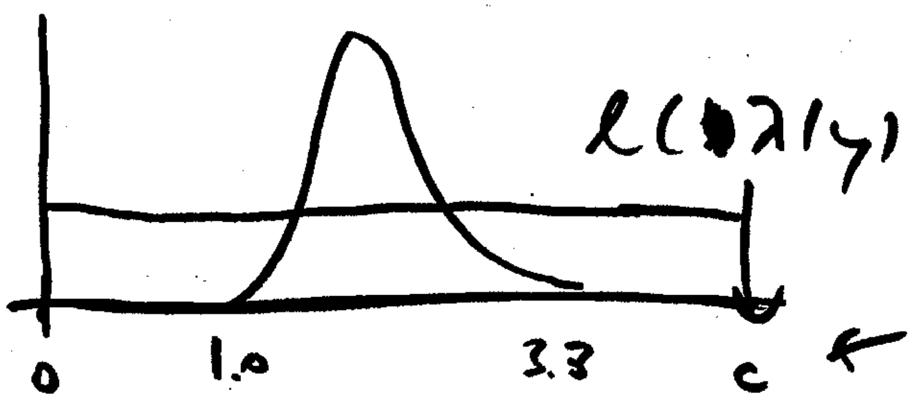
handwritten notes, Feb ①

part 2



$U(0, c)$

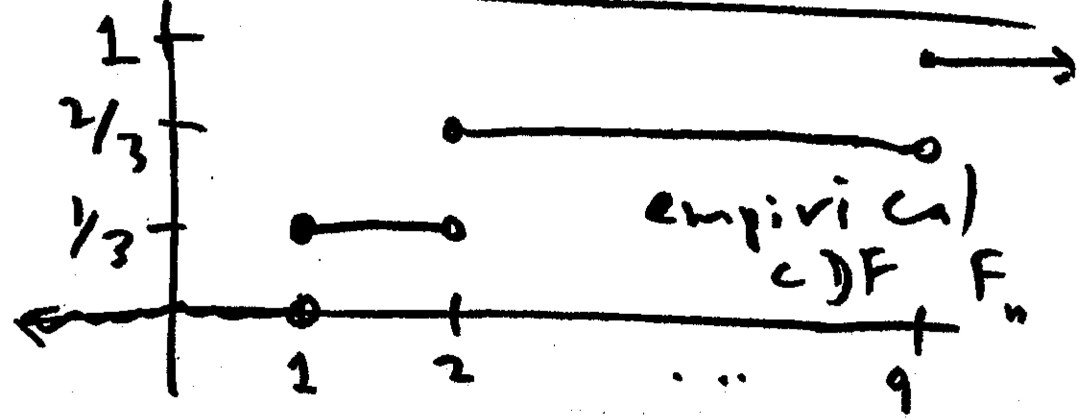
← avoid truncation of likelihood

$y = (y_1, \ldots, y_n)$, no other info (no predictor variables)

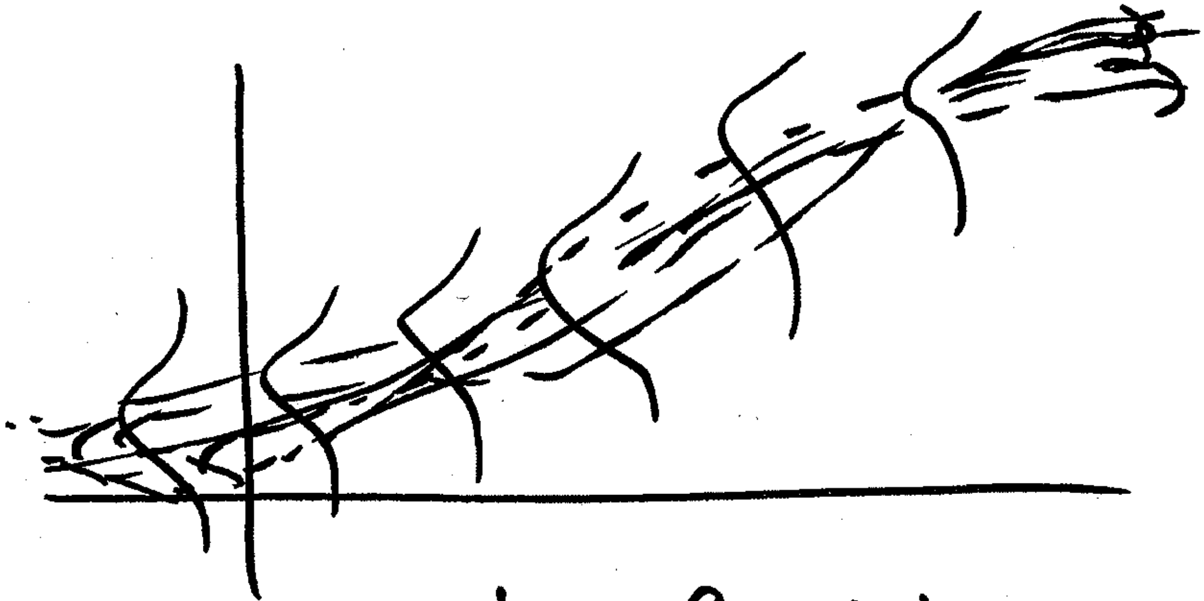
(continuous on $\mathbb{R}$)

still exchangeable

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$$



empirical CDF F_n



dist. on f'n: Bayesian nonparametric methods

(1970s)

UCLA Tom Ferguson

UCB David Freedman

(1990s) (MCMC) → full Bayesian nonparametrics prior model: $N(\mu, \sigma^2)$ unknown

smoothed hists of y_i

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all prior specifications have 2 parts:
① prior estimate of unknown
② prior sample size

in Bayesian nonparametrics the analogue is
① prior est. of F
② prior sample size

"center" the nonparametric prior on $N(\mu, \sigma^2)$, prior sample size $c = 10$ or 15

sample size $n = 100$

$(Y_1, \ldots, Y_n) \sim$ (continuous)

on $(0, \infty)$

$N(\mu, \sigma^2)$

? symmetric & unimodal

(1905)

Gosset

(Guinness brewery)

Biometrika

state of art

$$t = \frac{\bar{y} - c}{s/\sqrt{n}} \sim \cancel{N(0,1)} \quad \left(\frac{\text{signal}}{\text{noise}} \right)$$

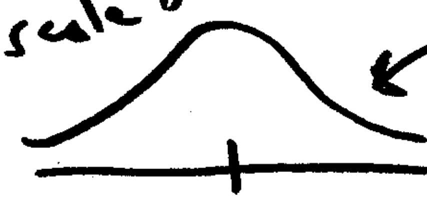
$(Y_i \mid \mu, \sigma^2) \stackrel{iid}{\sim} N(\mu, \sigma^2)$

unknown

n small

$H_0 : \mu = c$

scale σ



μ

tail weight

ν

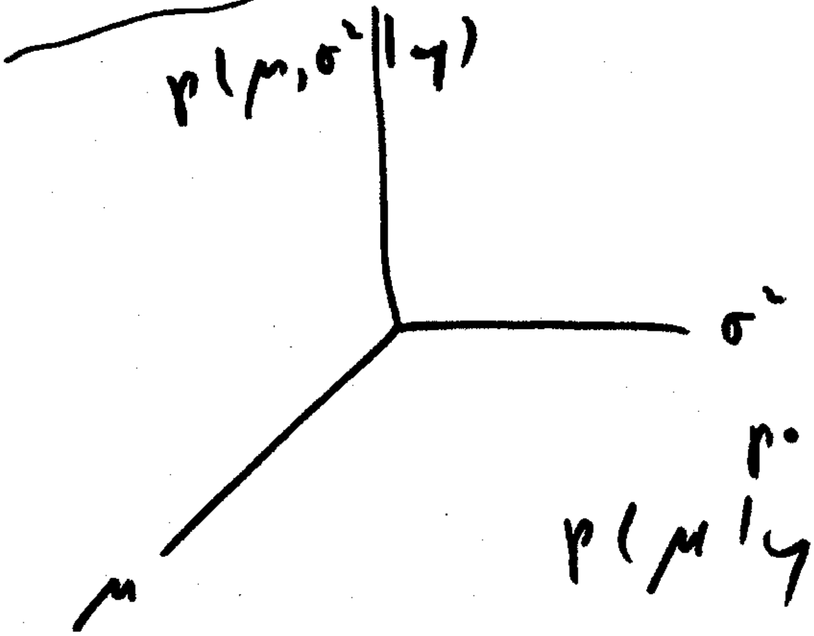
(degrees of freedom)

$(Y_1, \ldots, Y_n \mid \mu, \sigma^2, \nu) \stackrel{iid}{\sim} t_\nu(\mu, \sigma^2)$

$$\frac{1}{\sigma_*^2} = \frac{1}{\sigma_\mu^2} + \frac{n}{\sigma_0^2}$$

$$\begin{pmatrix}\text{post.}\\ \text{prec.}\end{pmatrix} = \begin{pmatrix}\text{prior}\\ \text{prec.}\end{pmatrix} + \begin{pmatrix}\text{lik.}\\ \text{prec.}\end{pmatrix}$$

$$p(\theta \mid y) = p(\mu, \sigma^2 \mid y)$$



even more interesting are <u>marginal</u> post. dist.

$$p(\mu \mid y) = \int_0^\infty p(\mu, \sigma^2 \mid y)\, d\sigma^2$$

& $$p(\sigma^2 \mid y) = \int_{-\infty}^{\infty} p(\mu, \sigma^2 \mid y)\, d\mu$$

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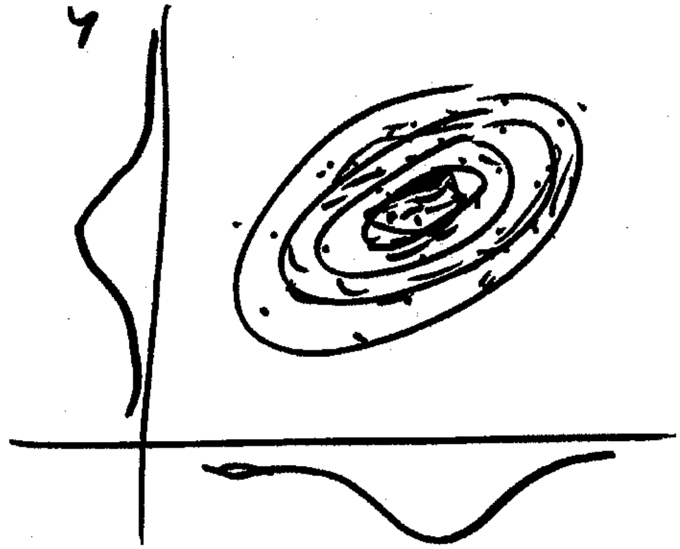
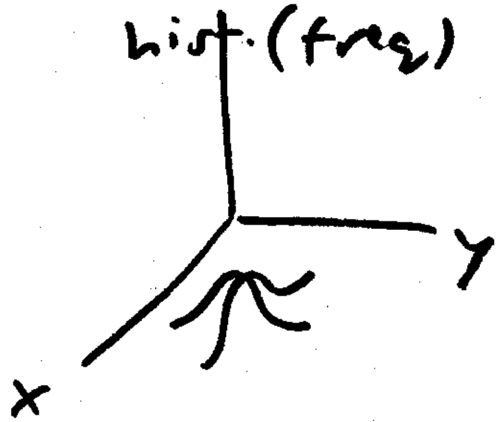
$$\begin{bmatrix} y_1 & x_1 \\ \vdots & \vdots \\ y_n & x_n \end{bmatrix}$$



scatterplot

hist. of y



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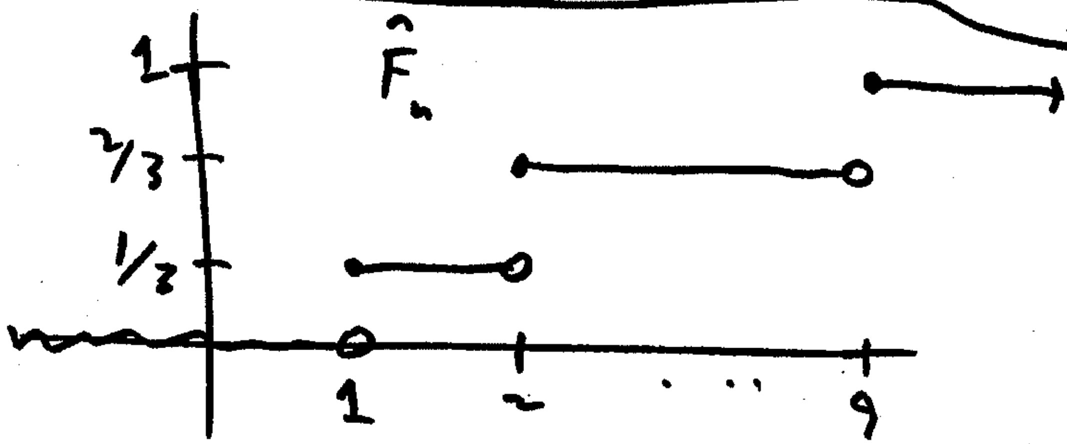
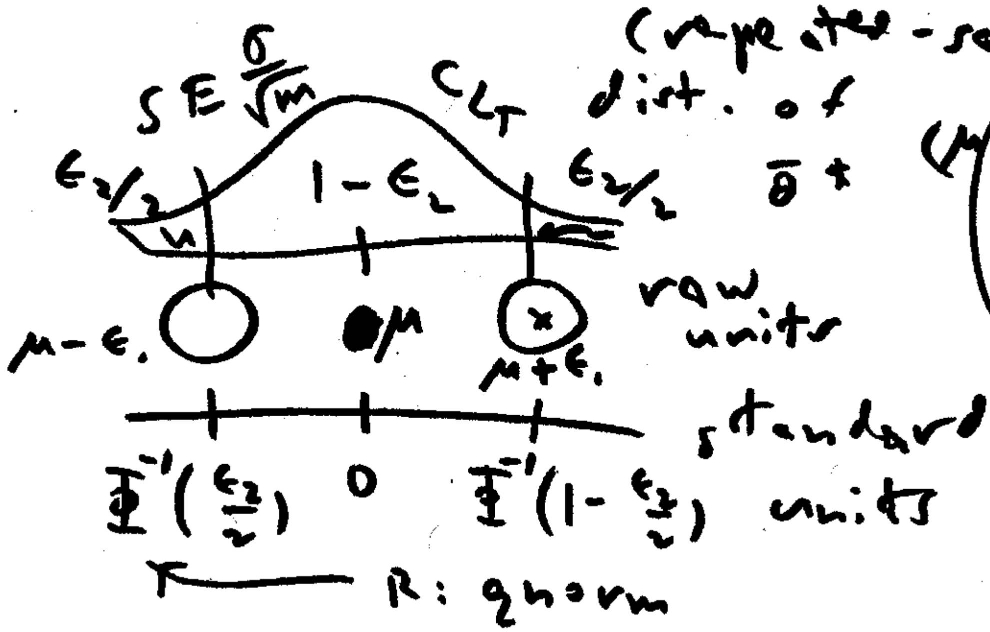
$$\begin{bmatrix}1\\2\\9\end{bmatrix}$$



(repeated-sampling) for the



$$(\mu+\epsilon_1)-\mu$$
$$\downarrow$$
$$\frac{x-\mu}{\sigma/\sqrt{m}} = \frac{\#-\text{mean}}{SE} = \Phi^{-1}\left(1-\frac{\epsilon_2}{2}\right)$$

CDF of standard normal dist $= \Phi$