

T can read S iff $L_S - O_T \subseteq L_T - O_T$. Because T can start T', we have that $O_{T'}$ is included in O_T and $L_T - O_T \subseteq L_{T'} - O_T$. Because T' can read S, we have that $L_S - O_{T'} \subseteq L_{T'} - O_{T'}$.

The conditions $L_T - O_T \subseteq L_{T'} - O_T$ and $L_S - O_{T'} \subseteq L_{T'} - O_{T'}$ **do not** imply that T can read S, which requires that $L_S - O_T \subseteq L_T - O_T$. For example, we could have two secrecy categories c and d, and $L_T = O_T = O_{T'} = \{c\}$ and $L_{T'} = L_S = \{c,d\}$, and then we satisfy the various hypotheses ($O_{T'}$ included in O_T , $L_T - O_T \subseteq L_{T'} - O_T$, and $L_S - O_{T'} \subseteq L_{T'} - O_{T'}$) but do not have the required $L_S - O_T \subseteq L_T - O_T$. Therefore: In general, T cannot always read S. Note that, in the counterexample above, we have that $O_T = O_{T'}$ so this extra hypothesis would not help.

On the other hand, if $L_{T'} \subseteq_{O_T} L_T$ then (by definition) $L_{T'} - O_T \subseteq L_T - O_T$. Moreover, $L_S - O_{T'} \subseteq L_{T'} - O_{T'}$ implies $L_S - O_{T'} - O_T \subseteq L_{T'} - O_{T'} - O_T$, by monotonicity, and since $O_{T'}$ is included in O_T this says that $L_S - O_T \subseteq L_{T'} - O_T$. By transitivity, $L_S - O_T \subseteq L_T - O_T$, as desired. So the read is possible when $L_{T'} \subseteq_{O_T} L_T$.