvering is no longer sufficient to maintain separation, the algorithm will attempt a cooperative maneuver. In order to find this point, the key parameter time to first LOS is varied from three minutes down to 3 seconds. At each time to first LOS, the simulation is run ten times. Each run is set at the default values found in Table 3.4 with an aircraft density of 25, equivalent to current air traffic density. The delay before the maneuver is set to zero seconds causing both aircraft to maneuver immediately after conflict detection. In addition, this simulation is repeated for each of the three bank angles 15°, 20°, and 25°. The mean of the data for 10 simulations for each time to first LOS and bank angle is used to produce Figures 4.37, 4.38, and 4.39. This same method is used for all plots in this chapter.

By examination of the turn radius formula in Equation 2.4, the time when cooperative maneuvers are required is reduced as the bank angle increases. The greater the bank angle, the smaller the circle's radius becomes, increasing the potential distance between two resolving aircraft. An improvement to the algorithm is to allow it to increase the bank angle when all cooperative maneuvers at a low bank angle result in LOS[1].

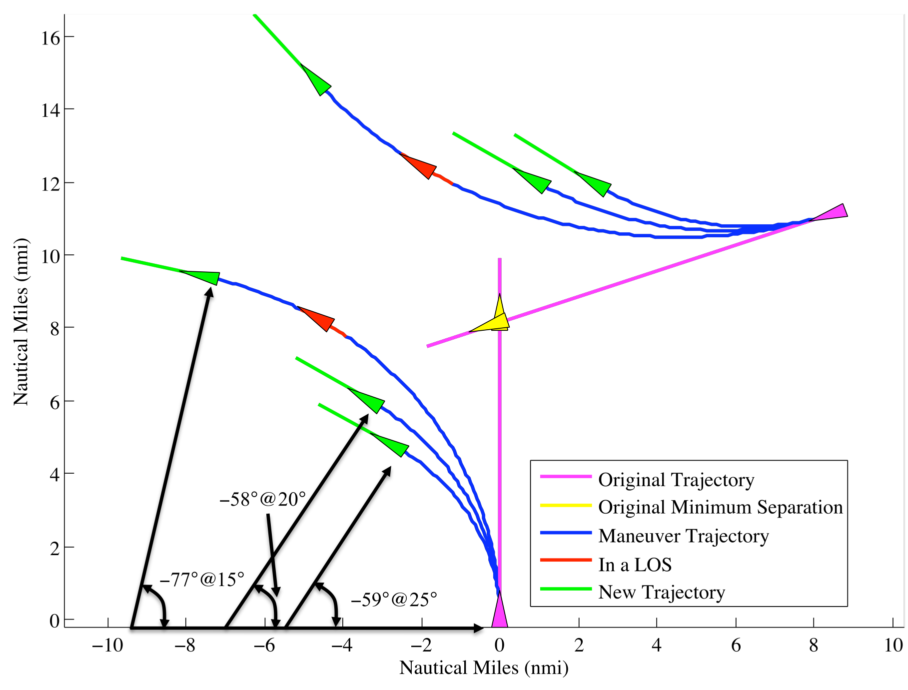

Figure 4.37: Shows the "turn angle @ bank angle" for one particular conflict. At a 15° bank angle, LOS is unavoidable for all maneuvers, while at greater bank angles separation is assured. These are minimum time maneuvers for 20° and 25° bank angles.

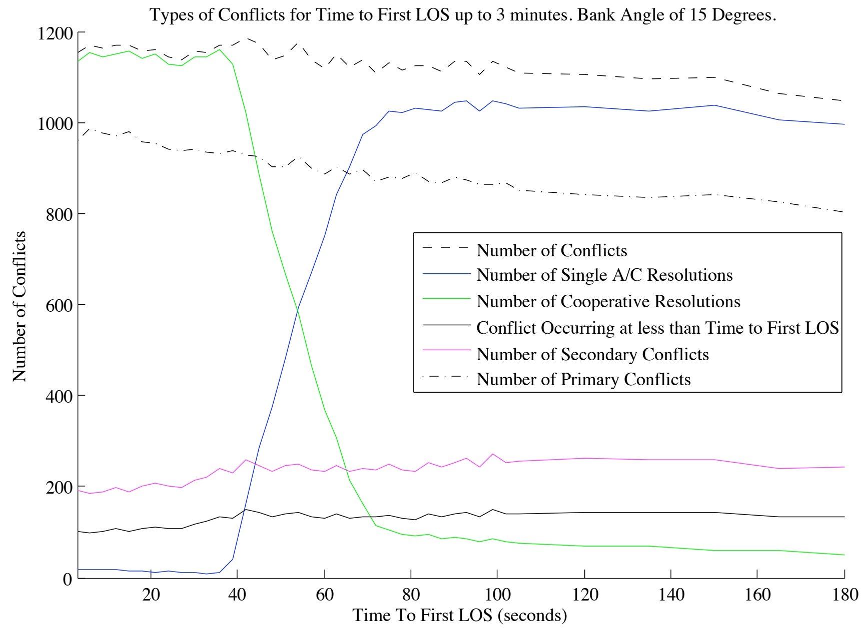

Figure 4.38: The number of cooperative and single aircraft maneuvers flip and become approximately constant as the time to first LOS increases. For a 15° bank angle, primary conflicts are resolved by single aircraft maneuvers at greater than 72 seconds.

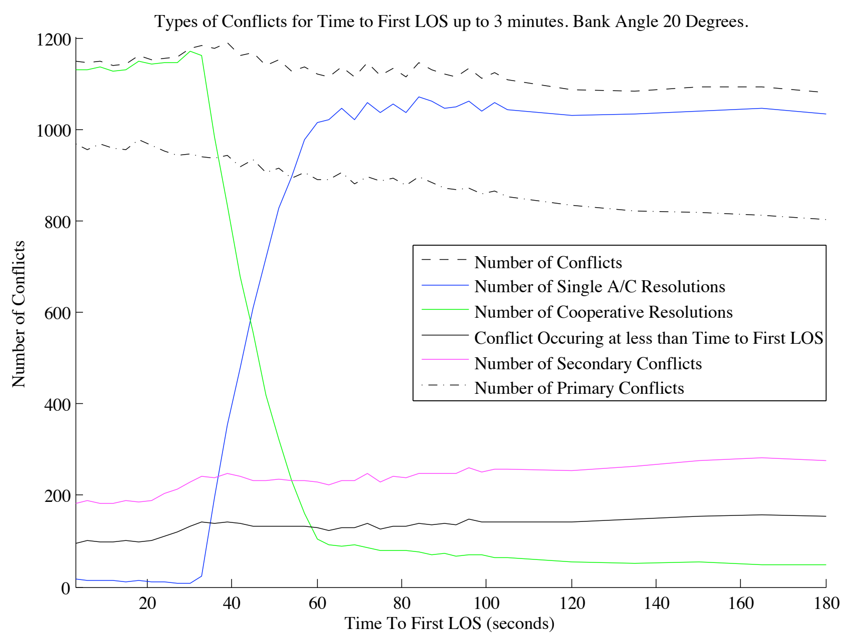

Figure 4.39: The number of cooperative and single aircraft maneuvers flip and become approximately constant as the time to first LOS increases. For a 20° bank angle, primary conflicts are resolved by single aircraft maneuvers at greater than 57 seconds.

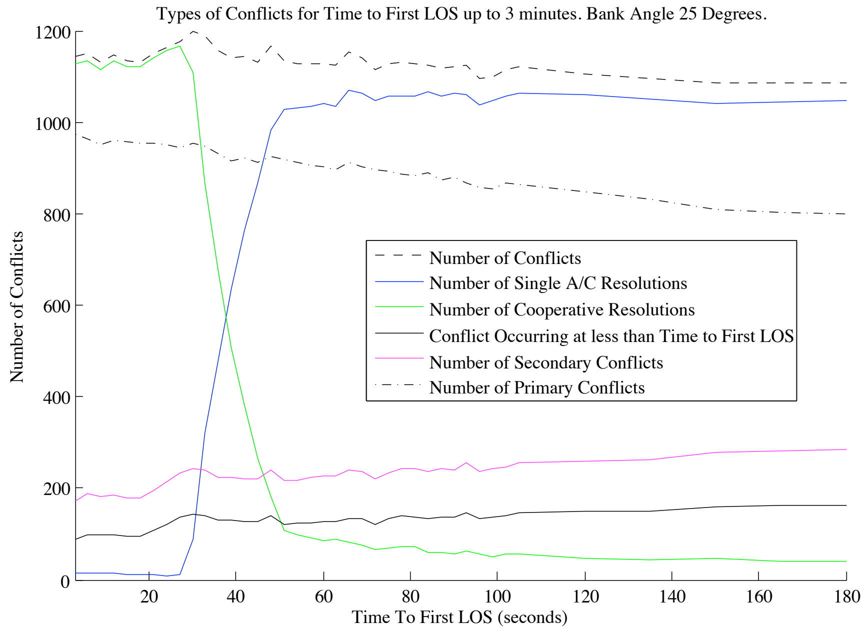

Figure 4.40: The number of cooperative and single aircraft maneuvers flip and become approximately constant as the time to first LOS increases. For a 25° bank angle, primary conflicts are resolved by single aircraft maneuvers at greater than 51 seconds.

A good indicator that primary conflicts are no longer being resolved by cooperative maneuvers is when the number of cooperative maneuvers drop below the number of conflicts detected at less than the time to first LOS parameter. One may note the number of secondary conflicts looks very similar to the number of conflicts detected at less than the time to first LOS. This is because all conflicts detected at less than the time to first LOS must be an immediate secondary conflict detected at the end of a maneuver. However, not all secondary conflicts are detected at less than the time to first LOS parameter, especially at higher times to first LOS.

Next, as the time to first LOS approaches zero seconds, there will be a point when primary conflicts result in unavoidable LOS. By counting the types of LOS that are occurring at each time to first LOS, the results will show when the algorithm predicts separation cannot be assured for primary conflicts using cooperative maneuvers. Figures 4.41, 4.42, and 4.43 show this threshold for different bank angles.

Once primary conflicts no longer result in LOS, the other two types of LOS are the result of more than two aircraft in a local area. As two aircraft finish a maneuver, each of them may become in an immediate secondary conflict, potentially resulting in an unavoidable LOS. As two aircraft maneuver to avoid each other, it is possible one of them will lose separation with a third aircraft. This third aircraft may even be maneuvering to avoid its own conflict. As the time to first LOS increases, it is more likely that the time when a secondary conflict is detected allows it to be resolved safely. This will reduce the number of LOS by immediate secondary conflicts. However, the total number of LOS is remaining approximately constant as the time to first LOS grows. The number of LOS by an uninvolved third aircraft increases in frequency. This increase is likely caused by maneuvering aircraft having a greater opportunity to breach the maneuver space of other primary conflicts. At a higher time to first LOS, most primary conflicts are resolved by a single aircraft maneuver. Recall both maneuvering aircraft will ignore other aircraft even during a single aircraft maneuver.

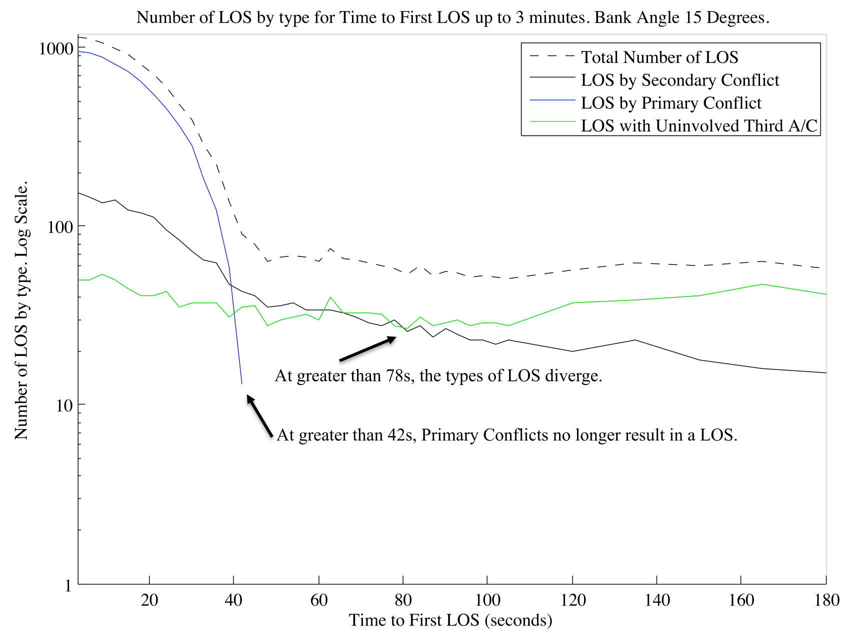

Figure 4.41: Primary conflicts may result in unavoidable LOS for a 15° bank angle when time to first LOS is less than 42 seconds.

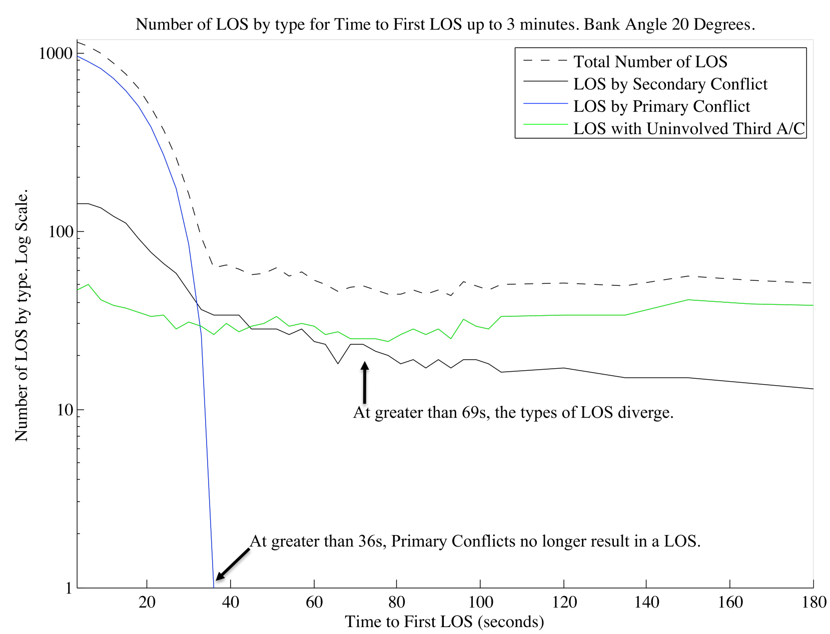

Figure 4.42: Primary conflicts may result in unavoidable LOS for a 20° bank angle when time to first LOS is less than 36 seconds.

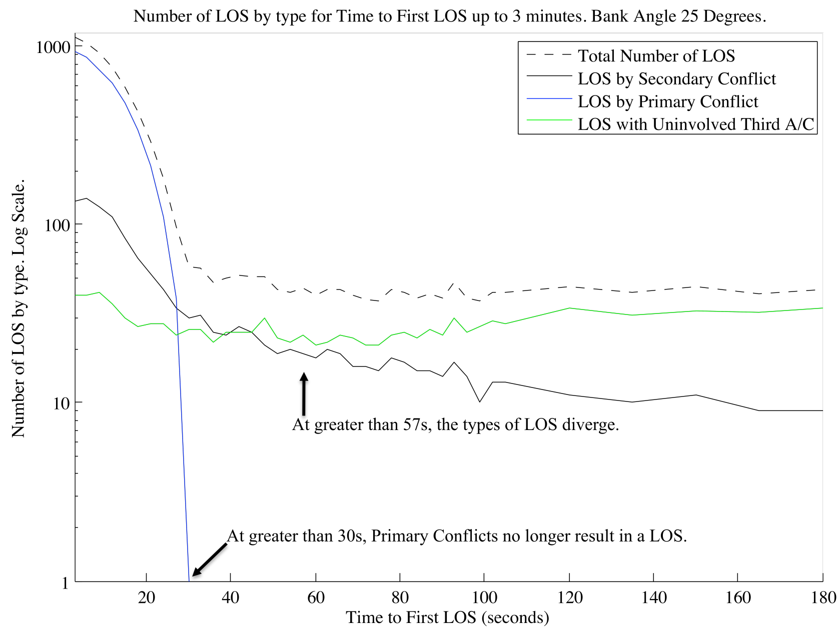

Figure 4.43: Primary conflicts may result in unavoidable LOS for a 25° bank angle when time to first LOS is less than 30 seconds.

As time to first LOS is increasing, the number of LOS with aircraft un-involved in the pair-wise conflict is becoming more frequent. This trend supports an improvement to this algorithm. Both aircraft involved in a pair-wise conflict will generate the eight possible maneuvers. By using the same assumptions from the algorithm, each aircraft can check each of the generated maneuvers against all other aircraft within its detection range. This is a secondary conflict detection algorithm along each of the potential maneuvers trajectories. Once a set of safe maneuvers has been found, each aircraft can communicate its intent to all other aircraft and start performing the maneuver. If any other aircraft need to perform a conflict resolution, they will be able to use the same secondary conflict detection and see any aircraft's intent, whether it be flying straight or turning. By looking ahead, a secondary conflict resolution algorithm can avoid many LOS with third aircraft un-involved in the initial pair wise conflict. This algorithm is part of the future work by Professor Heinz Erzberger.

A final test for this section is to increase the aircraft density by a factor of two. At a bank angle of 20 degrees, the same trends occur for two times density in Figures 4.44 and 4.45 as seen at one times density in Figures 4.39 and 4.42. Further exploration of increasing air traffic density is presented in the next section

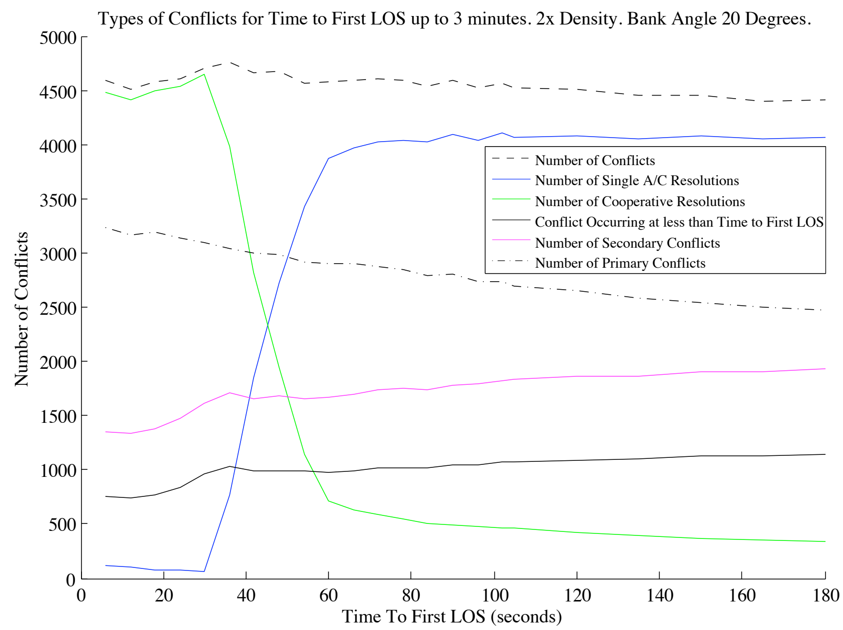

Figure 4.44: The same trends occur at two times density as in Figure 4.38 at one times density. Few primary conflicts before 57 seconds require cooperative maneuvers.

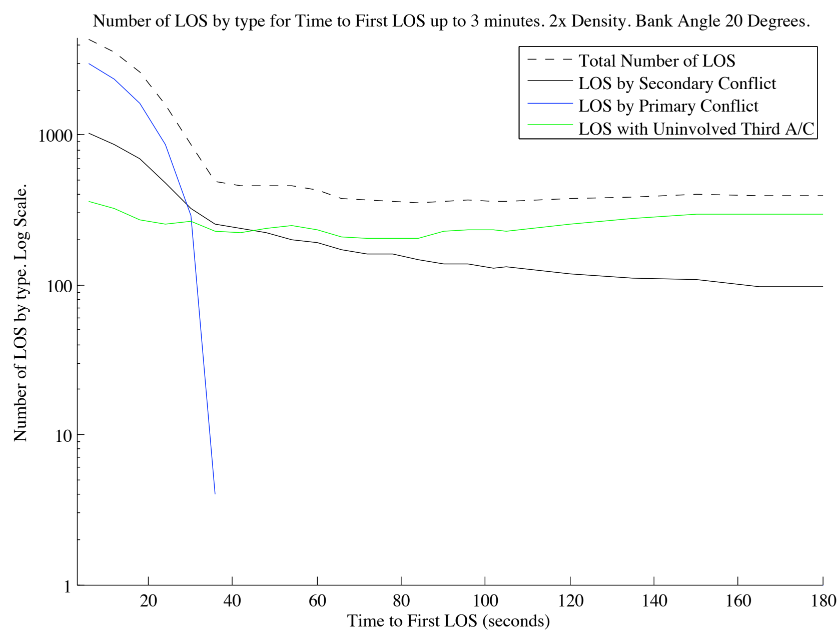

Figure 4.45: The same trends occur at two times density as in Figure 4.42 at one times density. Primary conflicts no longer result in LOS after 36 second.

Increasing Aircraft Density

One goal of implementing this simulation was to be able to check the effects of increased aircraft density upon this pair-wise separation assurance algorithm. The simulation is run thirty times at each aircraft density, ten times for one, two, and three minutes time to first LOS. The default values are used from Table 3.4. Figures 3.22 through 3.25 have already presented some effects of increased air traffic density. Figures 4.46 through 4.48 show the number of each type of conflicts. As air traffic density increases, the total number of conflicts grows quadratically, the percentage of secondary conflicts increases, and the percentage of conflicts resolved by cooperative maneuvers also increases. However, as the time to first LOS increases, the number of conflicts requiring a cooperative maneuver is reduced, even though the percentage of conflicts resolved by a cooperative maneuver is higher at three times density (75) than at one times density (25). This is a result of primary conflicts no longer using cooperative maneuvers and more secondary conflicts being detected at the max time to first LOS range.

As the aircraft density grows, Figures 4.49 through 4.51 show the number of LOS by an uninvolved, third aircraft grows faster than the number of immediate secondary conflicts resulting in an unavoidable LOS. As there are more aircraft in the airspace, the chance of running into other aircraft increases, causing more LOS by aircraft not involved in the conflict. This again is showing secondary conflict detection and resolution is an important next step in developing this algorithm.

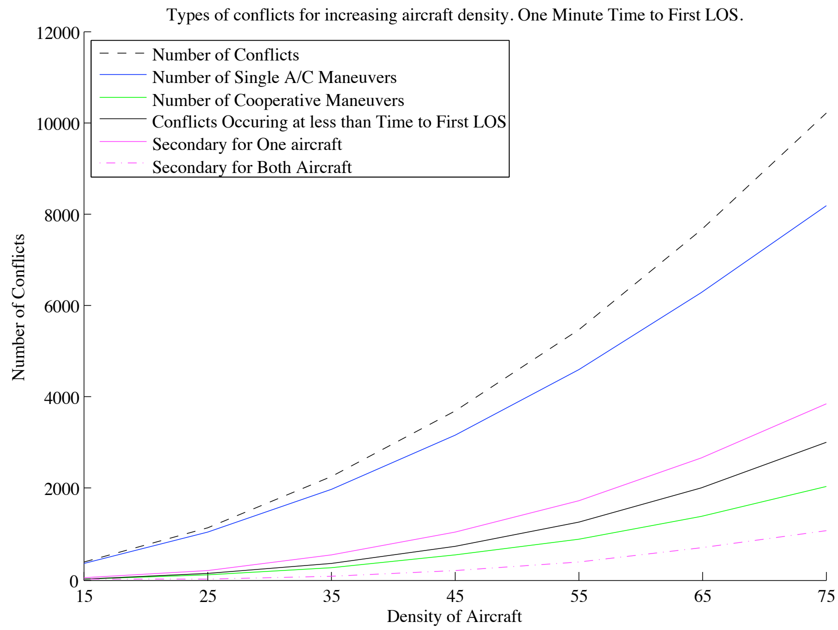

Figure 4.46: As aircraft density increases, the percentage of secondary conflicts increases. At one-minute time to first LOS and a 20° bank angle, there are few primary conflicts requiring a cooperative solution.

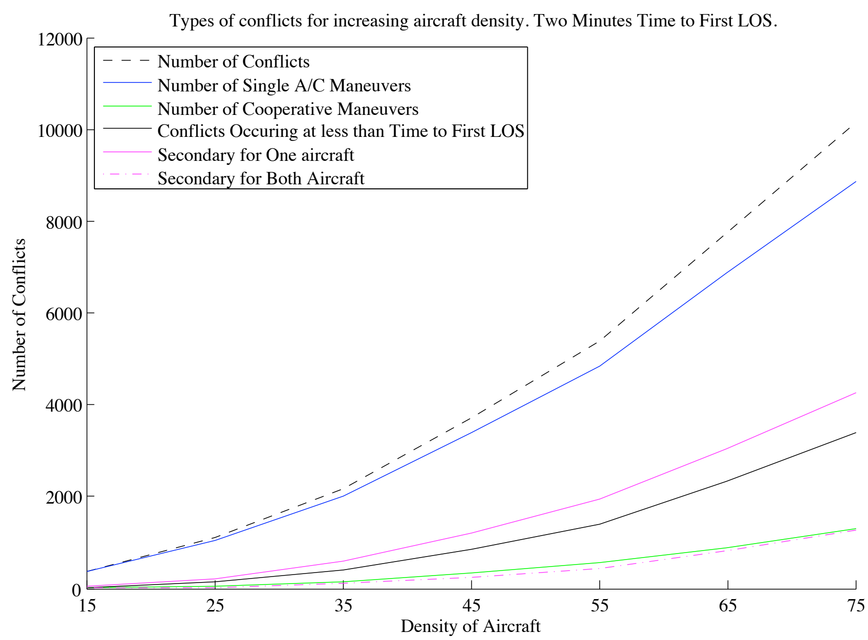

Figure 4.47: At two minutes, the percentage of conflicts that are secondary and the number of cooperative maneuvers is smaller than for one minute time to first LOS.

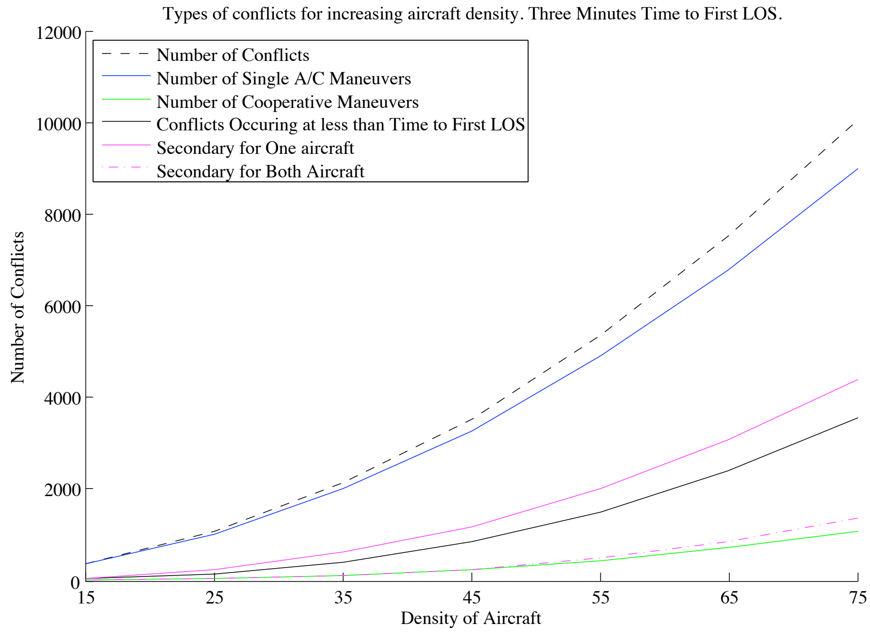

Figure 4.48: As aircraft density increases, the percentage of secondary conflicts is even smaller for three minutes time to first LOS and fewer conflicts require a cooperative maneuver.

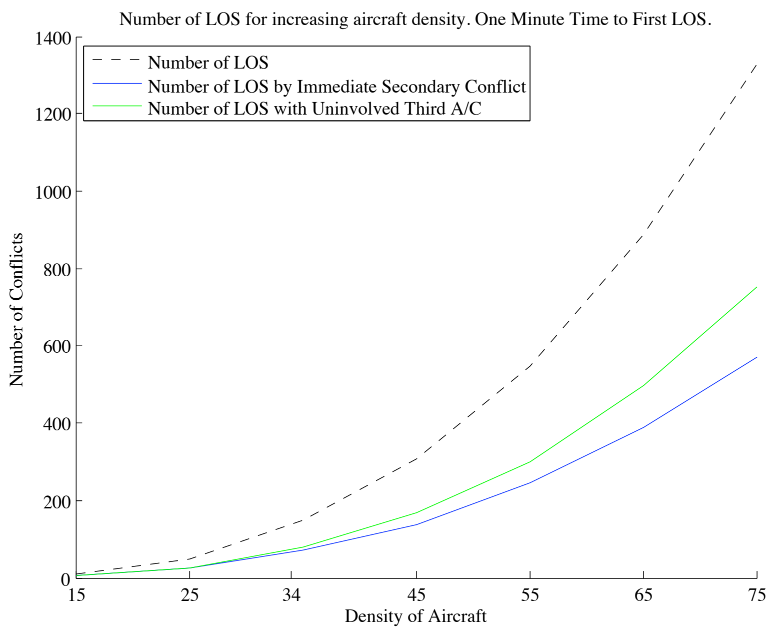

Figure 4.49: As aircraft density increases, the number of LOS due to an aircraft not involved in the conflict is increasing faster than the number of LOS by immediate secondary conflict.

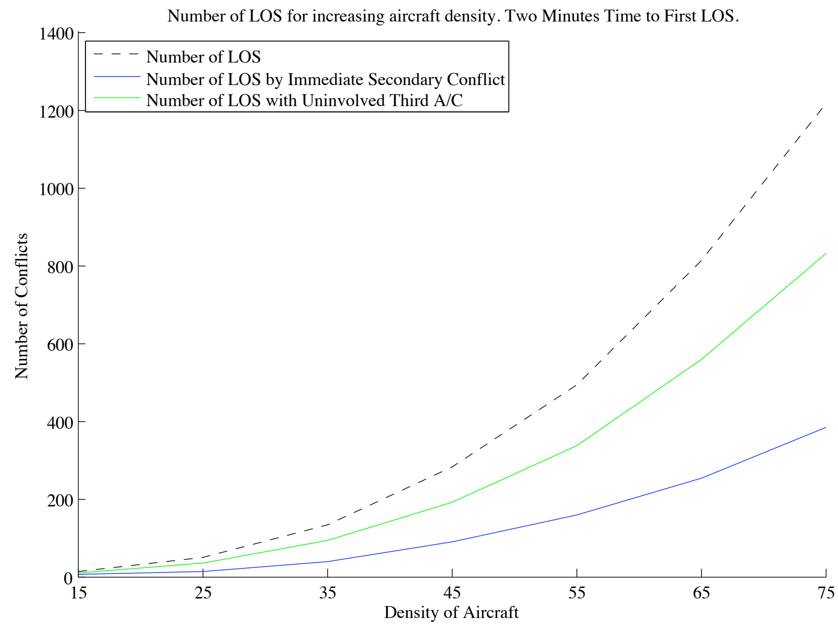

Figure 4.50: As aircraft density increases, the number of LOS due to an aircraft not involved in the conflict is increasing faster than the number of LOS by immediate secondary conflict.

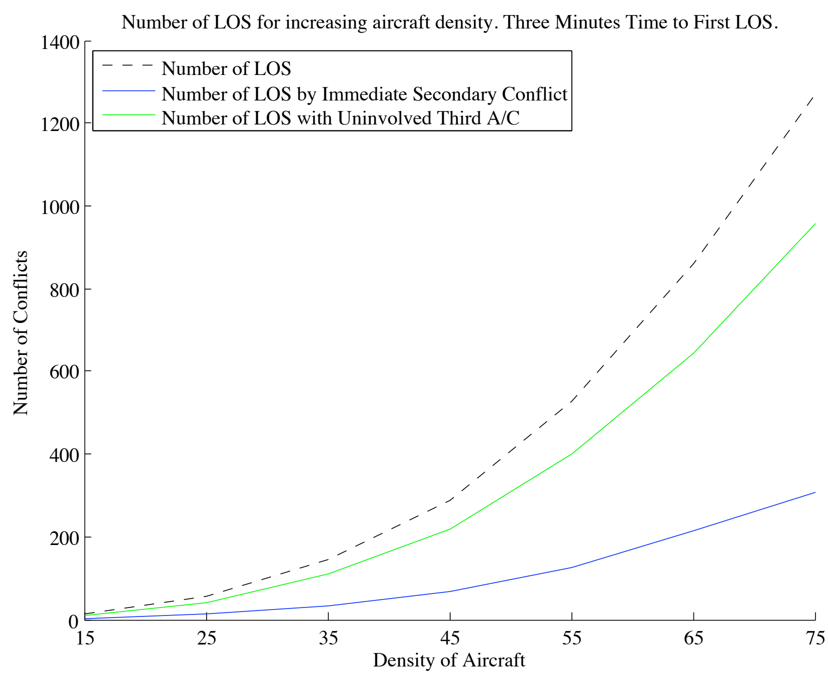

Figure 4.51: As aircraft density increases, the number of LOS due to an aircraft not involved in the conflict is increasing faster than the number of LOS by immediate secondary conflict.

Varying Detection Range

The detection radius of an aircraft defines the area in which it can see other aircraft and obtain their speed, heading, and position. By default this radius is 50 nmi. In a distributed manner, each aircraft updates its set of aircraft within its detection radius at each time step of the simulation. As the detection radius approaches the minimum separation required between two aircraft, the number of unavoidable LOS will approach the total number of conflicts detected. The detection radius has been varied from 100 down to 5 nmi using a two minutes time to first LOS and otherwise default values from Table 3.4. For each detection radius, the simulation is run ten times and the mean is taken to produce the following figures.

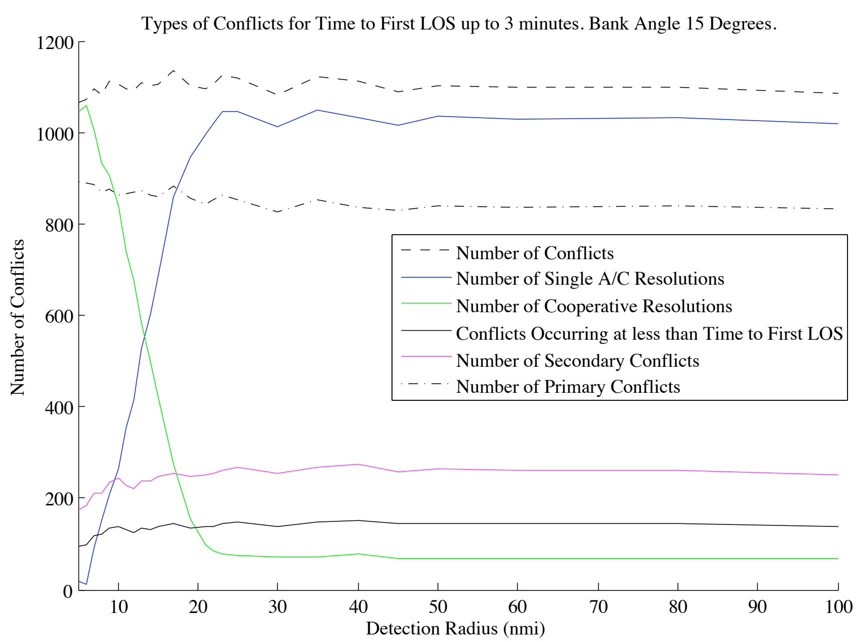

Figure 4.52: Number of conflicts as the detection radius approaches zero for a 15° bank angle. Primary conflicts do not require cooperative maneuvers after a 22 NMI detection radius.

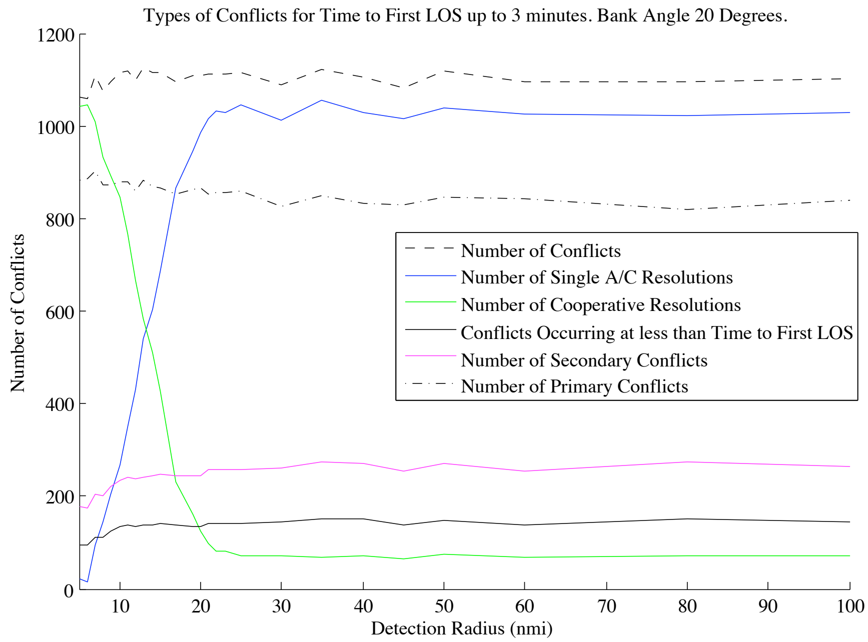

Figure 4.53: Number of conflicts as the detection radius approaches zero for a 20° bank angle. Primary conflicts do not require cooperative maneuvers after a 21 NMI detection radius.

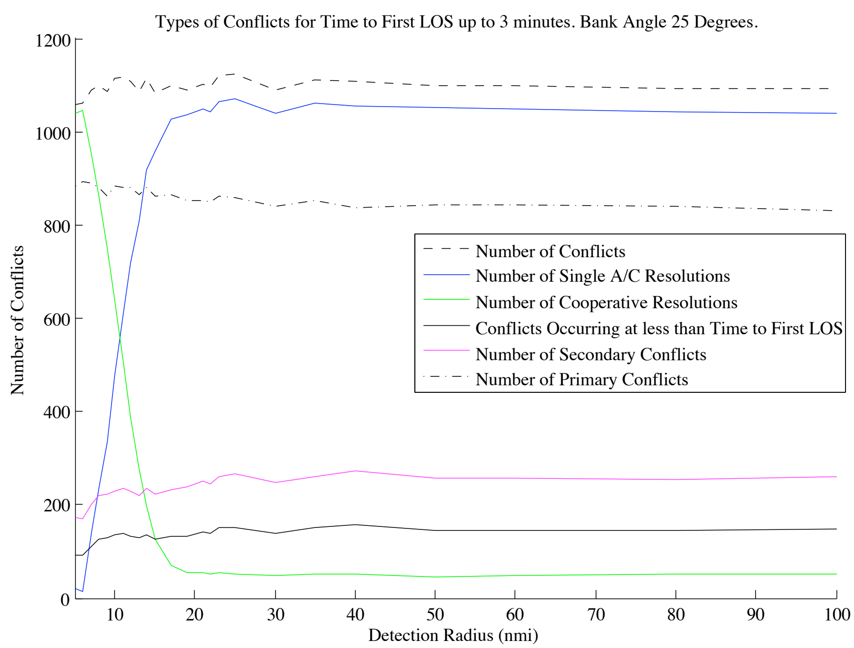

Figure 4.54: Number of conflicts as the detection radius approaches zero for a 25° bank angle. Primary conflicts do not require cooperative maneuvers after a 17 NMI detection radius.

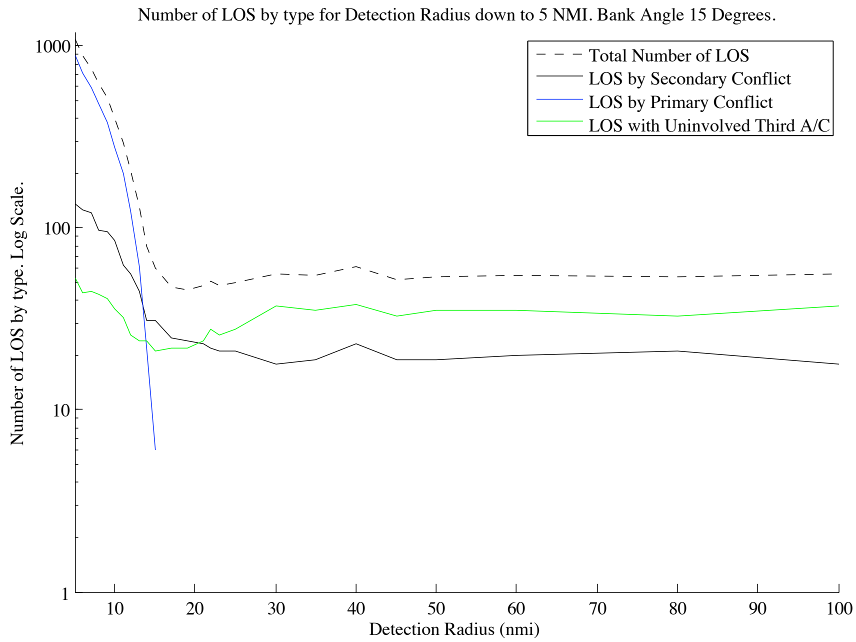

Figure 4.55: At a 15° bank angle, primary conflicts no longer lose separation after a 16 nmi detection radius.

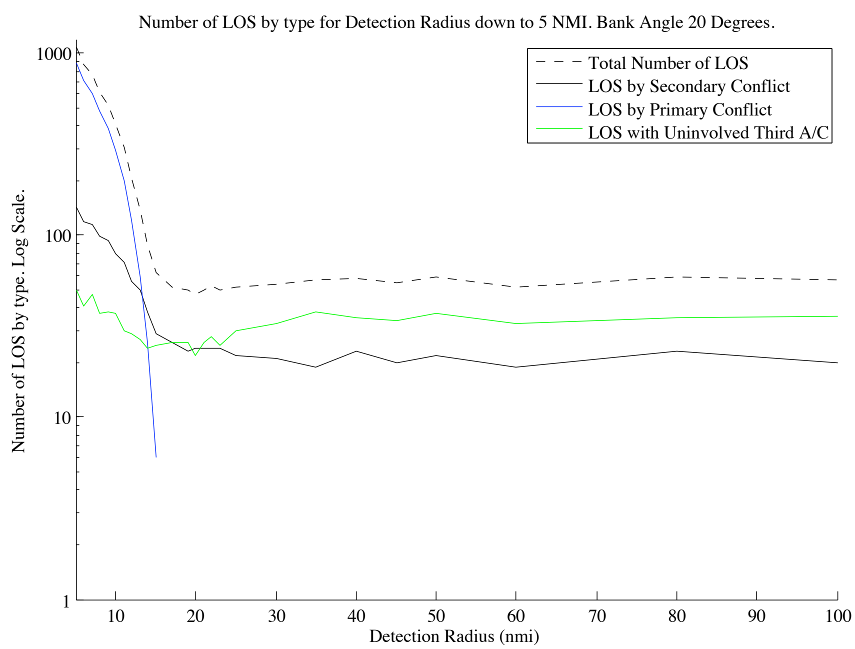

Figure 4.56: At a 20° bank angle, primary conflicts no longer lose separation after a 15 nmi detection radius.

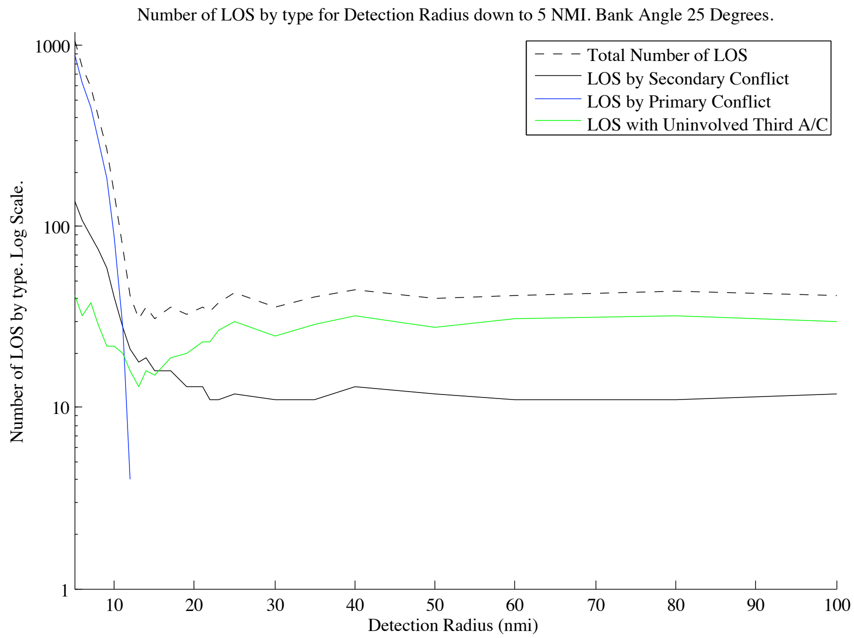

Figure 4.57: At a 25° bank angle, primary conflicts no longer lose separation after a 12 nmi detection radius.

Type of Relation	Bank Angle	Minimum Time to First LOS	Minimum Detection Radius
Primary conflicts no longer require cooperative resolutions.	15°	72s	22 nmi
	20°	57s	21 nmi
	25°	51s	17 nmi
Primary conflicts result in unavoidable LOS.	15°	42s	16 nmi
	20°	36s	15 nmi
	25°	30s	12 nmi
Number of LOS because of a third aircraft not involved in the pair-wise conflict.	15°	78s	21 nmi
	20°	69s	20 nmi
	25°	57s	16 nmi

Table 4.8: The relationship between bank angle, time, and detection radius showing the performance of the separation assurance algorithm.

As the detection range approaches zero, the same trends occur as when the time to first LOS approaches zero. This allows a relation between distance and time given varying air speeds shown in Table 4.8. Time was measured at a higher resolution than distance causing the relationship to be an approximation.

Constraints on the Separation Assurance Algorithm

This implementation of the separation assurance algorithm tries to minimize total time to minimum separation without consideration of returning to original path or for secondary conflicts. It will first select a single aircraft maneuver if it assures separation, else it will use a cooperative maneuver. While an aircraft is performing a maneuver, either turning or flying straight, it will ignore all other conflicts until it has reached minimum separation.

Ignoring all other aircraft until it has completed a maneuver will cause many immediate secondary conflicts and LOS with uninvolved third aircraft. Secondary conflict detection and resolution (CD&R) solves many of the problems found in the previous sections. However there will still be cases where a secondary CD&R algorithm will not be able to avoid a LOS. This may occur because of two conflicts resolving simultaneously in the same local area, or perhaps there are simply too many aircraft and resolving the situation is beyond the capabilities of pair-wise secondary CD&R.

When the algorithm minimizes for total time, it will usually select a larger turn angle for which the minimum separation occurs at the end of the turn. When the algorithm chooses a single aircraft maneuver, it is possible it could take well over double the time a cooperative maneuver would have used. This is an operational issue where the pilots, air traffic controllers, implementation choices, and other factors may all play roles in choosing the maneuver.

There is a possibility aircraft will in a distributed manner come to different conclusions about which of the eight maneuvers to perform. This may result in a LOS because of a failure to agree on the same maneuver. However, the occurrence of these failures is relatively small when compared to the number of conflicts resolved. All aircraft can be synchronized on the same maneuver through a communication step as discussed in Chapter 3. Figure 4.58 shows most failures would have resulted in a LOS, thus communication for the sake of agreement must be taken into consideration during any future work.

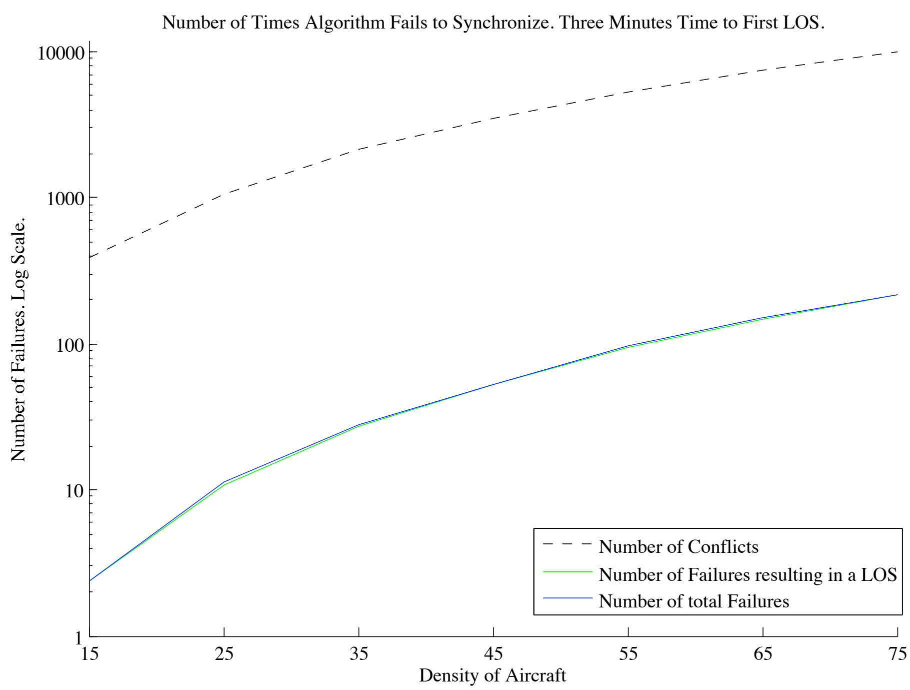

Figure 4.58: The number of times two aircraft fail to choose the same maneuver is relatively small compared to the number of total conflicts as aircraft density increases. All failures here are due to discrete calculation differences.

Chapter Five: Summary, Conclusions, and Future Work

A horizontal separation assurance algorithm that explicitly accounts for aircraft turn dynamics is explored in [3]. This algorithm is an improvement upon the current method [2] for resolving short-range conflict detections. A pair-wise, procedural approach to conflict detection and resolution is an important first step in developing a distributed separation assurance algorithm.

A simulation to test this algorithm's dynamics was developed. The simulation is built upon the turn dynamics also used in the separation assurance algorithm. Important performance thresholds of the algorithm have been found by varying key parameters to the simulation and measuring their effects upon the algorithms performance. These thresholds include the time when one aircraft maneuvering is no longer sufficient for maintaining separation, and the time when primary conflicts may result in a LOS even with both aircraft maneuvering. Types of conflicts and LOS that can occur when two maneuvering aircraft ignore all other aircraft have also been observed and categorized. Counting and classifying these types of LOS are the basis for continued development of this algorithm.

Horizontal secondary conflict detection and resolution is the primary next step in developing a distributed separation assurance algorithm. By first looking ahead at the other aircraft in the local area, it is possible to choose from the available maneuvers that either avoid secondary conflicts entirely, or maximizes the time at which a secondary conflict is detected if one is unavoidable. Once a maneuver has been decided upon, each aircraft could broadcast its intentions to every other aircraft. Given known intent, it is likely possible to avoid many of the observed secondary conflicts.

Another future step is to evaluate the algorithm's robustness to a delay before the maneuver begins. If one aircraft decides to start the maneuver later than expected, the amount of delay allowed until a LOS would occur on the expected maneuver must be determined.

Given the structure of the algorithm and the option of broadcasting intent, it may be possible to predict conflict detections for other aircraft. This is important if two pair-wise conflicts occur in a local area at the same time, such as in Figure 3.29. If they can predict each other's conflict, they could communicate intent to help resolve any secondary conflicts or LOS that may otherwise occur without coordination. The method of a timely, accurate, and secure communication is future work.

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