extra notes 18 Jan 13

1

$C < V$

population size

when $n << N$, SRS $\doteq$ IID

sample size

is a lot smaller than

Rasmussen Report (nuclear power safety)

$$P(A_1 \text{ and } A_2 \text{ and } \ldots \text{ and } A_k)$$

$$= P(A_1)\, P(A_2 \mid A_1)\, P(A_3 \mid A_1 A_2) \cdots$$

naive Rasm. $= P(A_1)\, P(A_2)\, P(A_3) \cdots P(A_k)$

independence: slippery frequentist concept

freq at design time (before data): Y_1, Y_2 indep iff

$$p_{Y_1, Y_2}(y_1, y_2) = p_{Y_1}(y_1) \cdot p_{Y_2}(y_2)$$

after data? frequentist story unclear

before data

	fixed	"random"
freq	θ	Y
Bayes	—	θ, y

after data

	fixed	"random"
freq	θ	?
Bayes	y	θ

y?

θ unknown

y data

$\{y_1, \dots, y_n, \dots\} = P$

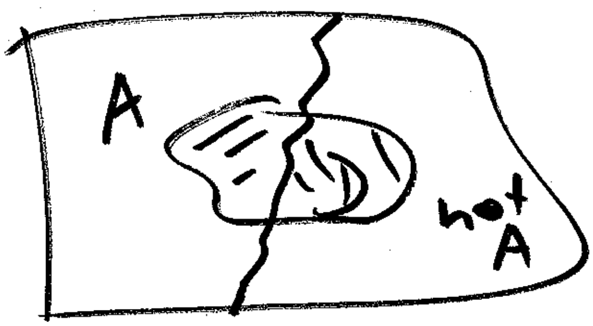
extending exch ~~tt~~

thinking of $y_1, \dots, y_n$

as like a random

sample from P

$$P(A \mid D) = \frac{P(A) \cdot P(D \mid A)}{P(D)}$$



$P(D) = ?$

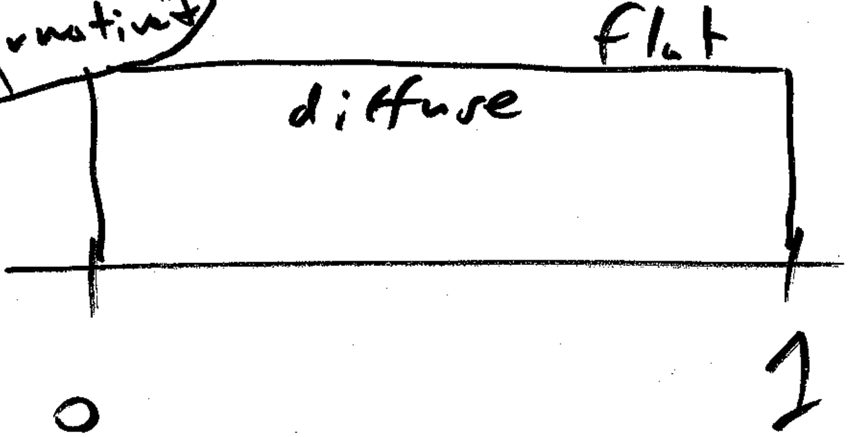
ELISA +

$$P(D) = P(D \text{ and } A) + P(D \text{ and not } A)$$

$$= \underset{.01}{P(A)}\, \underset{.95}{P(D \mid A)} + \underset{.99}{P(\text{not } A)}\, \underset{(1-.98)}{P(D \mid \text{not } A)}$$

don't know much about θ

"noninformative"



$p(\theta)$

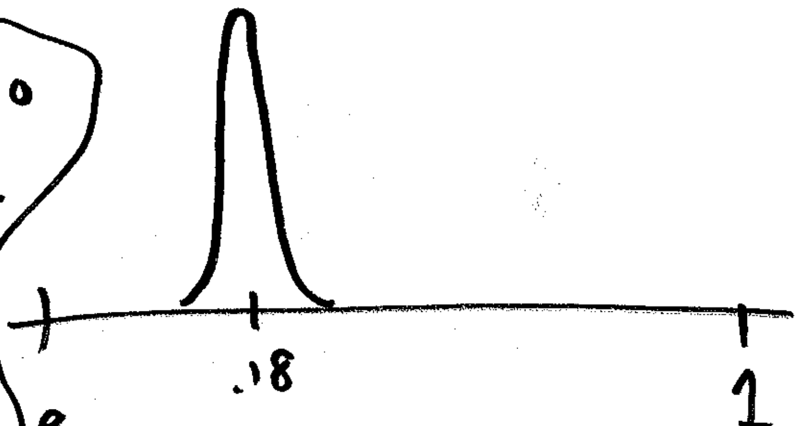
Fisher "fiducial inference"

$n = 400$

$s = 72$

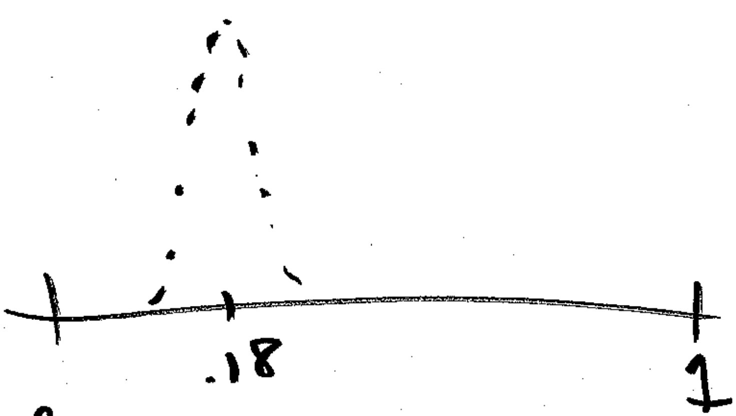
$\bar{y} = \frac{s}{n}$

$= 0.18$



$\ell(\theta | y) =$

$c \cdot p(y | \theta)$



$p(\theta | y)$

$$p(y_{m+1},\dots,y_n \mid y_1,\dots,y_m) =$$

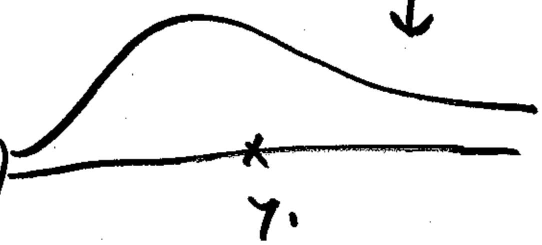
$$= \int_0^1 p(y_{m+1},\dots,y_n, \theta \mid y_1,\dots,y_m)\, d\theta$$

$$= \int_0^1 p(y_{m+1},\dots,y_n \mid \theta, \cancel{y_1,\dots,y_m}) \cdot \boxed{p(\theta \mid y_1,\dots,y_m)}\, d\theta$$

$$= \int_0^1 \underbrace{p(y_{m+1},\dots,y_n \mid \theta)}_{\substack{\text{sampling dist} \\ \text{for } y_{m+1},\dots,y_n}} \underbrace{p(\theta \mid y_1,\dots,y_m)}_{\substack{\text{post dist} \\ \text{for } \theta \\ \text{given} \\ y_1,\dots,y_m}} d\theta$$

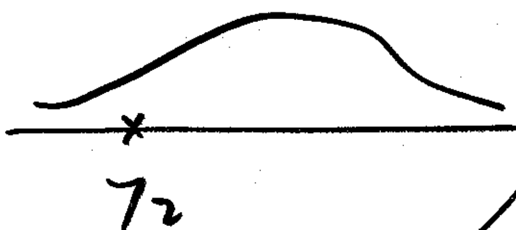
$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \in \mathbb{R}^n \qquad \begin{matrix} y_2 \\ \vdots \\ y_n \end{matrix}$$

$$p(y_1 \mid y_2, \dots, y_n)$$



$$\boxed{y_{-k} = (y_1, \dots, y_{k-1}, y_{k+1}, \dots, y_n)}$$

$$p(y_2 \mid y_{-2})$$



$$\ell(\theta \mid y) = \theta^{s} (1-\theta)^{n-s}$$

$$p(\theta \mid y) = c\; p(\theta) \cdot \ell(\theta \mid y)$$

$$= c\; p(\theta) \boxed{\theta^{s} (1-\theta)^{n-s}}$$

$$\underbrace{\theta \qquad (1-\theta)}_{\text{Beta}} \quad = \quad \underbrace{c\, \theta^{\alpha-1} (1-\theta)^{\beta-1}}_{\text{Beta}} \quad \underline{\qquad\quad} \;\text{Beta}$$

⑦

prior sample size

$$\begin{bmatrix} 1 \\ \vdots \\ 1 \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} \alpha = 4.5 \\ \\ \beta = 25.5 \end{array}$$

Beta(α, β) prior

$n_0 = (\alpha+\beta)$ mean $\frac{\alpha}{\alpha+\beta}$

$n_0 = 30$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \; n = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} s \\ \\ n-s \end{array}$$

$n = 400$

data

mean $\frac{s}{n}$ mean $\frac{s}{s+(n-s)} = \frac{s}{n}$

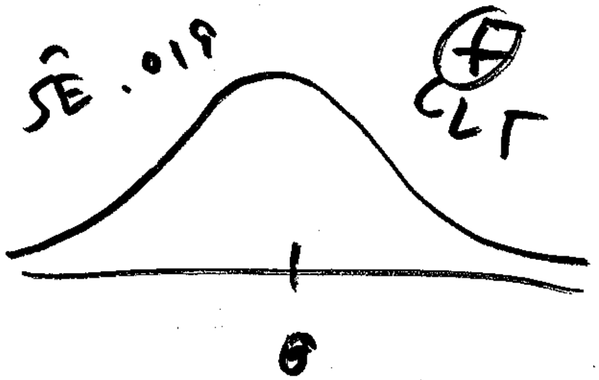
Analysis 1: Beta(α, β) prior, Ber. likelihood, Bayesian posterior

Analysis 2: merge prior, sample data, do likelihood analysis on merged sample

Analysis 1 = Analysis 2

(8)

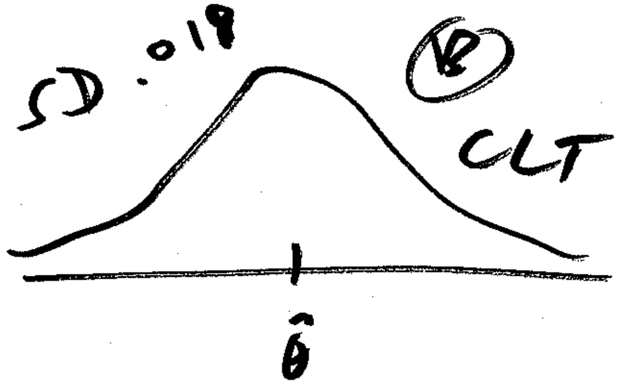
(F)



(F) CLT

repeated-sampling dist. of $\hat{\theta}$

(B)

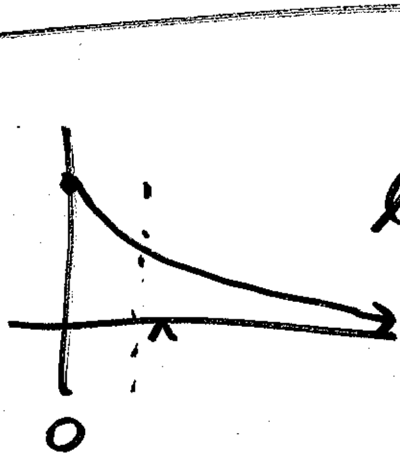


(B) CLT

post. dist for θ given y & relatively diffuse prior info

(F) $$c_1 e^{-c_2(\hat{\theta}-\theta)^2}$$ = (B) $$c_1 e^{-c_2(\theta-\hat{\theta})^2}$$

Bernstein - Von Mises Thm.



$\ell(\sigma^2 | y)$

$\hat{\sigma}^2_{MLE} = 0$

variance components models

$\ell(\sigma^2 | y)$