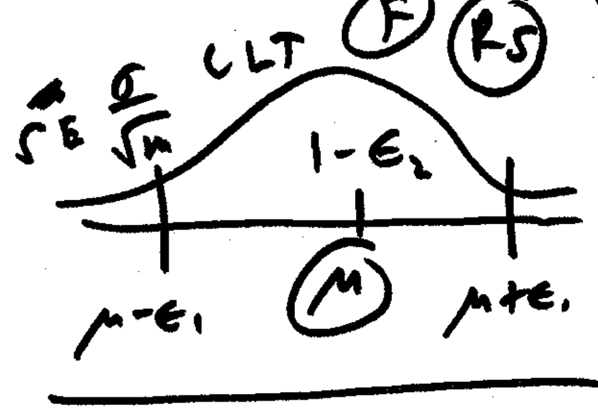
can read ch. 2 of Draper now

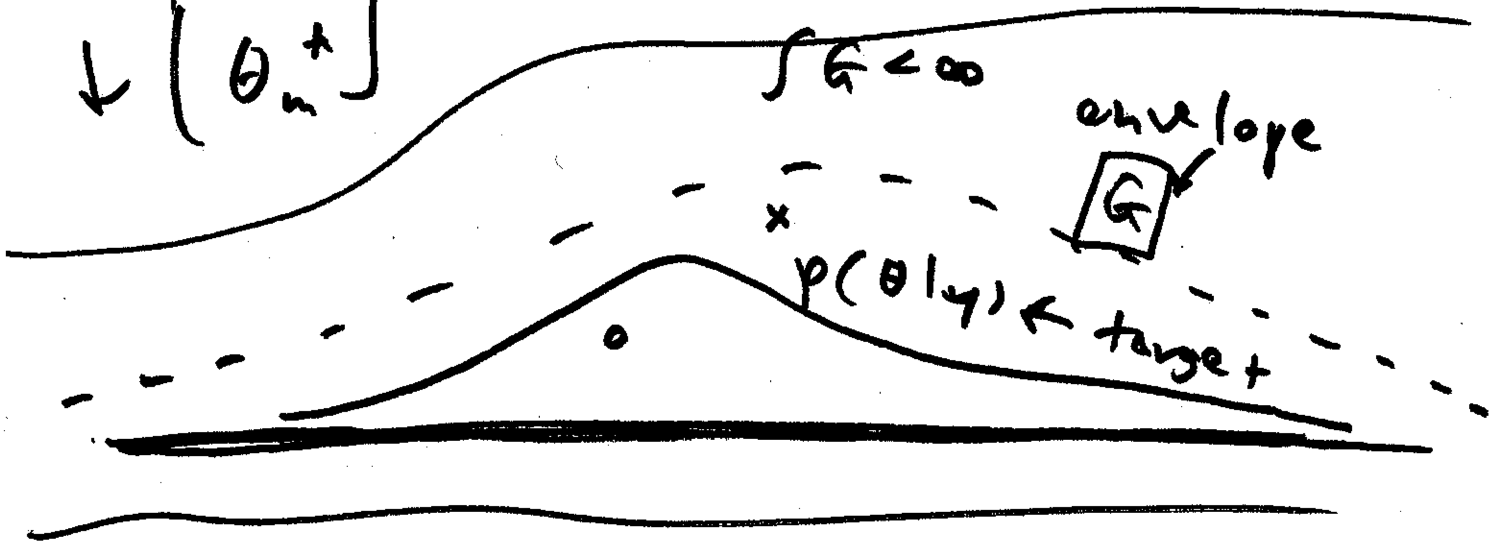
extra notes 8 Feb

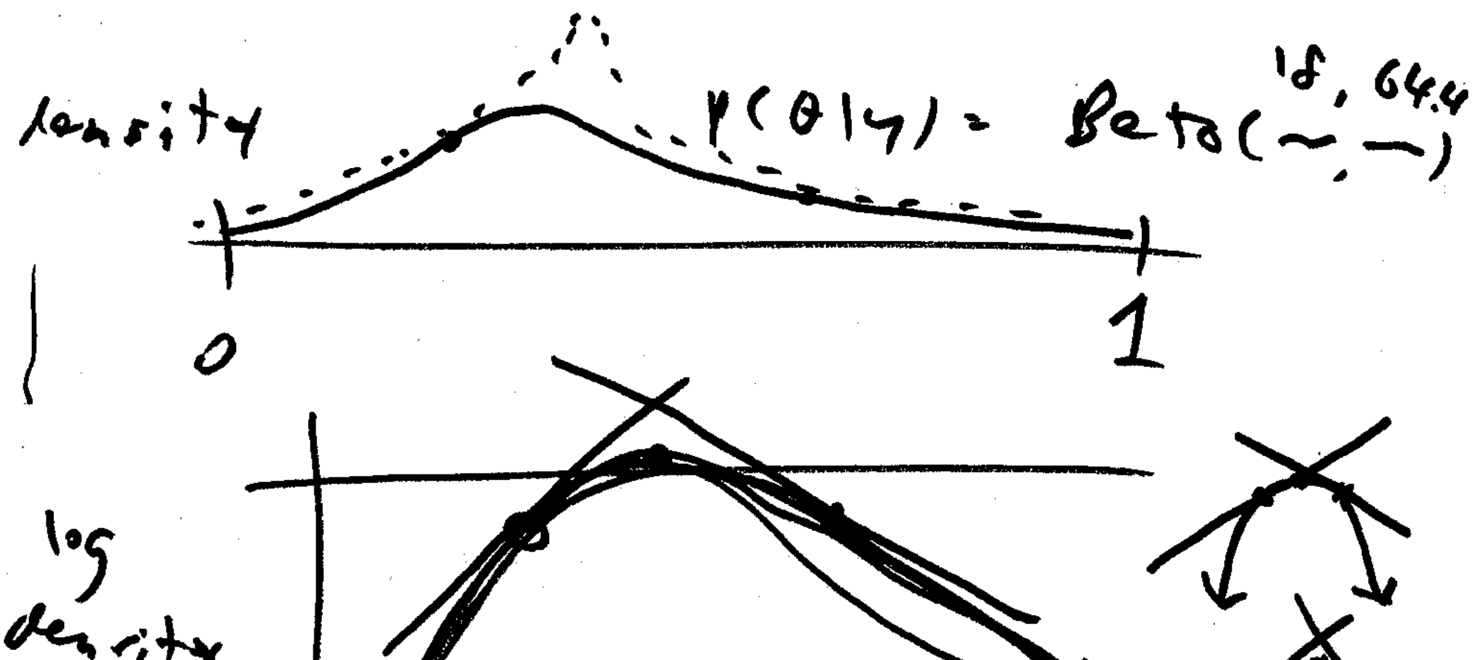
①

$$\theta \qquad m \left\{ \begin{bmatrix} \theta_1^* \\ \theta_2^* \\ \vdots \\ \theta_m^* \end{bmatrix} \right.$$

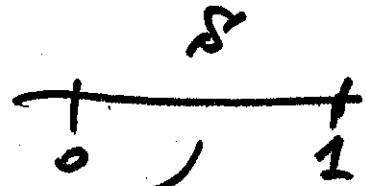


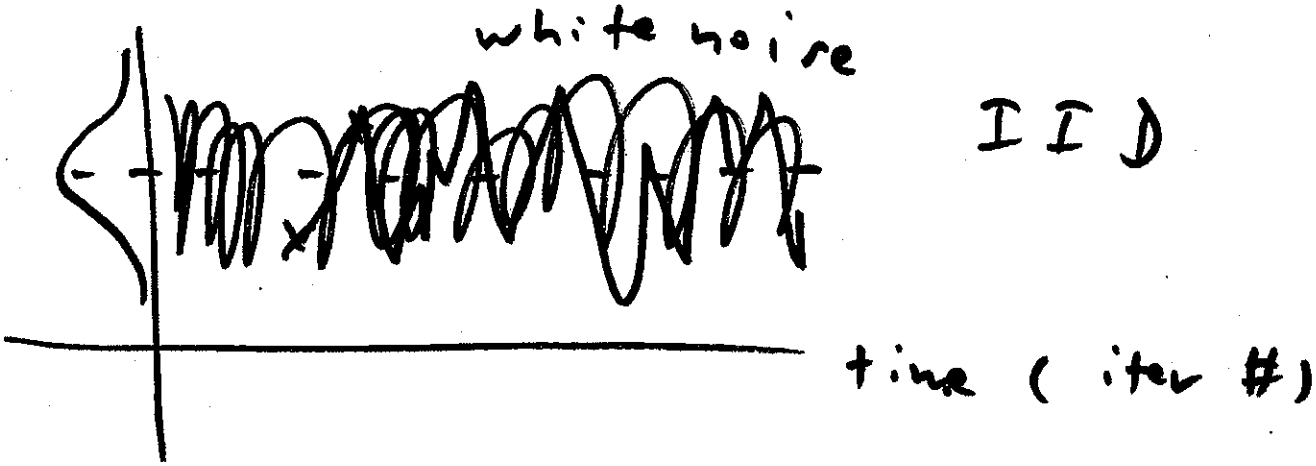
dist of $\bar{\theta}^*$





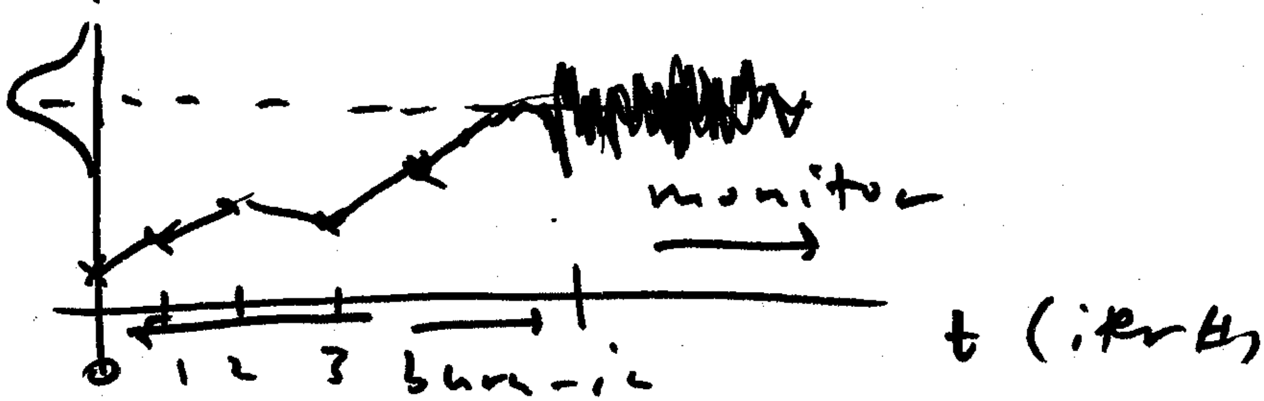
$$m \begin{bmatrix} \theta_1^* & I(\theta_1^* \le .15) \\ \theta_2^* & I(\theta_2^* \le .15) \\ \vdots & \vdots \\ \theta_m^* & \vdots \end{bmatrix}$$





IID

serially correlated



$$\boxed{p(\theta \mid y)} = \cancel{c}\; p(\theta)\; \ell(\theta \mid y)$$

vector length k (k big)

try not to have to evaluate

convert qual. & quant. info into dist.

sampling dist

theory (ex. binomial) and/or cheating (look at data)

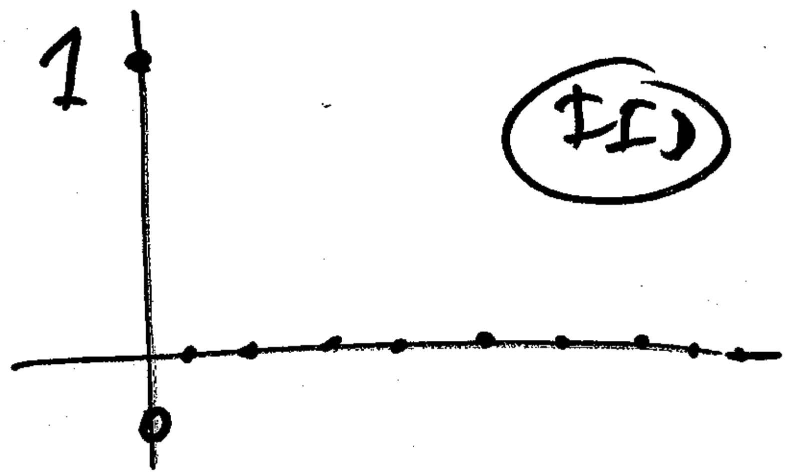
idea: restrict attention to Markov chains that have stationary behavior & try to force the chain to have $p(\theta \mid y)$ as its stationary dist.

$$\begin{array}{ccc} 1 & 2 & 3 \\ \cancel{\theta_1^*} & & \\ \theta_2^* & \theta_1^* & \\ \theta_3^* & \theta_2^* & \theta_1^* \\ \theta_4^* & \vdots & \tilde{\theta}_1^* \\ \theta_m^* & \theta_{m-1}^* & \end{array}$$

cor(1, 2) = autocor at lag 1

cor(1, 3) = ac at lag 2

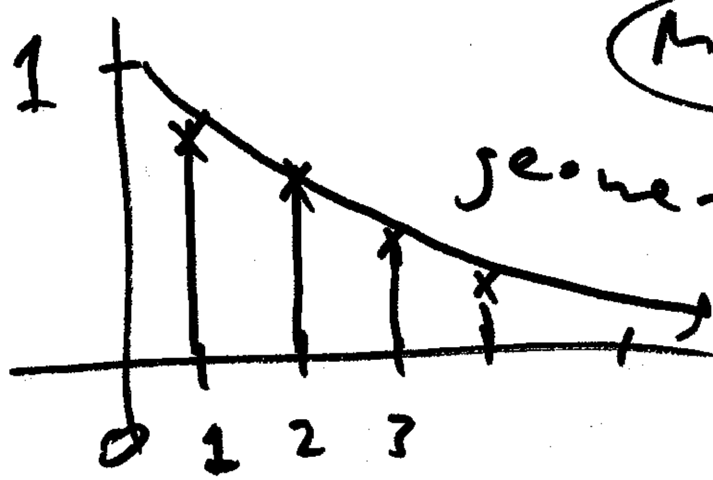
etc.



IID

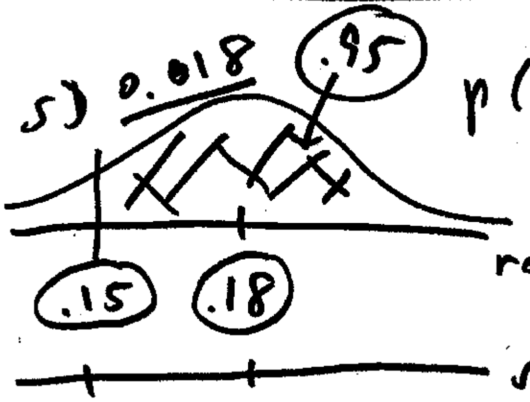
ACF
autocorrelation function

MCMC



geometric ACF decay

sample size determination in inference



$p(\theta \mid y)$ in AMI case study ($n = \underline{400}$)

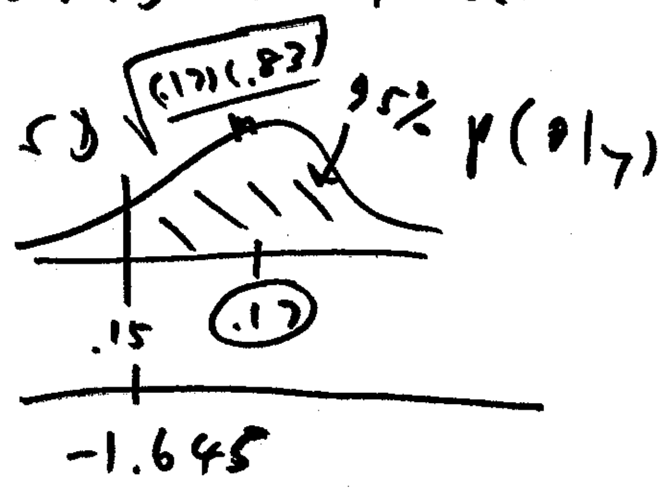
$$\frac{.15 - .18}{.018} = -1.67$$

$$p(\text{prima facie d of C problem} \mid y)$$

$$= p(\theta > .15 \mid y) \doteq \underline{95\%}$$

spse wanted, if underlying θ were θ_0 say (ex. 0.17), to demonstrate that this post. prob. was high:

$$p(\theta > \underset{0.15}{\theta_s} \mid y) = 0.95 = 1 - \alpha$$



$$\frac{.15 - .17}{\sqrt{\frac{(.17)(.83)}{n}}} = -1.645$$

$$\rightarrow \quad n = 954$$

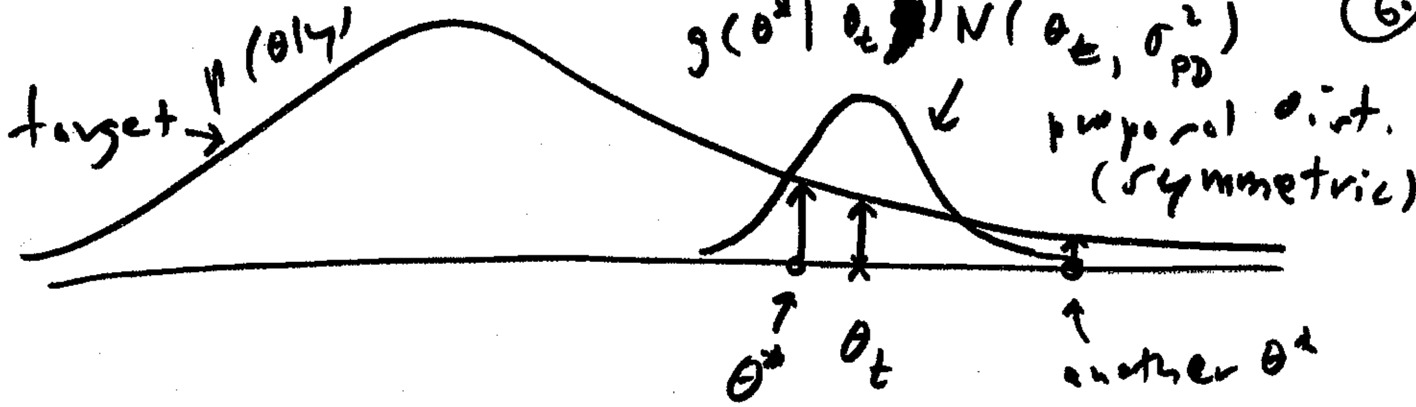
⑥

in general

$$n = \frac{\theta_0(1-\theta_0)\left[\Phi^{-1}(\alpha)\right]^2}{(\theta_s-\theta_0)^2}$$

$(\theta_s = 0.15)$

θ_0	α	n
.17	.95	95~~4~~
.17	.99	1909
.16	.95	3636

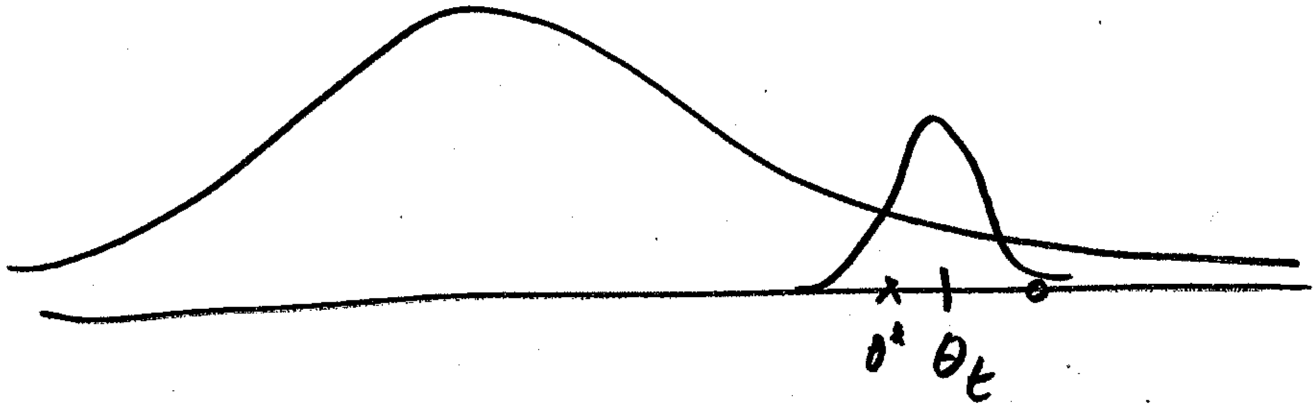


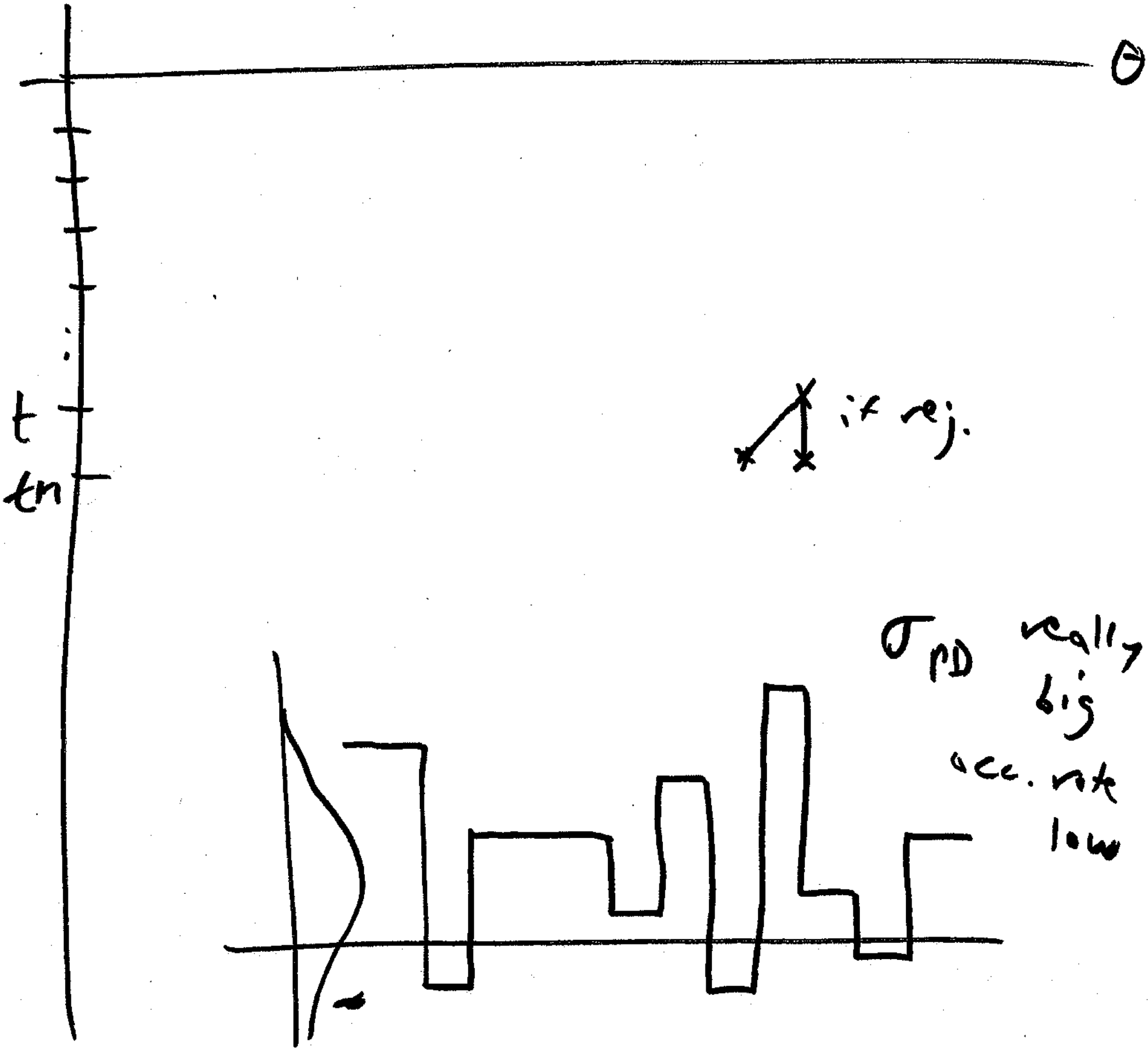
if $\dfrac{p(\theta^*|y)}{p(\theta_t|y)} \geq 1$ accept with prob 1
accept all (uphill moves)

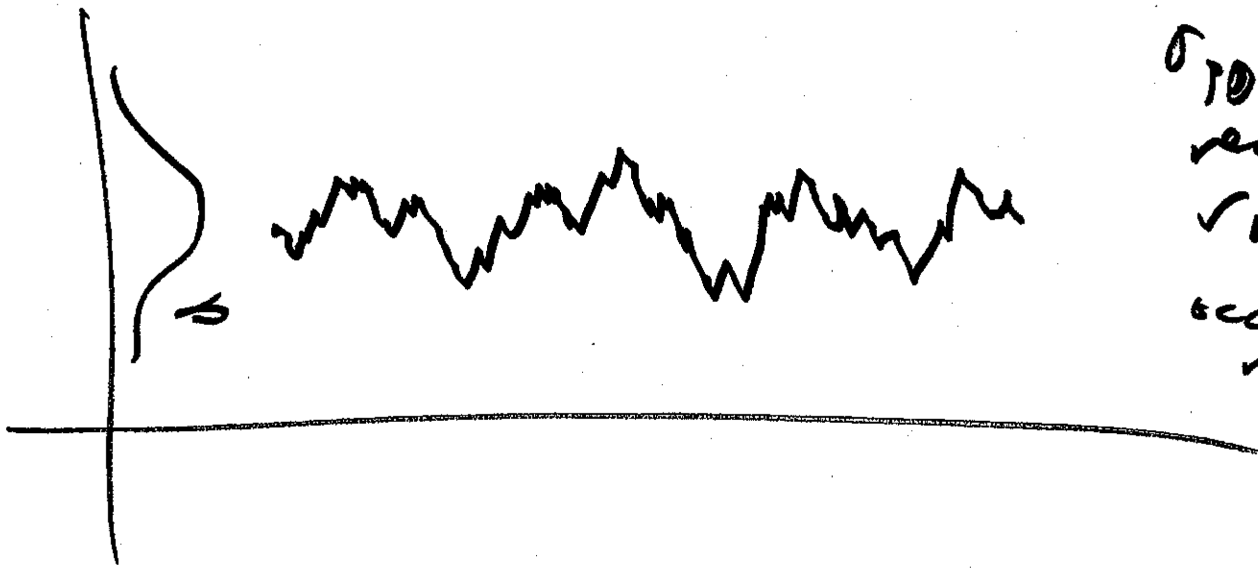
if $\dfrac{p(\theta^*|y)}{p(\theta_t|y)} < 1$ accept with prob $\dfrac{p(\theta^*|y)}{p(\theta_t|y)}$

if accept $\theta_{t+1} = \theta^*$

if reject $\theta_{t+1} = \theta_t$ (!) stay where you are

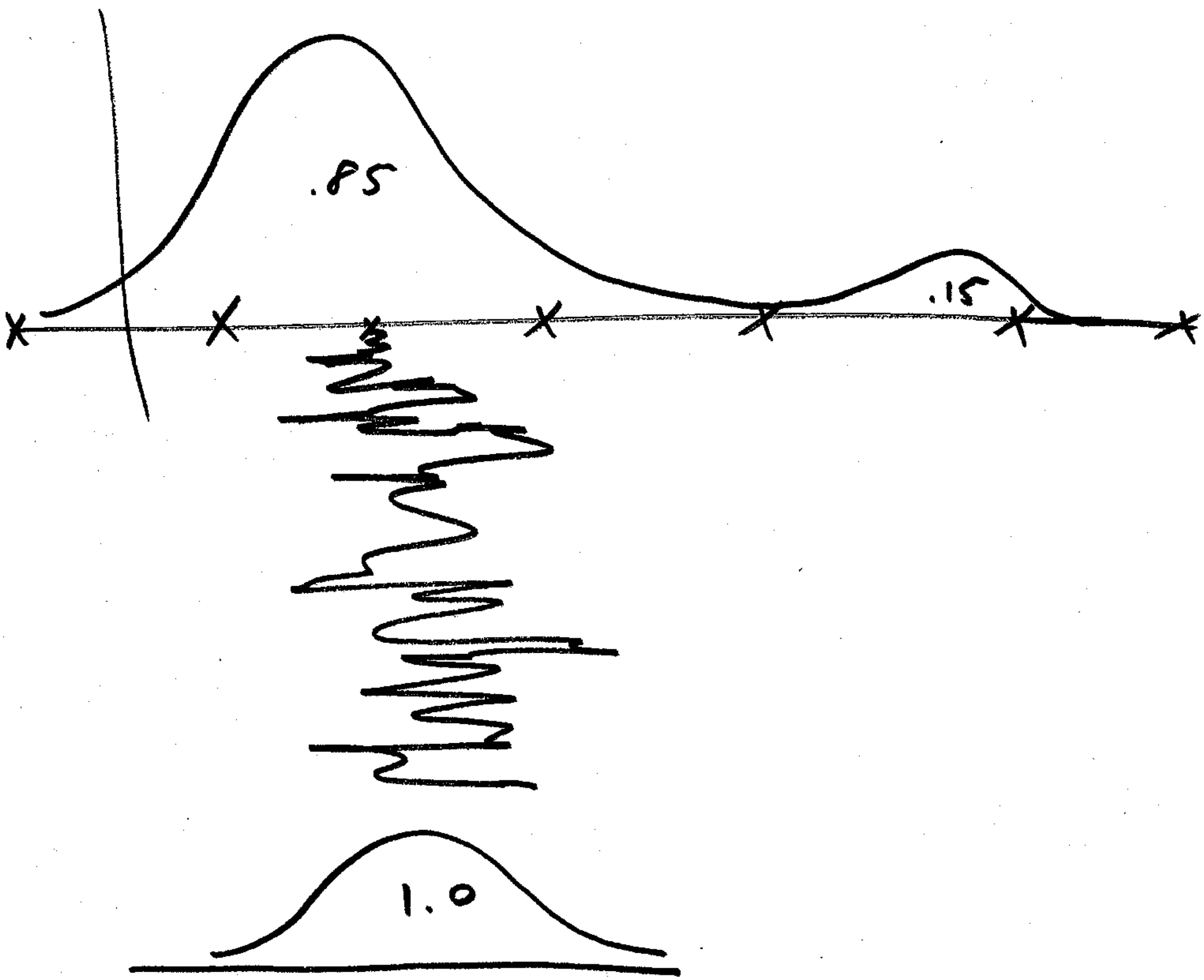

θ*
θt


θ
t
tn
if rej.
σPD really big
acc. rate low



σ_{TD} really small

acc. rate high

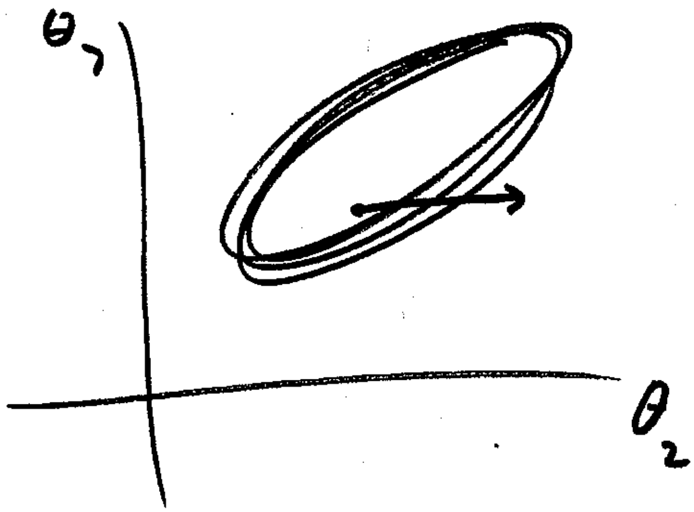


$\theta = (\mu \;\; \sigma_b^2 \;\; \sigma_e^2 \mid b_1 \;\; b_2 \; \cdots \; b_I)$

MCMC data set

1 by 1 : single-scan MH

block updating



$\theta = (\mu, \sigma^2, \tau)$

$(\mu \mid \underline{\underline{\sigma^2, \tau}})$

$\sigma^2 \mid \mu, \tau$

$\tau \mid \mu, \sigma^2$

$p(\theta \mid y) = p(\mu, \sigma^2, \tau \mid y)$

Gibbs sampling

$$\left.\begin{array}{l} p(\mu \mid y, \sigma^2, \tau) \\ p(\sigma^2 \mid y, \mu, \tau) \\ p(\tau \mid y, \mu, \sigma^2) \end{array}\right\} \text{full conditionals}$$

	μ	σ^2	r
0	μ_0	σ_0^2	r_0
1	μ_1^*	$(\sigma_1^2)^*$	r_1^*
	⋮	⋮	⋮

$$\mu_1^* \sim p(\mu \mid y, \sigma_0^2, r_0)$$

$$(\sigma_1^2)^* \sim p(\sigma^2 \mid y, \mu_1^*, r_0)$$

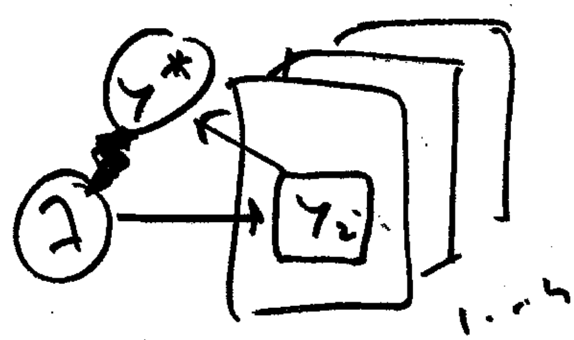
$$r_1^* \sim p(r \mid y, \mu_1^*, (\sigma_1^2)^*)$$

$$p(y^* \mid y)$$

$$\boxed{(\lambda \mid y)}$$

$$(y^* \mid y) \leftrightarrow (y^* \mid \lambda, \cancel{y})$$

λ	y^*



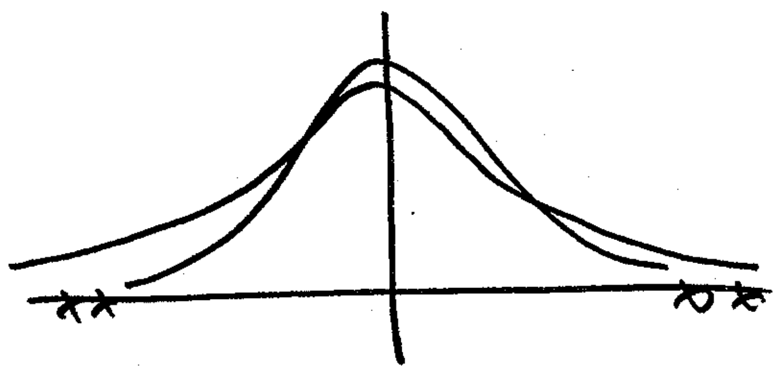
model 1 $\quad \beta \sim p(\beta)$

$(Y_i \mid \beta) \stackrel{iid}{\sim} \mathcal{E}\left(\frac{1}{\beta}\right)$

model 2 $\quad (\alpha, \beta) \sim p(\alpha, \beta)$

$(Y_i \mid \alpha, \beta) \stackrel{iid}{\sim} \Gamma(\alpha, \beta)$

1 is special case of 2 with $\alpha = 1$



latent variables

$(\lambda_i \mid \nu) \stackrel{iid}{\sim} \Gamma^{-1}\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$

$(Y_i \mid \mu, \sigma^2, \lambda_i) \stackrel{indep}{\sim} N(\mu, \underline{\sigma^2 \lambda_i})$