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Model predictive control of coordinated multi-vehicle formations[1]

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Abstract: *A generalized model predictive control (MPC) formulation is derived that extends the existing theory to a multi-vehicle formation stabilization problem. The vehicles are individually governed by nonlinear and constrained dynamics. The extension considers formation stabilization to a set of permissible equilibria, rather than a unique equilibrium. Simulations for three vehicle formations with input constrained dynamics on configuration space $SE(2)$ are performed using a nonlinear trajectory generation (NTG) software package developed at Caltech. Preliminary results and an outline of future work for scaling/decentralizing the MPC approach and applying it to an emerging experimental testbed are given.*

1 Introduction

Interest in stabilizing and maneuvering a formation of multiple vehicles has grown in recent years. Application areas include grid searching by coordinating robots, surveillance using multiple unmanned air or ground vehicles, and synthetic aperture imaging with clusters of micro-satellites. The existing literature contains methods for pre-computing control laws to achieve coordinated objectives. Methods utilizing potential functions for coordinating formations include [5] and [10], where graph theoretic tools are also effectively used in the latter reference. In both cases, individual vehicle dynamics correspond to fully actuated second order point masses. The same individual vehicle dynamics are considered by Young et al [13], where a leader-follower architecture is experimentally validated on wheeled robots. Stability and controllability by distributed local feedbacks is examined by Yamaguchi et al [12] for formations of kinematic robots. A contribution of our paper is that the individual vehicles may be governed by nonlinear and constrained dynamics.

A generalized multi-vehicle formation stabilization problem, free from a leader-follower architecture, is defined in this paper. The problem is similar, in the formation definition and use of a virtual leader, to that given by Egerstedt and Hu [4] who consider velocity control of kinematic robots. The difference is that the desired formation here is not necessarily a unique state for each vehicle in the formation. Moreover, we provide the necessary definitions and appropriate proofs for stability to a set of equilibria. The results in this paper are new in that model predictive control (MPC) is applied to the formation stabilization problem that we define. Another optimization based approach is explored in [11], where the problem is cast as a linear program subject to mixed integer constraints.

MPC is the most natural and in some cases the only methodology for control of systems that are governed by constrained dynamics. A recent thorough survey of nonlinear MPC stability theory is given by Mayne et al in [7]. The generalized formulation and conditions for stability stated in [7, 6] are used as a guide for the formulation here. Controller design for a multi-vehicle experimental testbed being developed at Caltech, where the individual vehicle dynamics are nonlinear and constrained, is a key motivation for this paper [2].

The organization of the paper is as follows. Section 2 details the MPC formulations in a generalized multi-vehicle settings and Section 3 focuses on multi-vehicle simulation examples. The first example considers the constrained dynamics of the testbed vehicles described in [2] and the second example considers simplified dynamics but examines distributed, synchronized MPC computations with the effects of model error between vehicles. The software used in the simulations is the Nonlinear Trajectory Generation (NTG) software package [8]. Conclusive remarks are given in Section 4.

2 MPC for vehicle formation stabilization

In this section, a multi-vehicle problem is posed and a general MPC formulation is stated as a solution. The formation problem definition is motivated by an objective of stability to a set of equilibria and by the requirement that all vehicles have equivalent roles relative to the formation, i.e. there is no leader/follower architecture. MPC stability results given in [7] are then extended to this new objective. The extension is much like the discrete-time robust MPC (to bounded disturbances) formulated in [6], where stability is guaranteed to a control invariant set.

2.1 MPC Problem Statement

The model predictive formulation given in this section is generalized in the incorporation of constraints and/or costs to achieve nominal asymptotic stability for nonlinear constrained systems. Consider a common state space for each

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vehicle $\mathbb{X}$ as either $T(SE(2))$ or $\mathbb{R}^4$. More generally, $\mathbb{X}$ may be a constraint space that is a convex and closed subset of $\mathbb{R}^n$. Consider the k vehicles with system models $\dot{x}_i = f_i(x_i, u_i)$, given $x_i(t_0) = x_{i0},\ t \in [t_0, \infty)$ for $i = 1, ..., k$. The set $\mathbb{U}$ is the input space and f_i is a vector field on $\mathbb{X}$ for all $i = 1, ..., k$. For notational ease later, introduce the vector notation

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}), \quad \boldsymbol{x} \in \mathbb{X}^k, \quad \boldsymbol{u} \in \mathbb{U}^k. \tag{1}$$

The vehicles are dynamically decoupled, which permits the statement that $\boldsymbol{x}$ lives in the Cartesian product space $\mathbb{X}^k$. At any current time t and for current state $\boldsymbol{x}(t) = \boldsymbol{x}$, the general (multi-vehicle) optimal control problem $\mathbb{P}_T^{mv}(\boldsymbol{x})$ is

$$\inf_{\boldsymbol{u}(\cdot)} \int_t^{t+T} q(\boldsymbol{x}(\tau), \boldsymbol{u}(\tau))\, \mathrm{d}\tau + V(\boldsymbol{x}(t+T)) \tag{2}$$

subject to equation (1) and terminal constraint

$$\boldsymbol{x}^u(t+T; \boldsymbol{x}) \in X_f.$$

Inside X_f we assume there exist (decentralized) local stabilizing controllers $\boldsymbol{\kappa_f}(\boldsymbol{x}) = [\kappa_f^1(x_1), ..., \kappa_f^k(x_k)]$. The cost optimal cost and control trajectory are denoted $J^*(\boldsymbol{x}, T)$ and $\boldsymbol{u}^*(\tau; \boldsymbol{x}, T)$, $\tau \in [t, t+T]$, respectively. In equation (2) we can set $t = 0$ as $\boldsymbol{f}(\cdot),\ V(\cdot)$ and $q(\cdot)$ are time-invariant.

The *MPC Problem* is to: 1) solve $\mathbb{P}_T^{mv}(\cdot)$ from state $\boldsymbol{x}$ at current time t, 2) implement the optimal input trajectory $\boldsymbol{\kappa}(\tau; \boldsymbol{x}, T) \triangleq \boldsymbol{u}^*(\tau; \boldsymbol{x}, T)$ for $\tau \in [0, \delta]$, where $0 \le \delta < T$, and 3) repeat step 1) from state $\boldsymbol{x} \leftarrow \boldsymbol{x}^*(\delta; \boldsymbol{x}, T)$ at current time $t \leftarrow t + \delta$ until $\boldsymbol{x} \in X_f$. Henceforth we assume the following

- A1 The minimum of $J^*(\cdot, T)$, $T \ge 0$, is attained.
- A2 Perfect knowledge of each vehicles dynamics governed by equation (1) (including initial condition) is available to all other vehicles.
- A3 Computation times are negligible.

Assumption A1 does not imply uniqueness of the optimal solution, provided local minima have equal cost. Assumption A2 is typical and A3 almost universal in the MPC literature. By ignoring uncertainty (absence of disturbances included) we can proceed by incorporating all vehicles in one (centralized) optimization over each horizon. Having more than one copy of such an optimization, say one per vehicle, would be redundant since they would all produce the same result. In the limit, assumption A3 permits $\delta = 0$, in which case $\mathbb{P}_T^{mv}(\cdot)$ is continuously resolved. The MPC controller in this case is denoted $\boldsymbol{\kappa}(\boldsymbol{x}, T)$ and the closed-loop system becomes

$$\dot{\boldsymbol{x}} = \begin{cases} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\kappa}(\boldsymbol{x}, T)), \ \forall \boldsymbol{x} \in X_T - X_f \\ \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\kappa_f}(\boldsymbol{x})), \ \forall \boldsymbol{x} \in X_f \end{cases}, \tag{3}$$

where X_T denotes the set of states $\boldsymbol{x}$ that can be steered to X_f by an admissible control in time T. The theoretical results that follow are stated in terms of equation (3), but hold for practical MPC controllers ($\delta > 0$). In the next sections we define a multi-vehicle formation and give generalized conditions on (q, V, X_f) for proving stability.

2.2 Multi-vehicle formation objective

The control objective is to steer the set of states $\mathbb{R} \ni t \mapsto \{x_1, \ldots, x_k\} \in \mathbb{X}^k$ to an *equilibrium formation*, which will be defined. The general formation set includes non unique permissible states at any given t, rather than precise locations for each vehicle at any t. Naturally, the equilibrium formation satisfies equilibrium conditions for all of the vehicles in the formation. Although modelled separately, the vehicle (closed-loop) dynamics become coupled by virtue of the formation objective.

Partitioning the state vector in terms of position and velocity subvectors will be useful for notational reasons. When $\mathbb{X} = T(SE(2))$ or $\mathbb{X} \subseteq \mathbb{R}^4$ denote, respectively,

$$x_i = \left[z_i,\ \theta_i,\ \dot{z}_i,\ \dot{\theta}_i\right] \quad \text{or} \quad x_i = [z_i,\ \dot{z}_i]\,,\ \forall i = 1, ..., k,$$

where z_i and $\dot{z}_i$ live in $\mathbb{R}^2$ and $(\theta_i, \dot{\theta}_i) \in TS^1$.

A precise definition of a formation of vehicles is now given. A $\mathbb{U}^k$-controlled positively invariant set M of equation (1) defines a subset of $\mathbb{X}^k$ for which $\forall \boldsymbol{x}(t_0) \in M$, there exits $\boldsymbol{u}(t) \in \mathbb{U}^k, \forall t \ge t_0$ such that $\boldsymbol{x}(t) \in M, \forall t \ge t_0$.

Definition 1 *Given a $\mathbb{U}^k$-controlled positively invariant set $M \subset \mathbb{X}^k$ of equation (1) and a formation reference $\mathcal{X}_r(t) \in \mathbb{X}$, $\forall t \ge t_0$, a k-vehicle formation associated with equation (1) is denoted $\mathcal{F}(M, k, \mathcal{X}_r(t))$ and defined as*

$$\mathcal{F}(M, k, \mathcal{X}_r(t)) = \left\{\boldsymbol{x} \in \mathbb{X}^k \mid (\boldsymbol{x}(t) - \boldsymbol{\mathcal{X}_r}(t)) \in M,\ \forall t \ge t_0\right\}$$

where $\boldsymbol{\mathcal{X}_r}$ is a column vector with k copies of $\mathcal{X}_r$ for each component.

The formation reference can be considered a virtual leader [4, 3]. In sub-vector notation, $\mathcal{X}_r$ is denoted $(\mathcal{Z}_r, \theta_r, \dot{\mathcal{Z}}_r, \dot{\theta}_r)$ or $(\mathcal{Z}_r, \dot{\mathcal{Z}}_r)$ depending on $\mathbb{X}$. In general, the incorporation of $\mathcal{F}(M, k, \mathcal{X}_r)$ in $\mathbb{P}_T^{mv}$ may be done by enforcing constraints over the entire horizon time, by design of the integrated cost or a combination of both. To be consistent with the existing theory of stabilizing MPC, only the case of designing the integrated cost q to accommodate the desired formation is considered here. An appropriate set that includes the restriction of $\mathcal{F}(M, k, \mathcal{X}_r(t))$ to equilibrium conditions is now given.

Definition 2 *An equilibrium formation $S_{eq} \equiv S_{eq}^X \times S_{eq}^U$ associated with $\mathcal{F}(M, k, \mathcal{X}_r(t))$ is the subset of $\mathbb{X}^k \times \mathbb{U}^k$ defined as*

$$S_{eq} = \Big\{ (\boldsymbol{x}, \boldsymbol{u}) \in \mathbb{X}^k \times \mathbb{U}^k \mid \dot{\boldsymbol{x}} = \boldsymbol{0},\ \boldsymbol{x} \in \mathcal{F}(M, k, \mathcal{X}_r(t)), \quad \dot{\mathcal{Z}}_r(t) = \dot{\theta}_r(t) = 0,\ \forall t \ge t_0 \Big\} \tag{4}$$

By definition, S_{eq}^X is a S_{eq}^U-controlled invariant set with respect to equation (1). Either S_{eq}^X is M or, if the constants $(\mathcal{Z}_r, \theta_r)$ are nonzero, it is a translated and rotated version of M with respect to an inertial frame in $\mathbb{X}^k$. Thus, the design of the formation determines the invariant set M. Examples for M are given in Section 3.1 and [3].

2.3 Stability

MPC of constrained systems is nonlinear, warranting the use of Lyapunov stability theory. The value function is typically employed as a Lyapunov function for stability analysis for nonlinear (constrained or not) and constrained linear systems. The generalized conditions in [7] regarding the terminal cost $V(\cdot)$, terminal constraint set X_f and local controller $\boldsymbol{\kappa_f}(\cdot)$ are here used as a guide. We make the following assumptions

A4 $\boldsymbol{f}$ is C^2 and $f_i(x_i, u_i)$ linearized around any (x_i, u_i) in the equilibrium set is controllable for any $i = 1, ..., k$.

A5 For all t of interest, $\boldsymbol{u}(t) \in \mathbb{U}^k$, a convex compact subset of $\mathbb{R}^{km}$ containing S_{eq}^U and $S_{eq}^U \equiv \{\boldsymbol{0}\}$. If $\dot{\boldsymbol{x}} = \boldsymbol{0}$ requires a constant $\boldsymbol{u} \neq \boldsymbol{0}$, assume we can translate the equilibrium input to the origin.

A6 $S_{eq}^X \subset \mathbb{X}^k$ and for all t of interest $\boldsymbol{x}(t) \in \mathbb{X}^k$.

A7 $q(\cdot)$ is C^2 and $\boldsymbol{u} \mapsto q(\boldsymbol{x}, \boldsymbol{u})$ is convex for each $\boldsymbol{x} \in \mathbb{X}^k$.

A8 $q(\cdot)$ is positive definite in $\boldsymbol{u}$ and semi-definite in $\boldsymbol{x}$, satisfying $q(S_{eq}) = 0$.

From A5 and Definition 2, $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{0}) = \boldsymbol{0}$ for each $\boldsymbol{x} \in S_{eq}^X$. Since controllability is assumed, linear control techniques may be used for $\boldsymbol{\kappa_f}$ when X_f is local. The stability conditions will require a Lyapunov function with stabilizing properties toward a set. An appropriate lemma, combining Lyapunov's stability theorem and LaSalle's theorem, is now given for the system in equation (1) with closed loop state-feedback control

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{v}(\boldsymbol{x})). \tag{5}$$

Lemma 1 *Let M and Ω be positively invariant sets for equation (5) with $M \subset \Omega \subset \mathbb{X}^k$ and Ω is compact. Let $V : \Omega \to \mathbb{R}$ be a continuously differentiable function such that*

$$V(\boldsymbol{x}) = 0 \text{ in } M \text{ and } V(\boldsymbol{x}) > 0 \; \forall x \in \Omega - M \tag{6}$$

$$\dot{V}(\boldsymbol{x}) = 0 \text{ in } M \text{ and } \dot{V}(\boldsymbol{x}) < 0 \; \forall x \in \Omega - M \tag{7}$$

Then, M is an asymptotically stable invariant set.

An appropriate choice for Ω is the largest bounded level of V contained in $\mathbb{X}^k$, containing M and satisfying equation (7). We now state a theorem based upon conditions B1-B4 in [7] that incorporates a generalized MPC implementation for stabilization of multiple vehicles to an equilibrium formation.

Theorem 1 *Assume that $J^*(\cdot)$ is C^1 and that X_T is compact. If $(V(\cdot), X_f, \kappa_f)$ satisfy*

1. *$X_f \subset \mathbb{X}^k$, X_f compact, $S_{eq}^X \subseteq X_f$.*
2. *$\kappa_f^i(X_f) \subset \mathbb{U}$, $\forall i = 1, ..., k$.*
3. *X_f is positively invariant for $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\kappa_f}(\boldsymbol{x}))$.*
4. *$V : X_f \to \mathbb{R}$ is C^1, satisfies equation (6) with $M = S_{eq}^X$ and $\left[\dot{V} + q\right](\boldsymbol{x}, \boldsymbol{\kappa_f}(\boldsymbol{x})) < 0$, $\forall \boldsymbol{x} \in X_f - S_{eq}^X$,*

then S_{eq}^X is an asymptotically stable invariant set of equation (3) with region of attraction X_T.

The proof of this theorem and the various cost/constraint based variants of MPC contained in conditions 1-4 are detailed in [3]. Constraints may invalidate the assumption that J^* is C^1; there are proofs that do not require this assumption [1]. Also, if J^* is radially unbounded and X_T is taken as a large level set of J^* (subset of X_T originally defined), then X_T is compact and we can delete this assumption.

3 MPC Coordinated Multi-Vehicle Simulations

This section details simulation examples of multi-vehicle coordinated control problems solved using MPC. The first example considers vehicle dynamics on configuration space $SE(2)$ with constrained inputs. A simple formation reference and the effects of adding a local terminal cost to an integrated cost for stabilization of the formation are investigated. The second example considers vehicles with linear, 2-D second order dynamics with only an integrated cost and no constraints. This example investigates what happens when assumption A2 is no longer true, i.e. when the (neighboring) vehicle models are no longer perfect and the MPC computations are distributed on each vehicle. The simulations are done using the NTG software package developed at Caltech [8].

3.1 Desired Formation and Reference

In the examples, the following invariant sets will be referenced for $k = 3$ vehicles. For $\mathbb{X} = T(SE(2))$ or $\mathbb{X} \subseteq \mathbb{R}^4$,

$$M_3^1 = \left\{ \boldsymbol{x} \in \mathbb{X}^3 \;\middle|\; ||z_i|| = 1, \; ||z_i - z_j|| = \sqrt{3}, \; \dot{z}_i = 0, \; \forall i, j = 1, 2, 3, \; i \neq j \right\}.$$

For $\mathbb{X} = T(SE(2))$,

$$M_3^2 = \left\{ \boldsymbol{x} \in M_3^1 \;\middle|\; \dot{\theta}_i = 0, \; \theta_i = 0, \; \forall i = 1, 2, 3 \right\},$$
$$M_3^3 = \{\hat{\boldsymbol{x}}(\boldsymbol{\alpha}, \boldsymbol{\xi})\}, \text{ for some } \hat{\boldsymbol{x}} \in M_3^2,$$

where $(\boldsymbol{\alpha}, \boldsymbol{\xi})$ are scheduling parameters, defined in Section 3.2, based on the locations of the vehicles at the end of the optimization horizon. The state space $\mathbb{X}$ for each vehicle in the simulations is $T(SE(2))$ for Example 1 and $\mathbb{R}^4$ for Example 2.

3.2 Example 1

The dynamics of the individual vehicles are taken from the multi-vehicle wireless testbed [2] (schematic and pictures given). Denoting the configuration $(w, y, \theta) \in SE(2)$ and assuming viscous friction, the equations of motion of a vehicle are:

$$\begin{aligned} m\ddot{w} &= -\eta\dot{w} + (F_s + F_p)\cos\theta \\ m\ddot{y} &= -\eta\dot{y} + (F_s + F_p)\sin\theta \\ J\ddot{\theta} &= -\psi\dot{\theta} + (F_s - F_p)r. \end{aligned} \quad (8)$$

The starboard and port fan forces are denoted F_s and F_p, respectively, and r denotes the (common) moment arm of the forces. To match the previous notation, $z_i^T = [w_i, y_i]$. An equilibrium point for the dynamics in equation (8) is any constant position and orientation (w_c, y_c, θ_c) with zero velocity. However, the linearized dynamics are not controllable around any equilibrium (uncontrollable subspace has rank 2). To achieve controllability, we can look at the error dynamics around tracking a constant velocity $\dot{w}_{nom}$ and heading θ_{nom} reference, where $\mathcal{X}_r(t)$ is defined as

$$[w_r(t_0) + t\dot{w}_{nom},\ y_r(t_0) + t\dot{y}_{nom},\ \theta_{nom},\ \dot{w}_{nom},\ \dot{y}_{nom},\ 0],$$

$\dot{y}_{nom} = \dot{w}_{nom}\tan(\theta_{nom})$ and $t \geq t_0$. The error state and inputs are $(x_{ei}, F_{sei}, F_{pei}) = (x_i - \mathcal{X}_r, F_{si} - F_{nom}, F_{pi} - F_{nom})$, $i = 1, 2, 3$, and the error dynamics are

$$\begin{aligned} m\ddot{w}_{ei} &= -\eta(\dot{w}_{ei} + \dot{w}_{nom}) + (F_{si} + F_{pi})\cos(\theta_{ei} + \theta_{nom}) \\ m\ddot{y}_{ei} &= -\eta(\dot{y}_{ei} + \dot{y}_{nom}) + (F_{si} + F_{pi})\sin(\theta_{ei} + \theta_{nom}) \\ J\ddot{\theta}_{ei} &= -\psi\dot{\theta}_{ei} + (F_{sei} - F_{pei})r, \end{aligned} \quad (9)$$

with $F_{nom} = (\eta\dot{w}_{nom})/(2\cos\theta_{nom})$. No state constraints are enforced, so $\mathbb{X} = T(SE(2))$. The inputs (F_s, F_p) live in the constraint space $\mathbb{U} = [0, 6] \times [0, 6] \subset \mathbb{R}^2$. The reachable space of the inputs is used to determine that the controllable equilibrium of equation (9) is now any constant position (w_c, y_c) with θ and velocity equal to zero. For the desired formation with three vehicles, the integrated cost function $q(\boldsymbol{x}, \boldsymbol{u})$ is

$$\sum_{i=1}^{3} \left\{ W_i \left[\sqrt{w_{ei}^2 + y_{ei}^2} - 1 \right]^2 + V_i \left[\dot{w}_{ei}^2 + \dot{y}_{ei}^2 \right] + U_i \left[F_{sei}^2 + F_{pei}^2 \right] \right\} + \sum_{i,j=1, i\neq j}^{3} \frac{W_{ij}}{2} \left[\sqrt{(w_{ei} - w_{ej})^2 + (y_{ei} - y_{ej})^2} - \sqrt{3} \right]^2 \quad (10)$$

In the simulations, $W_i = V_i = 1.0$, $U_i = 0.05$, and $W_{ij} = W_{ji}$ is 1.0 or 2.0 for all i, j. With respect to assumption A7, q is not C^1 or C^2 at the origin. Replacing the distance error-squared penalties above with distance squared error squared penalties (4th-order), e.g. $(z^2 - \rho^2)^2$, restores continuity as is done in [4]. Instead, equation (10) was implemented with a small constant under every radical to satisfy assumption A7. It was noted that since the formation is not near the origin, problems were seldom encountered without the small constant term and that performance was superior to the cost with 4th order distance penalty. The cost q is positive in $(\mathbb{X}^3 - M_3^1) \times \mathbb{U}^3$. Any element of S_{eq} must be an equilibrium point of equation (9), i.e. of the form $((w_c, y_c, 0, 0, 0, 0), (0, 0))$, so an appropriate terminal cost function must be designed for stability.

3.2.1 A Formation Terminal Cost Function: A terminal cost that satisfies Theorem 1 with $(\Omega, M) = (X_f, M_3^3)$ is now given. For an LQR problem associated with the linearization of equation (9) around any equilibrium, denote the corresponding positive-definite Riccati matrix as P. The terminal cost designed for this formation is a (schedulable) quadratic penalty on an error state $\boldsymbol{e}_i$, for each vehicle $i = 1, 2, 3$, with P as a weighting matrix. Specifically

$$V(\boldsymbol{x}) = \gamma \left(\boldsymbol{e}_1^T P \boldsymbol{e}_1 + \boldsymbol{e}_1^T P \boldsymbol{e}_1 + \boldsymbol{e}_3^T P \boldsymbol{e}_3 \right), \quad (11)$$

where γ is a positive, scalar weighting and the error state $\boldsymbol{e}_i$ for vehicle $i = 1, 2, 3$ is

$$\left[w_{ei} - g_{i1}(w_i, y_i),\ y_{ei} - g_{i2}(w_i, y_i),\ \theta_{ei},\ \dot{w}_{ei},\ \dot{y}_{ei},\ \dot{\theta}_{ei} \right].$$

The functions $g_{i1},\ g_{i2}$ are defined as

$$g_{i1}(w_i, y_i) = \cos\left(\bar{\xi} + \alpha_i\right), \quad g_{i2}(w_i, y_i) = \sin\left(\bar{\xi} + \alpha_i\right),$$
$$\bar{\xi} = (\xi_1 + \xi_2 + \xi_3)/3, \quad \xi_i = \arctan\left(\frac{y_{ei}}{w_{ei}}\right),$$

for each $i = 1, 2, 3$ and the scheduling variable α_i is defined $(\forall i \neq j)$ as

$$\alpha_i = \begin{cases} 0, & \text{for } |\xi_i - \bar{\xi}| < |\xi_j - \bar{\xi}| \\ 2\pi/3, & \text{for } |\xi_i - (\bar{\xi} + 2\pi/3)| < |\xi_j - (\bar{\xi} + 2\pi/3)| \\ -2\pi/3, & \text{for } |\xi_i - (\bar{\xi} - 2\pi/3)| < |\xi_j - (\bar{\xi} - 2\pi/3)|. \end{cases} \quad (12)$$

If $\xi_i = \xi_j$ for some $i \neq j$, then $\alpha_k, k \notin \{i, j\}$ is identified according to equation (12) and α_i and α_j are arbitrarily chosen to be (distinctly) what is left of $\{0, 2\pi/3, -2\pi/3\}$. If $\xi_1 = \xi_2 = \xi_3$, then each α_i is arbitrarily chosen to be one of $\{0, 2\pi/3, -2\pi/3\}$, not equal to any other α_j.

The idea behind this terminal cost is as follows: at the end of each optimization horizon, the vehicles are in some location relative to each other and the formation set. Calculating the angle ξ_i for each vehicle gives its angular location in the relative formation set frame. Taking the average of these locations ($\bar{\xi}$), one desired vehicle state is on the circle with angular location $\bar{\xi}$ and all other state variables matching the reference. This is the desired state for the vehicle with angular location closest to $\bar{\xi}$, according to condition 1 for α_i in equation (12). The desired states for the other vehicles, equilaterally spaced on the set, are chosen also according to equation (12). Thus, the terminal cost penalizes the weighted 2-norm of the error between each vehicle's state and it's desired state. The cost V is C^1 as long as $(w_{ei}, y_{ei}) \neq (0, 0)$ for all $i = 1, 2, 3$, which can be incorporated in the domain X_f. Given the LQR-based design of the terminal cost above, there exists a corresponding local set X_f in which κ_f can be taken to be the LQR controller. Instead of estimating X_f and enforcing it as a terminal constraint set, the effects of the integrated cost with and without the terminal cost are investigated. Without enforcing a terminal constraint set, stability via Theorem 1 is guaranteed if T is large enough (see [3]) and, when the terminal cost in equation (11) is employed, if scheduling occurs only at the initial optimization. Provably stable *reconfigurable* MPC, e.g. that might incorporate equation (11) with α_i changing at each optimization, is also an interesting topic for future research. A reconfigurable MPC would be attractive for multi-vehicle missions [9].

3.2.2 Simulation Cases and Results: In the simulations, the horizon and update times are 5.0 and 1.0, respectively. In addition to matching the appropriate initial condition for the state at each update, continuity of the input forces is implemented. Most simulations with only an integrated cost were observed to be stabilizing without collisions of the vehicles. Figure 1 shows an initial condition that resulted in stability but two vehicles passed through each other (an unacceptable scenario resulting in collision for real vehicles). The

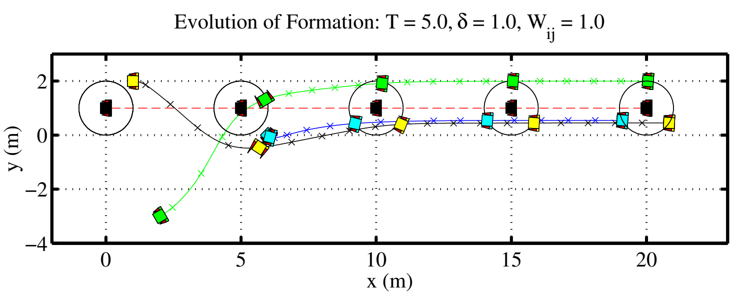

Figure 1: MPC formation with integrated cost: collision occurs.

(black) vehicle in the center of the circle represents the formation reference and the x's along the trajectories of the formation vehicles represent the updates in the MPC controller. The affect of adding the scheduled terminal cost in equation (11), keeping the relative weight at 1.0, is shown in Figure 2. Other simulations cases for various initial conditions and cost weight values are detailed in [3].

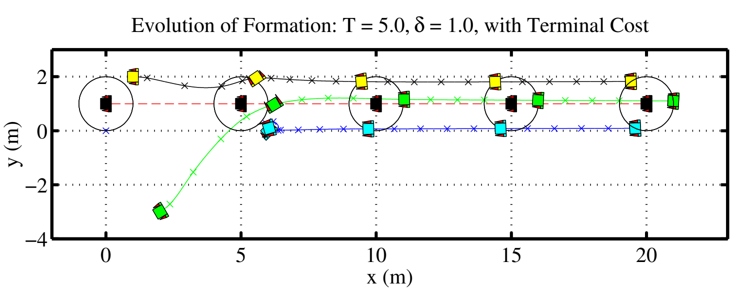

Figure 2: Addition of terminal cost to stabilize formation.

3.3 Example 2

In this example, individual vehicle dynamics are simplified to observe how a stabilizing global MPC policy is affected when it becomes distributed and there is model uncertainty between the vehicles in the formation. The dynamics for each (point mass) vehicle are linear, double-integrators on state space $\mathbb{X} = \mathbb{R}^4$ and input space $\mathbb{R}^2$. The same cost function in equation (10) is used. In the global version, one (centralized) optimization is performed to compute the MPC law for every vehicle at every update time δ. In the local version, there are 3 separate MPC optimizations at each update time, one for each vehicle. Each local optimization incorporates the correct model for the host vehicle and assumes that the other vehicles go in straight lines according to the shared initial conditions. In both cases, a nontrivial reference is implemented (see [3] for details).

The horizon length is $T = 2.0$ seconds and update time $\delta = 0.5$ seconds. In the plots, the vehicles are represented as triangles, where each triangle points in the direction of it's velocity vector, and the reference is represented as a red square. As before, the reference trajectory is marked by a dashed red line. The global (*full info*) MPC solution vehicles are represented by the three triangles in black, with colored squares at the center of each triangle. The *local info* vehicles are represented by triangles in full color, with matching colors corresponding to matching initial conditions for the first optimization. Each triangle's trajectory is marked by a line and each figure shows the formation at points in time along the entire time history. A global and local model predictive result is shown in Figure 3 for a velocity error weighting of $V_i = 1.0$, $i = 1, 2, 3$. The top plot corresponds (roughly) to the first half of the time history and the bottom plot shows the remaining portion. The global formation is stable throughout

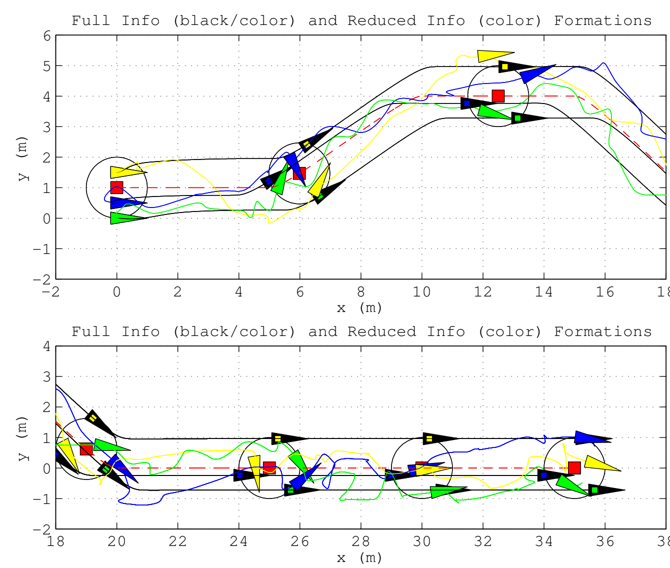

Figure 3: Global and local MPC simulations ($V_i = 1.0$)

the entire time history, while the local formation is very unstable. In fact, some of the local vehicles are observed to intersect each other, resulting in collision in a real implementation.

The response from the same set of initial conditions but with increased weight on the velocity error penalty (20.0) is shown in Figure 4. For the particular choice of local models (and formation reference) there is a large degree of sensitivity to

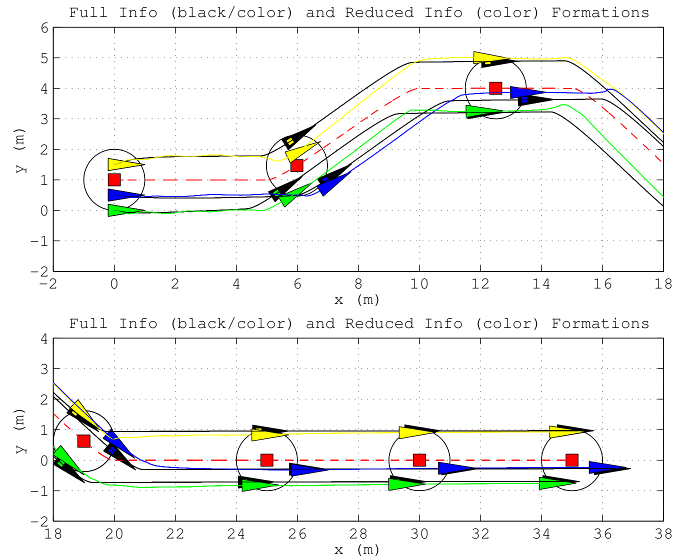

Figure 4: Global and local MPC simulations ($V_i = 20.0$)

such weighting changes, as is evident by the local formation responses in Figure 4. The global formation appears on the other hand to be insensitive to the weighting change. The degree of sensitivity of the local formation could likely be reduced by making a more "educated" model for the reduced order vehicles. A good choice might be to assume that the other vehicles travel at constant acceleration, perhaps equal to the known initial value of the reference acceleration. Development of the theory of distributed MPC would likely guide the model classes from which distributed models could be chosen, given the full (nominal) models of actual vehicles.

4 Conclusions and Extensions

A generalized constrained and nonlinear MPC formulation with guaranteed stability has been detailed in this chapter for asymptotic stabilization of multiple vehicles to an equilibrium formation. A multi-vehicle coordination problem that admits a generalized formation objective was then posed. The objective allows that vehicles are stabilized to a set of permissible equilibria, rather than a precise location for each vehicle in the formation. There is also no particular role assignment in this formulation, although a formation reference is defined and could be considered a virtual leader. The theory of MPC is continuing to branch out to address uncertain environments and recent results have investigated real-time issues associated with this methodology [3]. It is realistic to assume that computational tools for MPC will only improve with time. The extension of MPC to a distributed problem adds new elements of complexity to the theory, from which many new interesting problems can be examined. Of particular interest is computational uncertainty (distributed and local) and reduced order model effects of the environment, as explored in Section 3.

The unification of these topics is related to a multi-vehicle experimental testbed being developed at Caltech [2]. The individual vehicle dynamics and inputs are subject to constraints and the objectives include real-time formation maneuvers while avoiding collision and obstacles. The MPC framework outlined in this paper is thus a natural choice to meet these objectives. The testbed will be subject to the issues that arise from applying MPC real-time, e.g. model-uncertainty and non-trivial computation times. These topics as well as exploring other variants of MPC and the theoretic implications of distributing the computations over networks will be explored in future work.

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