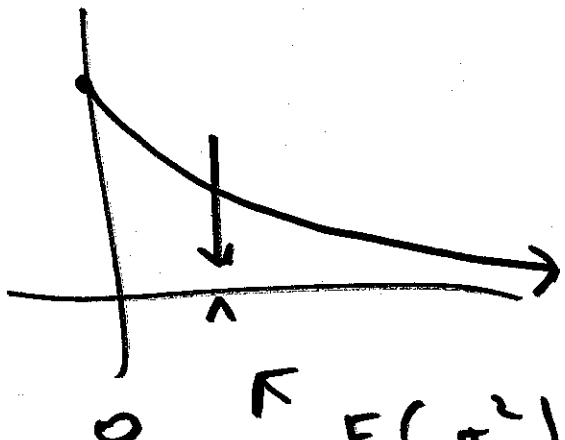
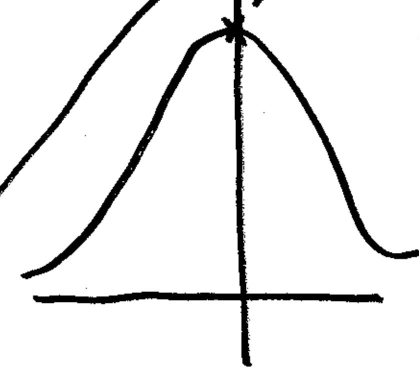
density of mixture of 2 normals $= p\, N(\mu_1, \sigma_1^2) + (1-p)\, N(\mu_2, \sigma_2^2)$



$p(\sigma^2 | y) \doteq \ell(\sigma^2 | y)$

$\hat{\sigma}^2_{MLE} = 0 \leftarrow$ poor inferential summary

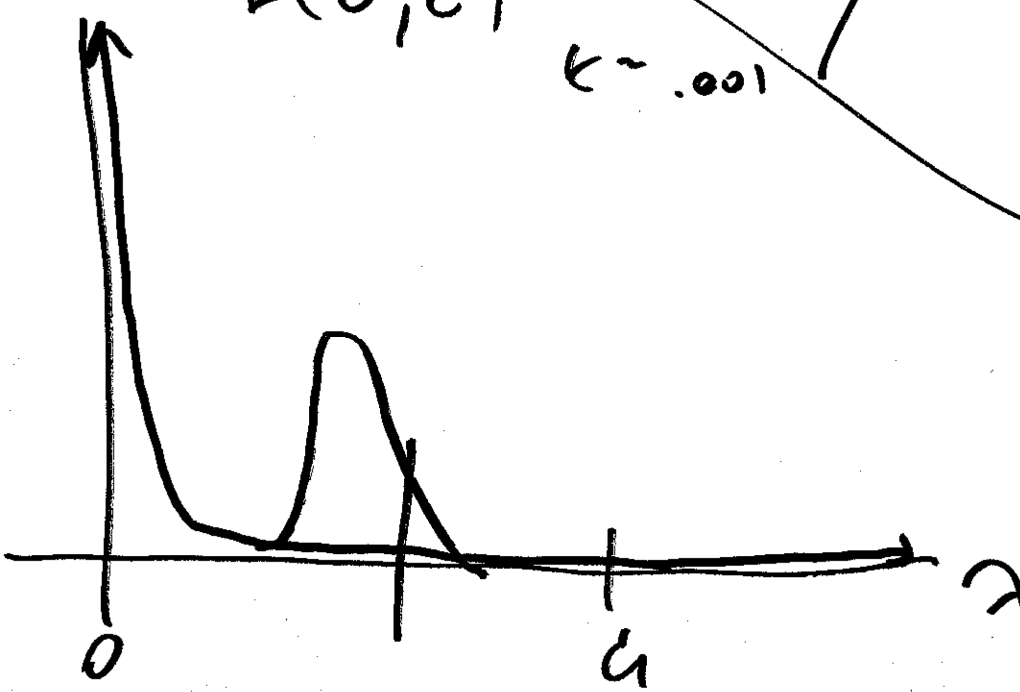
$E(\sigma^2 | DB)$ better

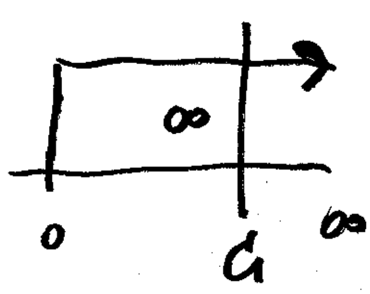


$E(\theta | DB) \doteq$ argmax lik $= \hat{\theta}_{MLE}$

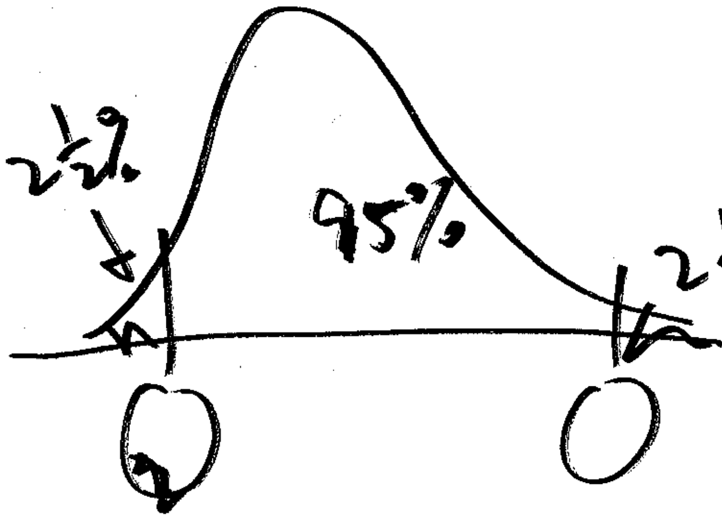
$\Gamma(\epsilon, \epsilon)$

$\epsilon \sim .001$



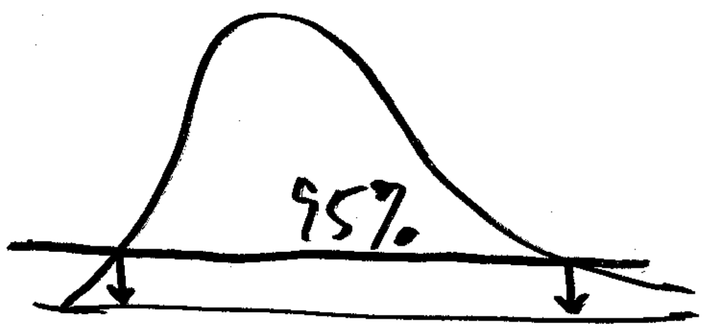


(2)



equal-tails

λ



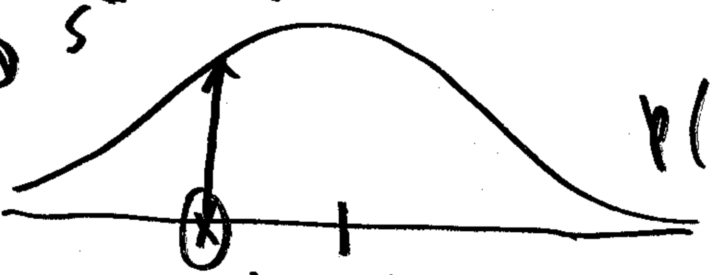
highest posterior density (HPD) interval (region)

λ

log scoring

$\boxed{\log p(y^* \mid y\, B)}$

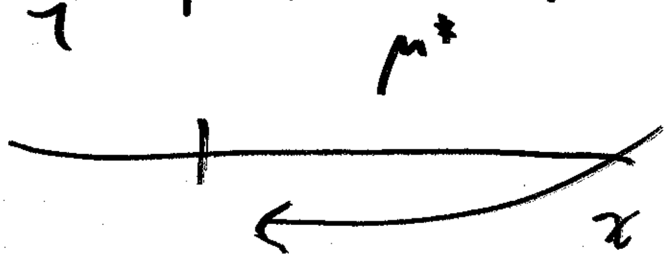
SD s^*



$p(y_{n+1} \mid y\, B)$

$y^* = y^*_{n+1}$ $E(y_{n+1} \mid y\, B)$

μ^*



$= \frac{y_{n+1} - \mu^*}{s^*}$

(optimal scoring

③

$Y_i \in \mathbb{R}$

$(Y_1, \dots, Y_n) \longrightarrow$ population P

$\downarrow$

$\underbrace{(Y_1, Y_2, \dots)}$

you see

CDF F

your uncertainty

of $(Y_1, Y_2, \dots)$

exchangeable

(pop. CDF)

prior on sampling dist

Bayesian nonparametrics (BNP)

$\theta \in (0,1)$

Y_i binary

$\downarrow$

$F \sim p(F \mid \mathcal{B})$

$\theta \sim p(\theta)$

(1937) $(Y_i \mid F) \overset{iid}{\sim} F$

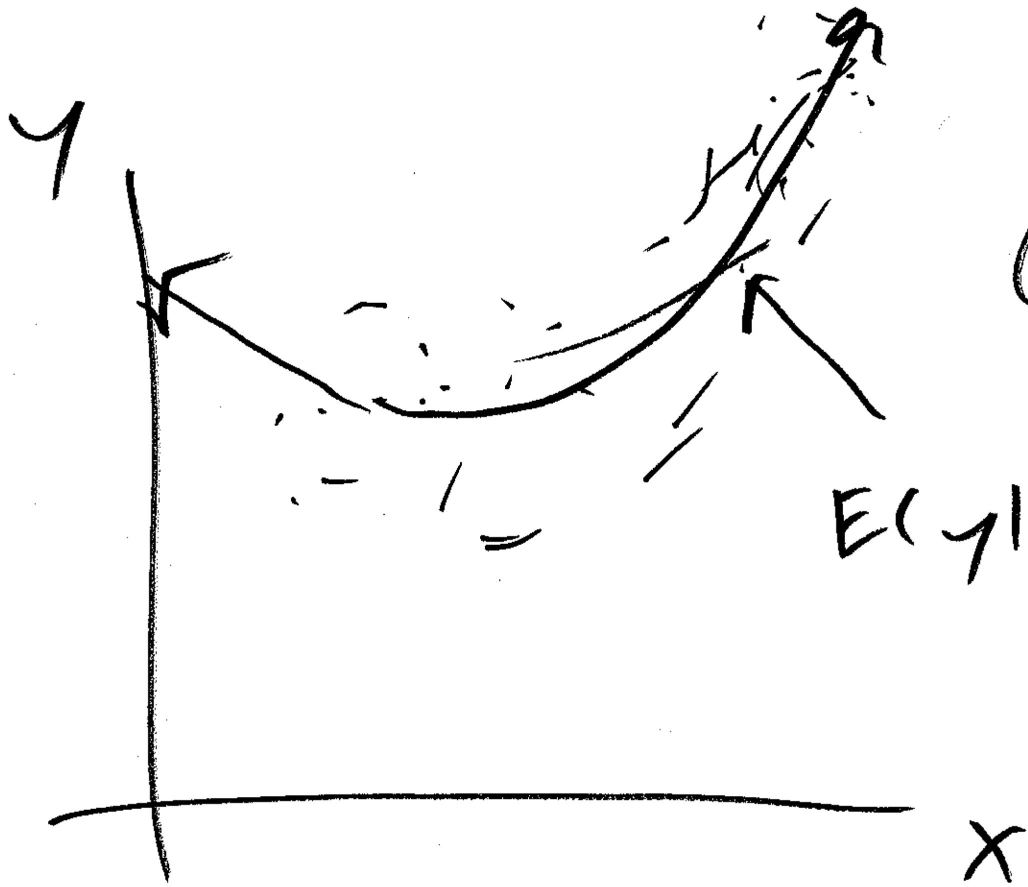
putting prior on function

$(Y_i \mid \theta) \overset{iid}{\sim}$ Bernoulli (θ)

① $(i = 1, \dots, n)$

uniquely specifies sampling dist

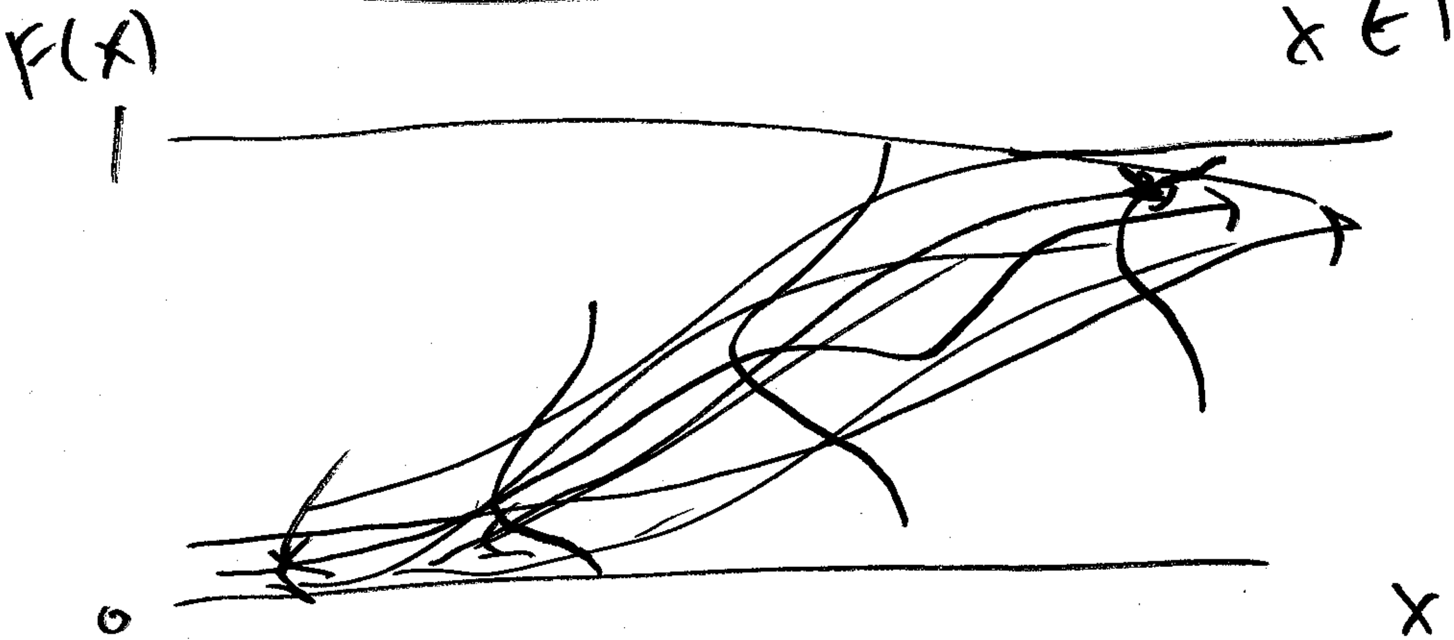
$\mathcal{F} = \{$all CDFs on $\mathbb{R}\}$



②

$E(y|x) = fn$
uncertain

$x \in \mathbb{R}$



1970
Tom Ferguson

$$p(F) = \{ N(\mu, \sigma^2) \text{ CDFs} : -\infty < \mu < \infty, \ \sigma^2 > 0 \}$$

Cromwell's Rule

if $P(B) = 0$ then $P(B \mid A) = 0$

unknown → B, data → A

1 $\qquad P(B \mid A) = 1$

~~B (when k > 1)~~

$p(\overbrace{\mu, \sigma, \nu} \mid y_n^B)$ hard to visualize $_n$

$$p(\mu \mid y\, B) = \iint p(\mu, \sigma, \nu \mid y\, B)\, d\sigma\, d\nu$$

W.S.
Gossett (1908) (frequentist) ⑥

(Guinness)

$$(Y_i \mid \mu, \sigma^2) \overset{\substack{i = 1, \dots, n \leftarrow \text{small} \\ \text{iid}}}{\sim} N(\mu, \sigma^2)$$

μ, σ^2 unknown

$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$	$s^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2$

Gossett wanted repeated-sampling

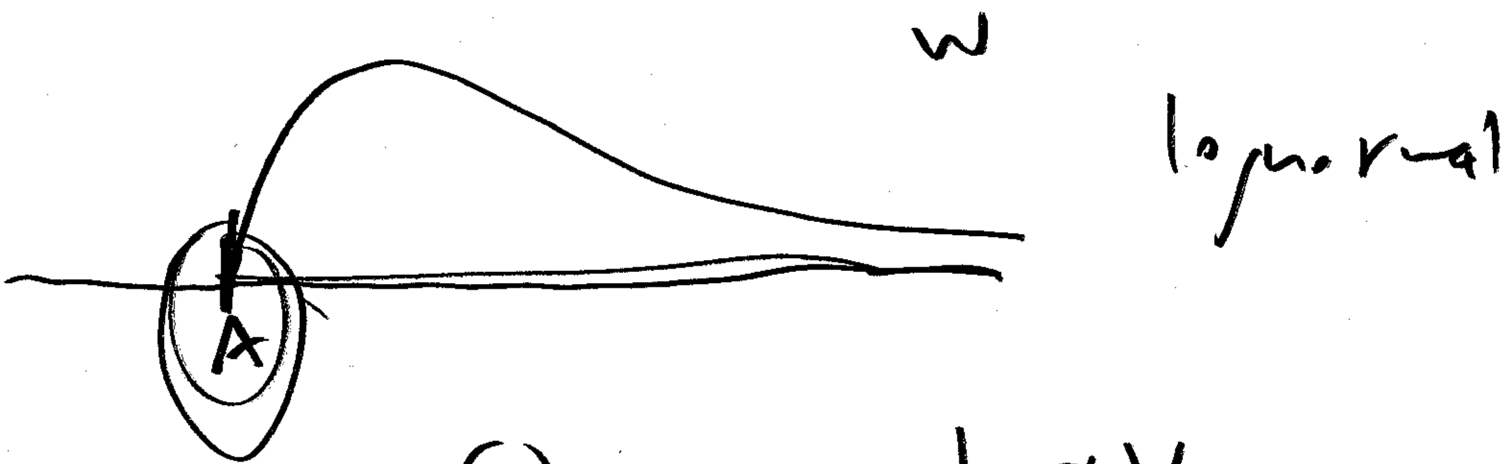
distribution of $\dfrac{\bar{y} - \mu}{s/\sqrt{n}}$

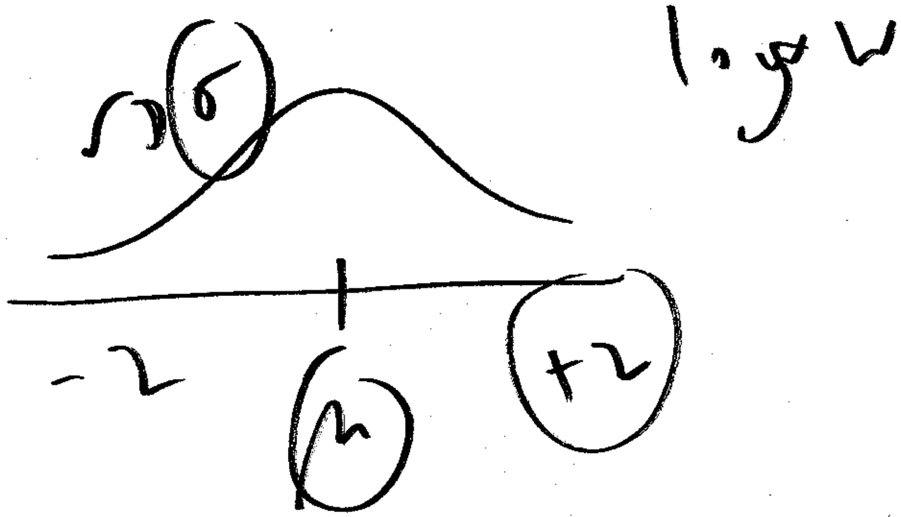
1942 Nick Metropolis, Stan Ulam

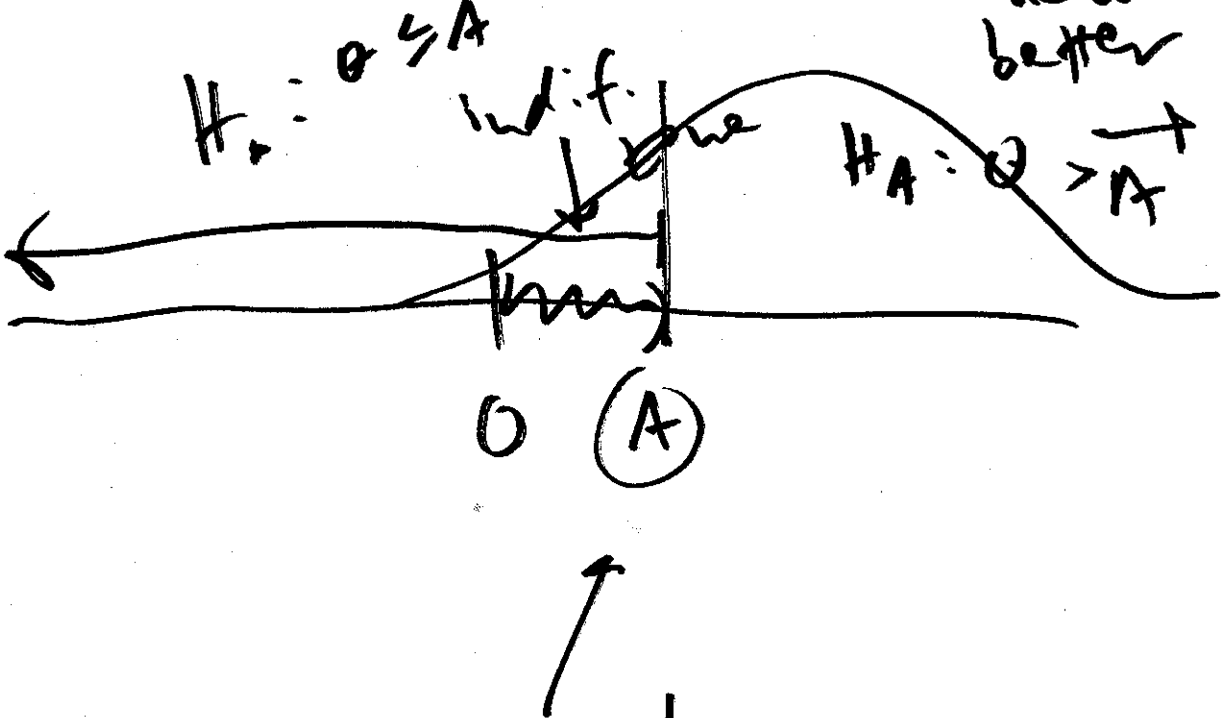
$$\left(\frac{\theta}{1-\theta}\right)^s = \exp\left[\log\left(\frac{\theta}{1-\theta}\right)^s\right]$$

$$= \exp\left[\underset{h_1(y)}{s} \boxed{\log\left(\frac{\theta}{1-\theta}\right)}\right]$$

$$w \sim LN(\mu, \sigma^2) \longleftrightarrow \log w \sim N(\mu, \sigma^2)$$







pract.
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