SRS = at random without repl.

IID = independent identically distributed = at random with repl.

extra notes 18 Jan [redacted]

pop

AMI patients

where?

when?

sample (AMI at DH)

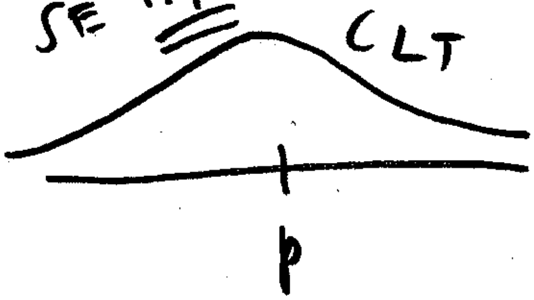
n = 400

F

(n "large")

$\hat{SE}$ 1.9%

CLT



long run histo (density) of $\hat{p}$ in repeated sampling

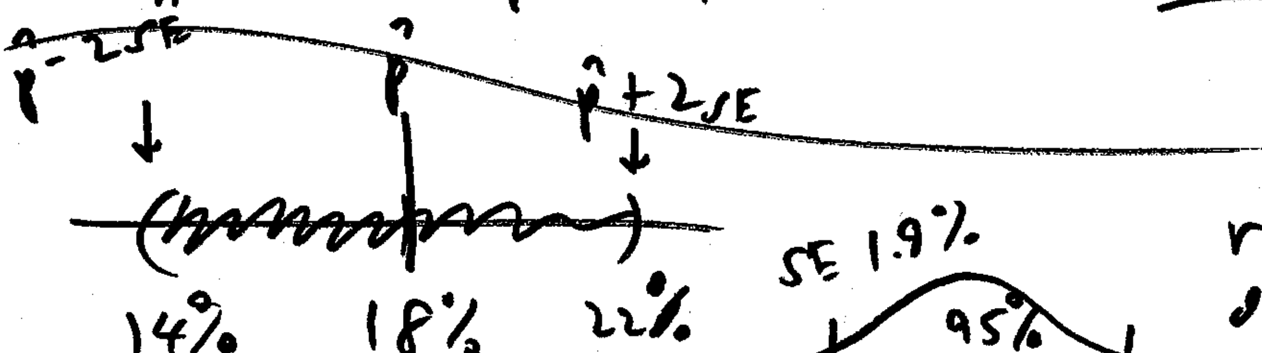
I think p is around 18%, give or take 1.9%

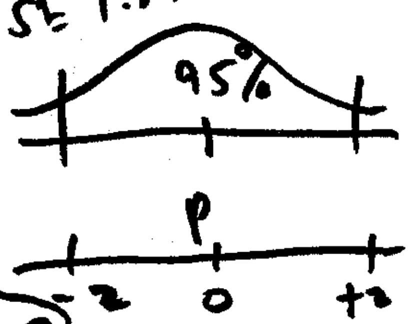
est SE

$$\boxed{\hat{SE}_{IID}(\bar{y}) = \frac{\hat{\sigma}}{\sqrt{n}}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\hat{SE}(\hat{\theta}_{MLE}) = \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}} \quad n \leftarrow 400$$

$$= \sqrt{\frac{(0.18)(0.82)}{400}} \doteq \underline{\underline{0.019}}$$



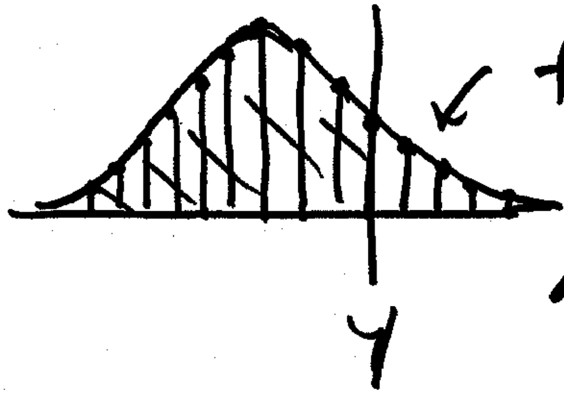


r.s. dist of $\hat{p}$

$$P_F\left(\boxed{\underline{p - 2\hat{SE}} \le \hat{p}} \le \underline{p + 2\hat{SE}}\right)$$

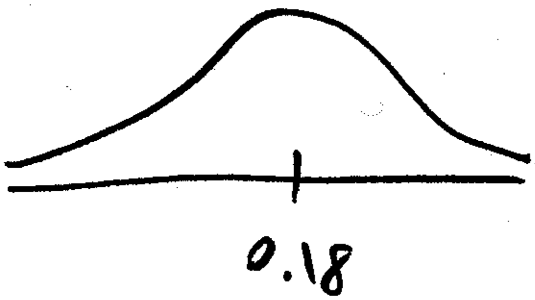
$$\doteq 0.95$$

$$= P_F\left(\hat{p} - 2\hat{SE} \le p \le \hat{p} + 2\hat{SE}\right)$$

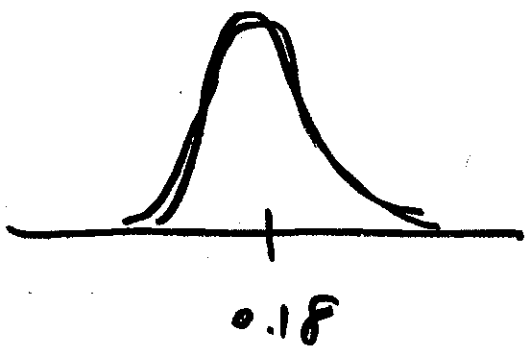


$f_Y(y)$: $\int_{-\infty}^{\infty} f_Y(y) = 1$

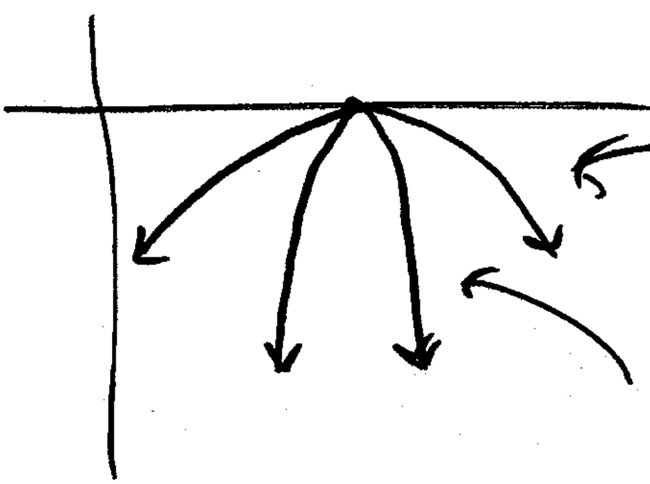
density of Y



$\ell(\theta \mid y)$ $(n = 400)$



$\ell(\theta \mid y)$ $(n = \underline{\underline{1600}})$



$n = 400$

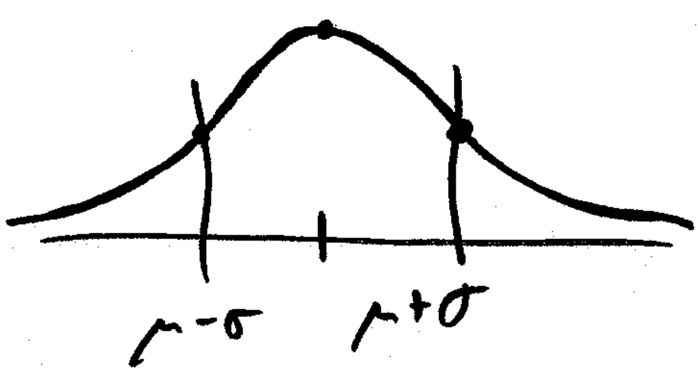
$\boxed{\ell\ell}$

$n = 1600$

$$\hat{V}(\hat{\theta}_{MLE}) = \frac{s(n-s)}{n^3}$$

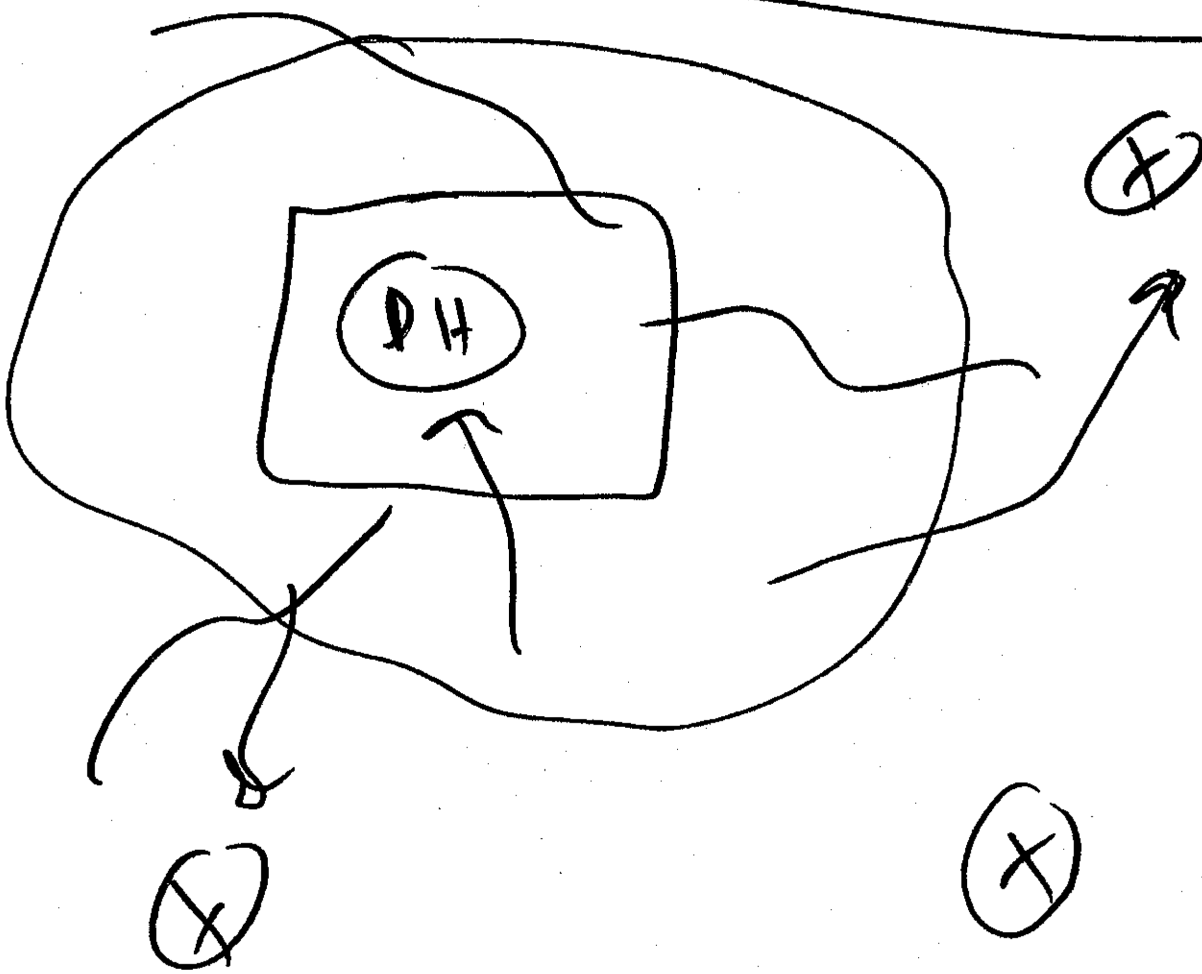
$$= \frac{s}{n} \frac{(n-s)}{n} \frac{1}{n}$$

$$= \frac{\hat{\theta}(1-\hat{\theta})}{n}$$



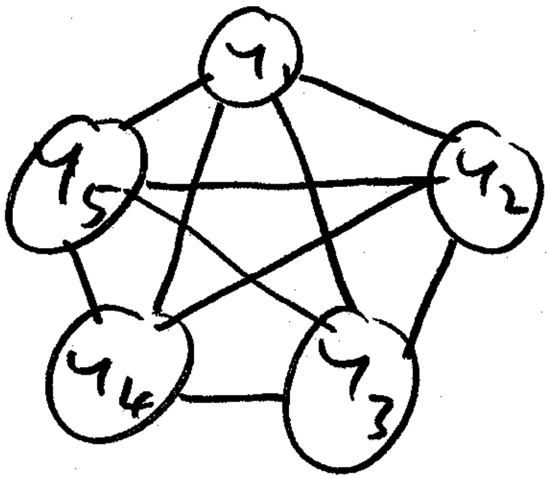
Y_i: part of inf. exch. collective

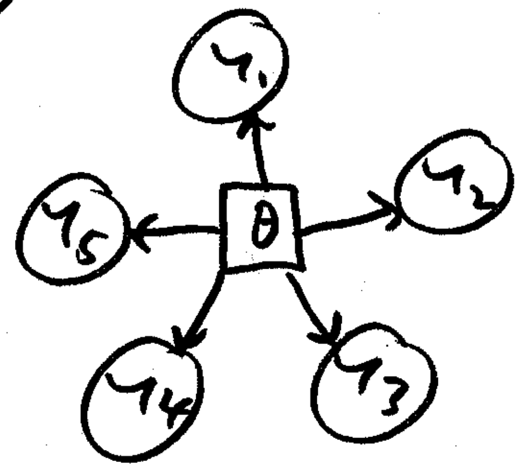
willing to think of Y_i as like a sample from a pip



the value of conditional independence (when it's appropriate to assume)

exch | $n = 5$ | (cond. ind. $\mid \theta$)





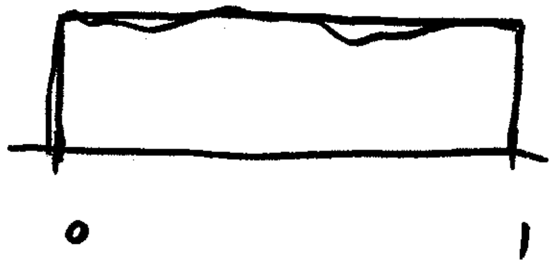
—— related

$A \longrightarrow B$ A "causes" B (knowing A allows me to simulate B)

$\binom{n}{2} = \frac{n(n+1)}{2} = 10 = \underline{\underline{O}}(n^2)$ links

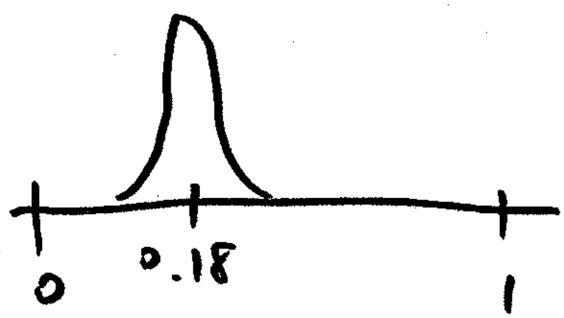
$\binom{n}{1} = n = 5 = \underline{\underline{O}}(n)$ links

θ

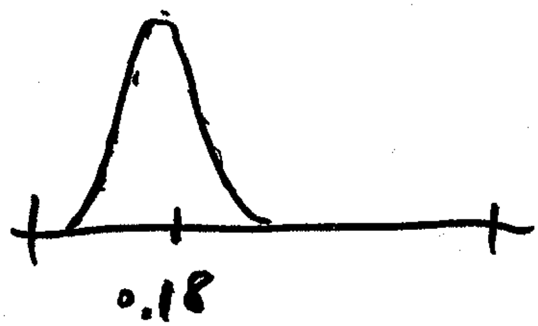


$p(\theta)$ prior

if prior = flat



$\ell(\theta | y)$ lik



then

$p(\theta | y)$ post

$\doteq \ell(\theta | y)$

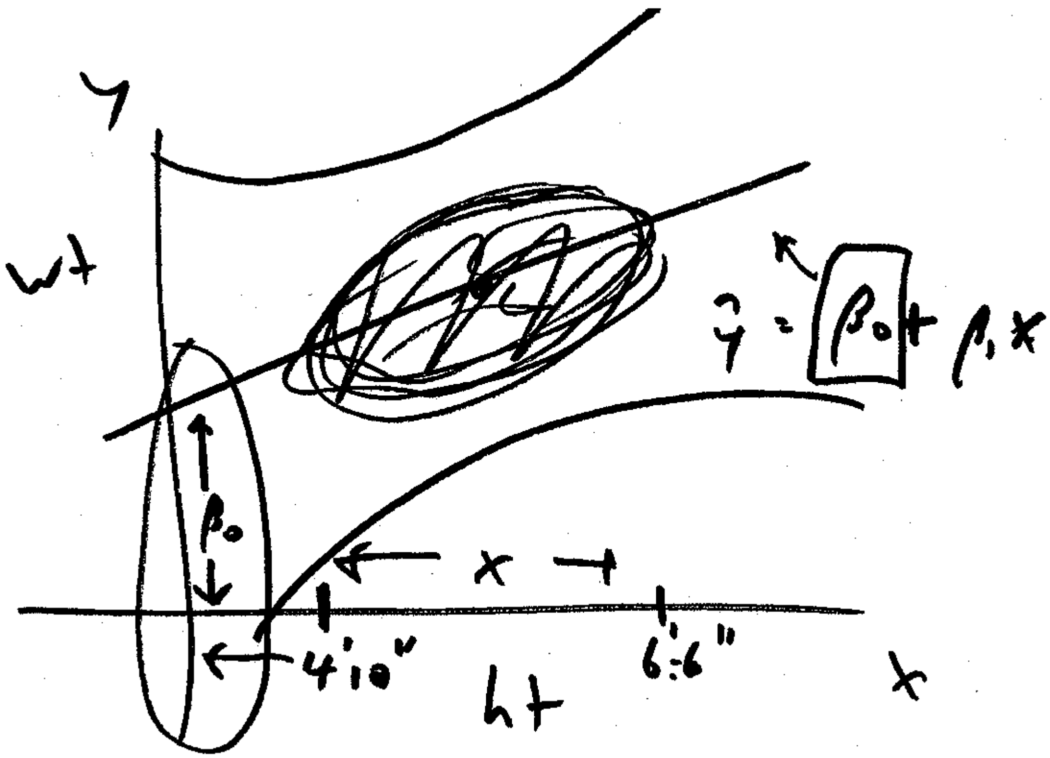
$$U(\theta, \hat{\theta}) = -(\theta - \hat{\theta})^2$$

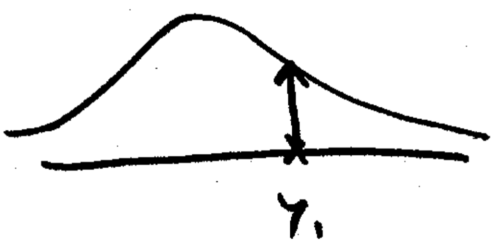
squared error loss $\rightarrow$

optimal Bayes est. is post. mean

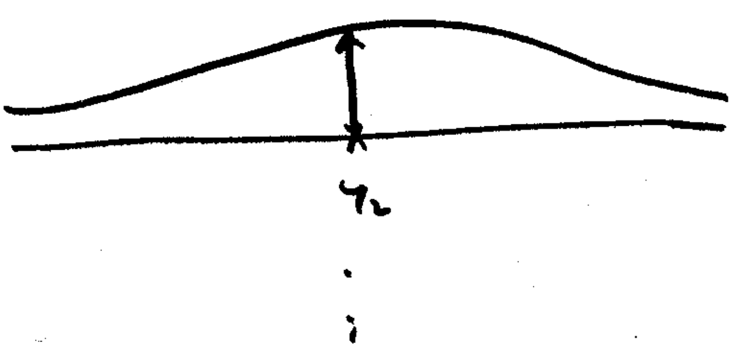
& prior info is flat

if lik is $\doteq N$ (n large) post. mean $\doteq$ MLE

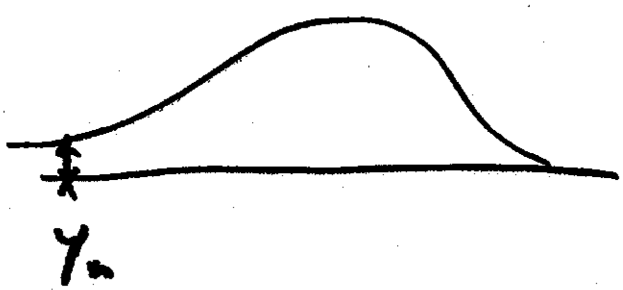




$p(y_1 \mid y_2, \dots, y_n, \underset{\sim}{x})$

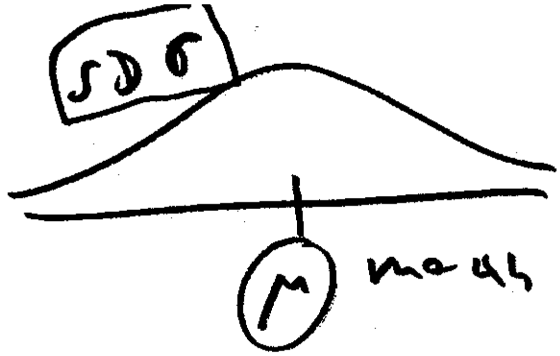


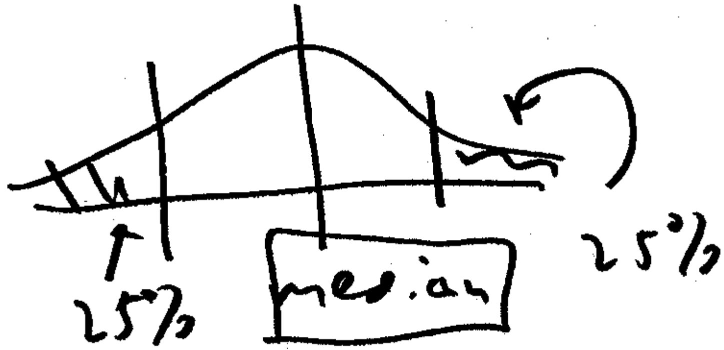
$p(\underline{y_2} \mid y_1, y_3, \dots, y_n, \underset{\sim}{x})$



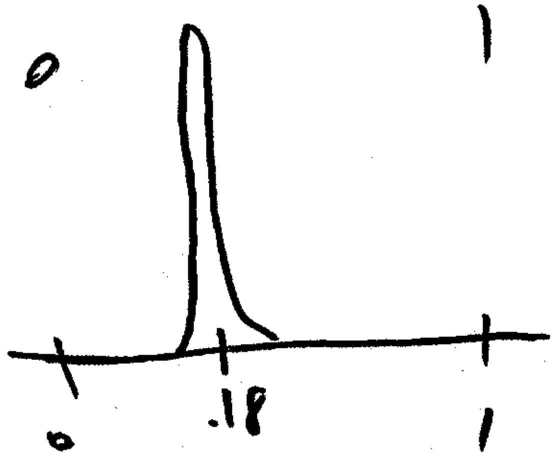
$p(y_n \mid y_1, \dots, y_{n-1})$

no





$p(\theta) = \underline{O}(1)$



$l(\theta | y) = \underline{O}(n)$

$$p(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1}$$

$$p(\theta|y) = c \; p(\theta) \, l(\theta|y)$$

$$= c \; \underline{\theta^{\alpha-1}(1-\theta)^{\beta-1}} \, \theta^{s}(1-\theta)^{n-s}$$

prior & post have same math. form → this prior is said to be <u>conjugate</u> to this lik.

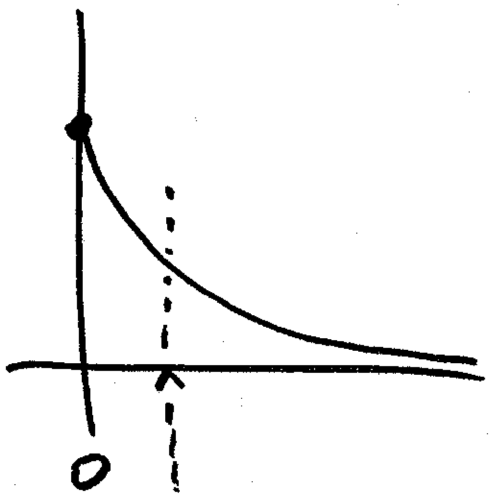
$$0.95 = \int_{0.05}^{0.30} \text{Beta}\left(\theta \,\middle|\, \alpha, \frac{17\alpha}{3}\right) d\theta$$

(90c)

$$Y_{ij} = \mu + a_i^{H} + a_{ij}^{P}$$

hosp. pt. grand mean

$a_i^{H} \overset{iid}{\sim} N(0, \underline{\underline{\sigma_H^2}})$ $\quad a_{ij}^{P} \overset{iid}{\sim} N(0, \underline{\underline{\sigma_P^2}})$



$\ell(\sigma_H^2 \mid y)$

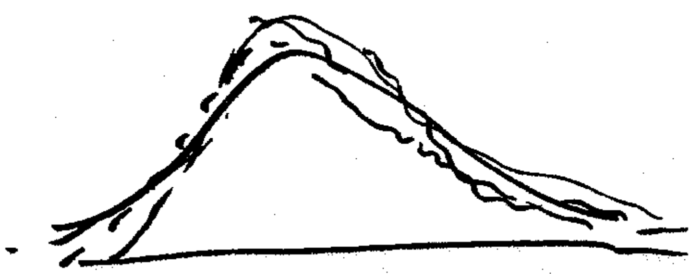
$\boxed{\hat{\sigma}_H^2 = 0}$

MLE

too small

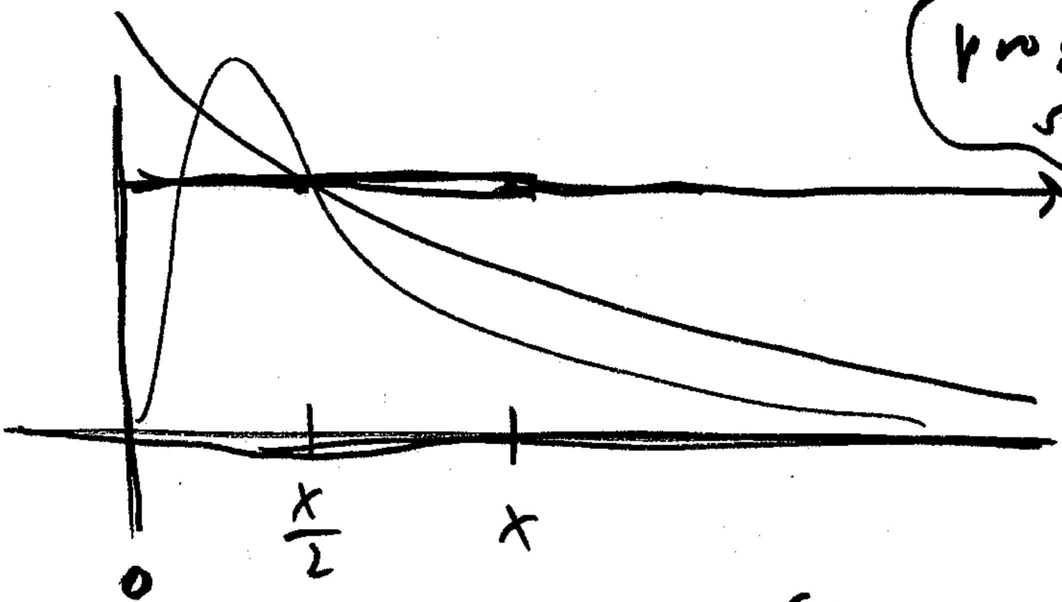
nom. 95% $\rightarrow$ $\boxed{\hat{\mu} \pm 2\,\widehat{SE}(\hat{\mu})}$

actual 65% - 70%



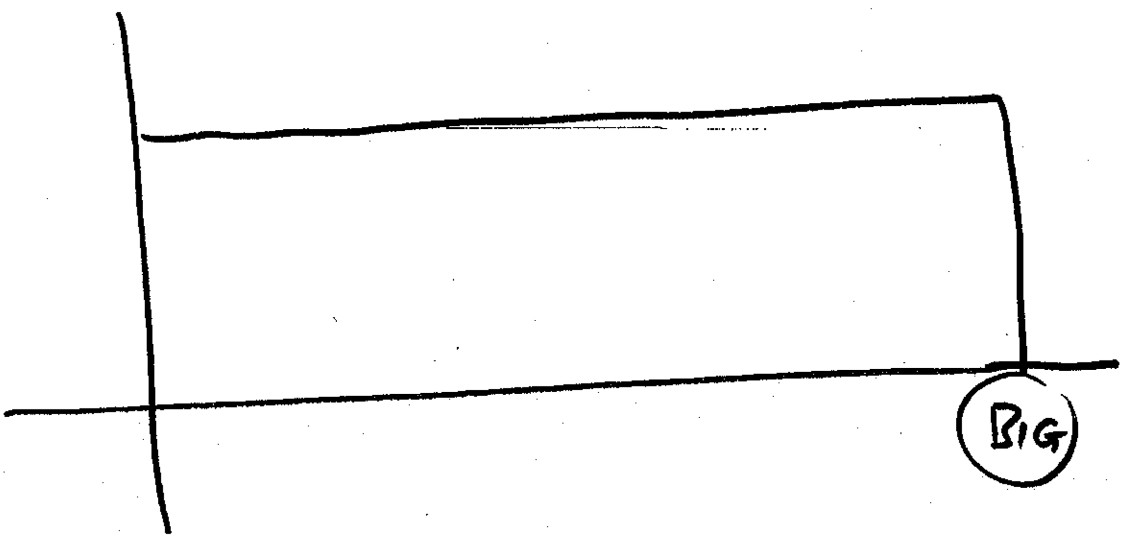
$p(\theta_3 \mid y)$

problem set 2 #1



prior with $\int = +\infty$

$\leftrightarrow$ <u>improper prior</u>



problem set 2 #2

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad n = 2$$

① with repl.: IID

② without repl.: ID? yes indep? no $\frac{1}{3} \cdot \frac{1}{2}$

without repl.

Y_1 \ Y_2	1	2	9	
1	X	1/6	1/6	1/3
2	1/6	X	1/6	1/3
9	1/6	1/6	X	1/3
	1/3	1/3	1/3	1

exch? yes

(a) IID

$Y_1 \backslash Y_2$	1	0	
1	θ^2	$\theta(1-\theta)$	θ
0	$\theta(1-\theta)$	$(1-\theta)^2$	$1-\theta$
	θ	$1-\theta$	1

$0 < \theta < 1$

(b) exch

$Y_1 \backslash Y_2$	1	0	
1	α	$\theta-\alpha$	θ
0	$\theta-\alpha$	$1-2\theta+\alpha$	$1-\theta$
	θ	$1-\theta$	1

exch but not IID: $\alpha \neq \theta^2$

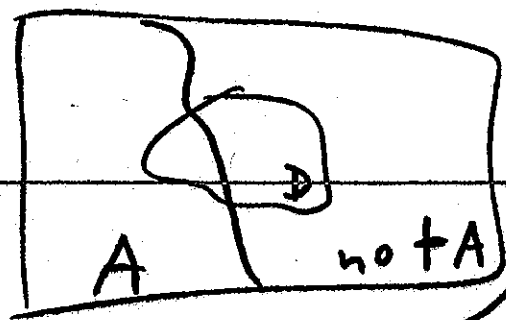
$< \alpha < \theta$

$\max(0, 2\theta-1)$

exch.

$P(Y_1 = y_1 \ \& \ Y_2 = y_2) =$

$$P(Y_1 = y_2 \ \& \ Y_2 = y_1)$$



$$P(D) = P(D \text{ and } A) + P(D \text{ and not } A)$$

$$= P(A)\,P(D \mid A) + P(\text{not } A)\,P(D \mid \text{not } A)$$

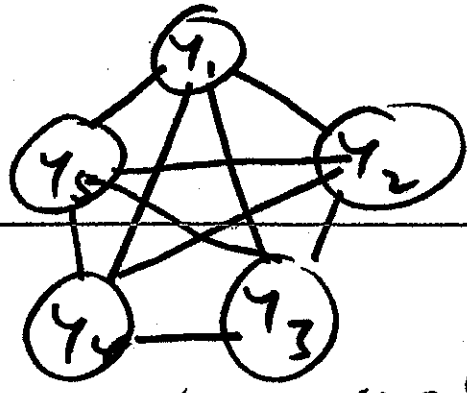
predictive distribution for future data

("the trick")

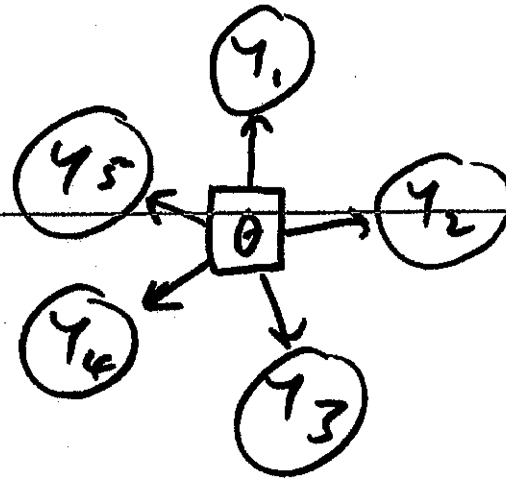
{weighted average} or {mixture}

$$p(y_1, \ldots, y_n) = \int_{-\infty}^{\infty} p(y_1, \ldots, y_n, \theta)\, d\theta$$

$$= \int_{-\infty}^{\infty} p(y_1, \ldots, y_n \mid \theta)\, p(\theta)\, d\theta$$



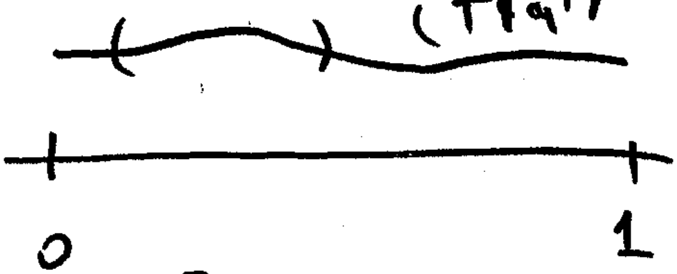
$(n=5)$



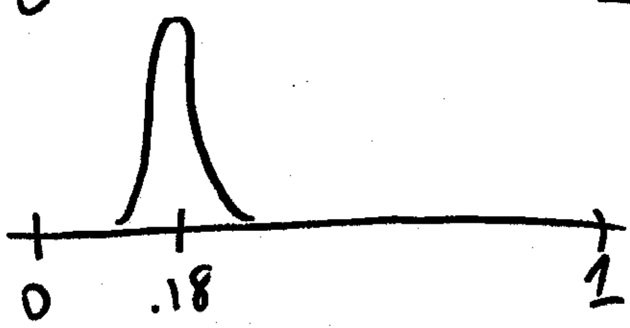
$$\binom{n}{2} = \frac{n(n+1)}{2} = \underline{O}(n^2)$$

$$\binom{n}{1} = n = \underline{O}(n)$$

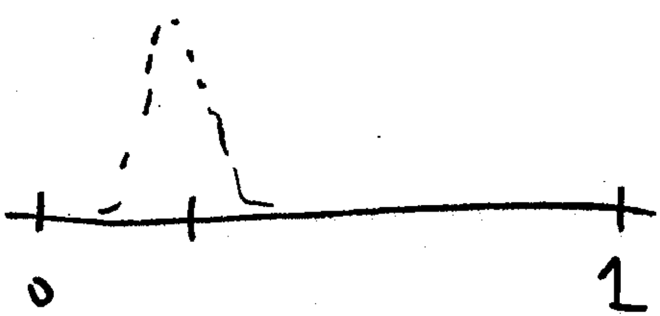
(diffuse)
(flat)



$p(\theta)$ little or no info about θ



$\ell(\theta \mid y)$



$p(\theta \mid y) \doteq \ell(\theta \mid y)$ in this case