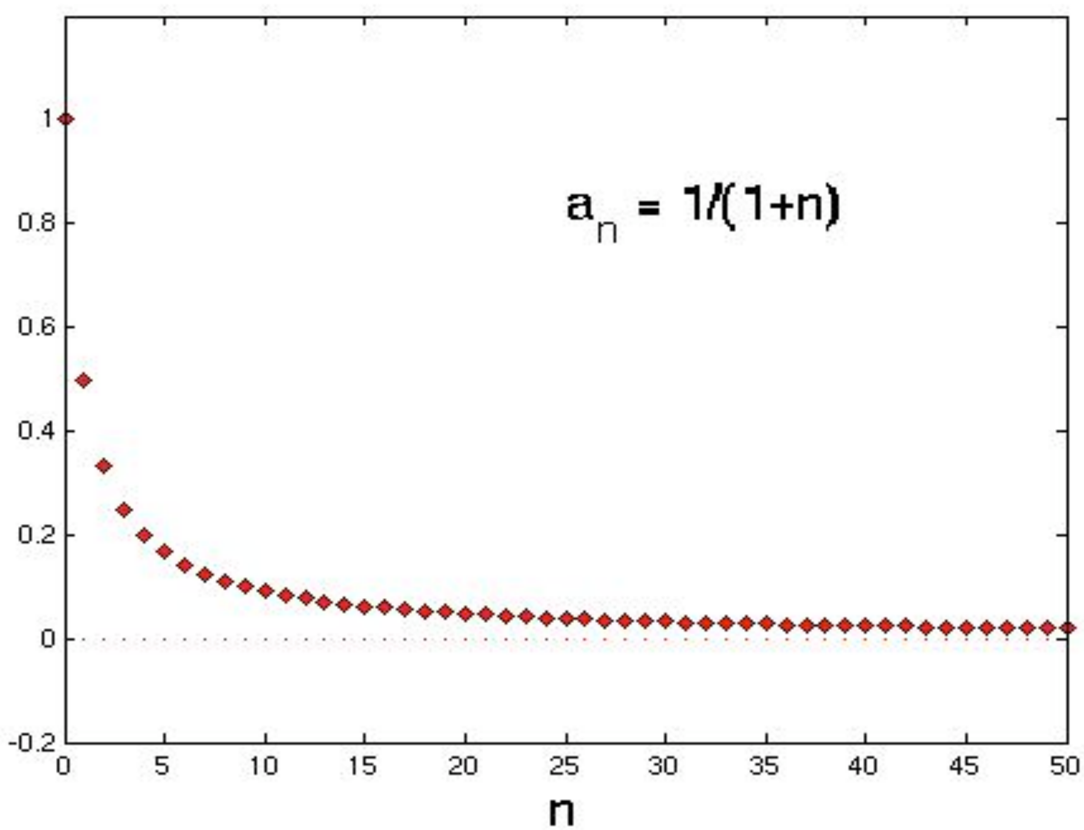
IV.4 Limits of sequences introduction

- What happens to a sequence a_n when n tends to infinity?

- A sequence can have one of three types of behaviour:
  - Convergence
  - Divergence
  - Neither of the above…

Graphical interpretation of limits for explicit sequences

Examples of converging sequences

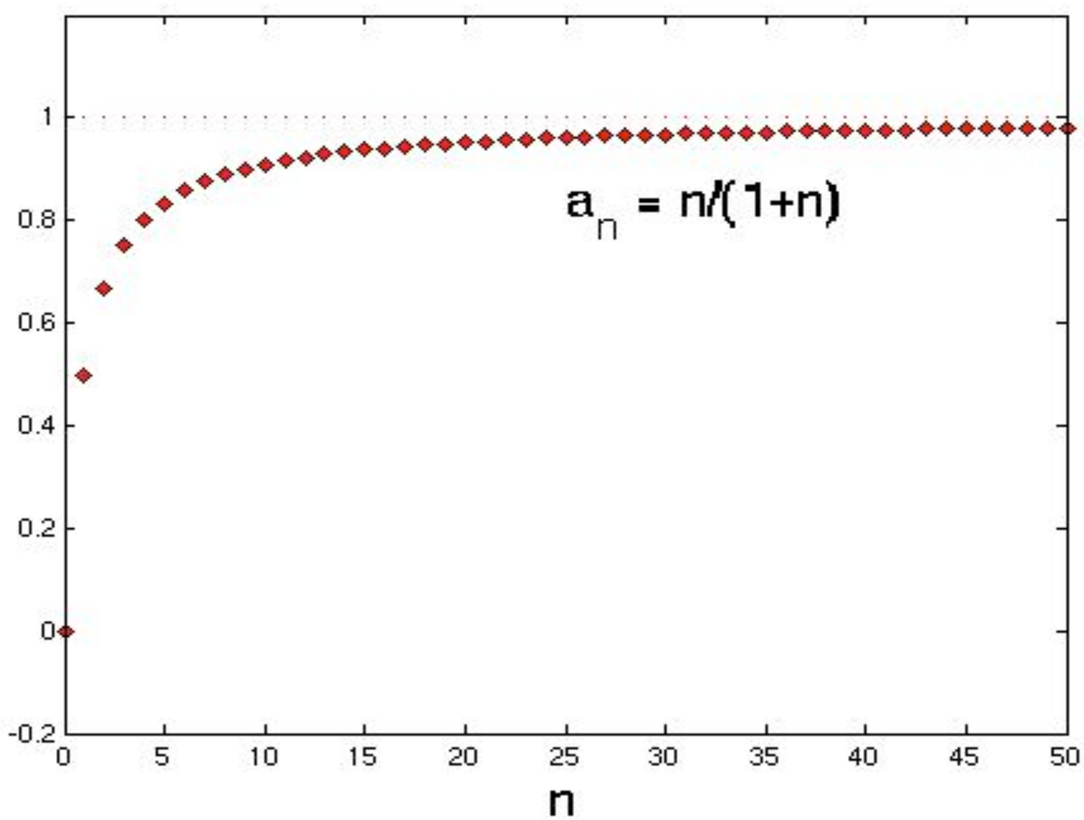


The sequence converges to a unique value: 0

We write this as

$\lim_{n \to \infty} a_n = 0$

Examples of converging sequences

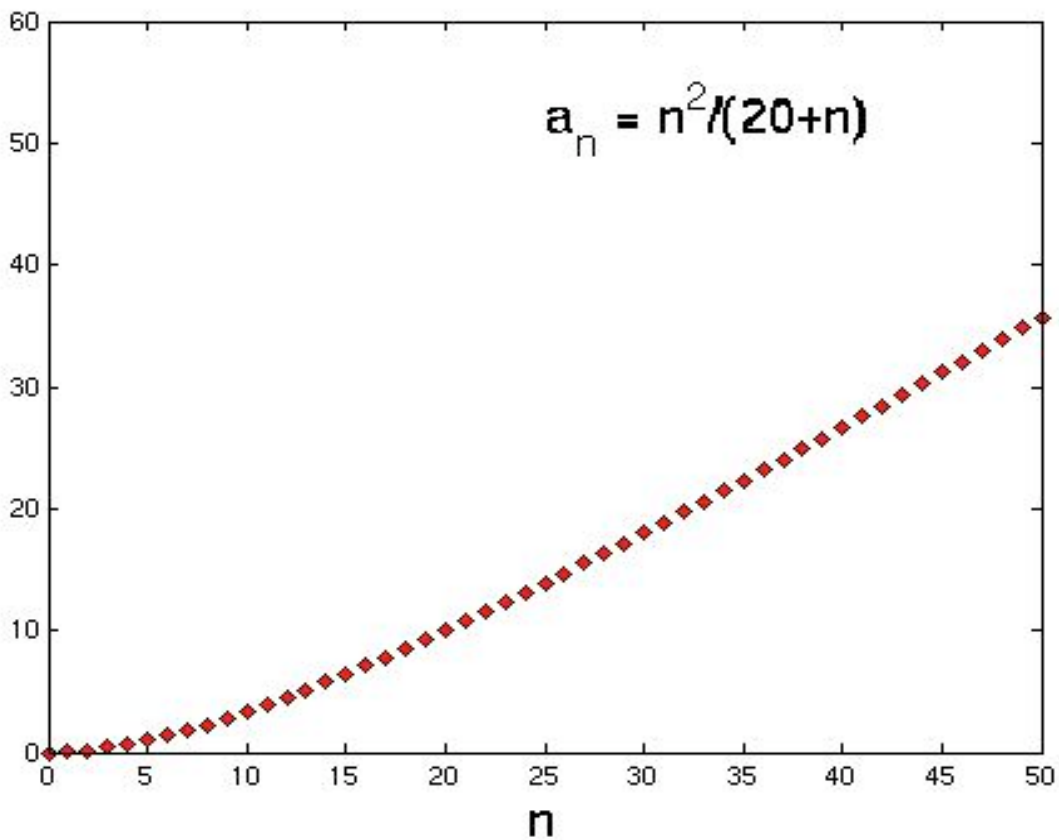


The sequence converges to a unique value: 1

We write this as

$\lim_{n \to \infty} a_n = 1$

Examples of diverging sequences

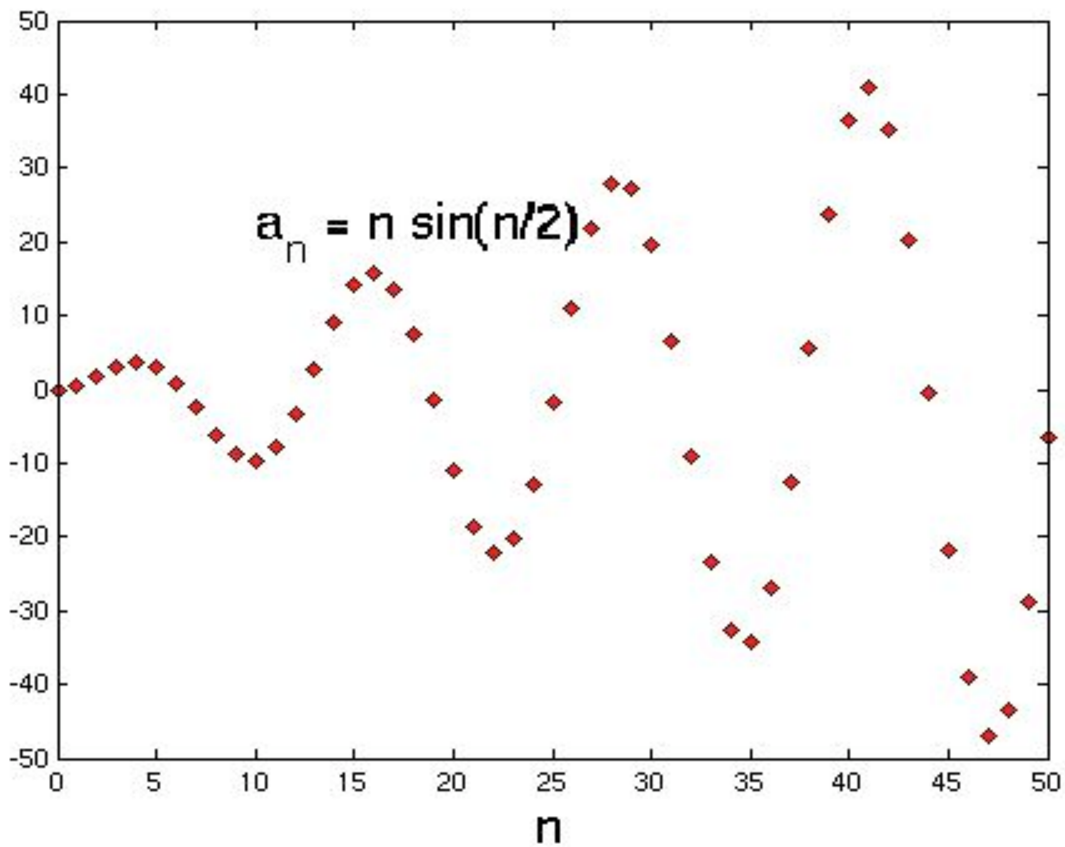


The sequence diverges, it has no real limit.

We can write this as

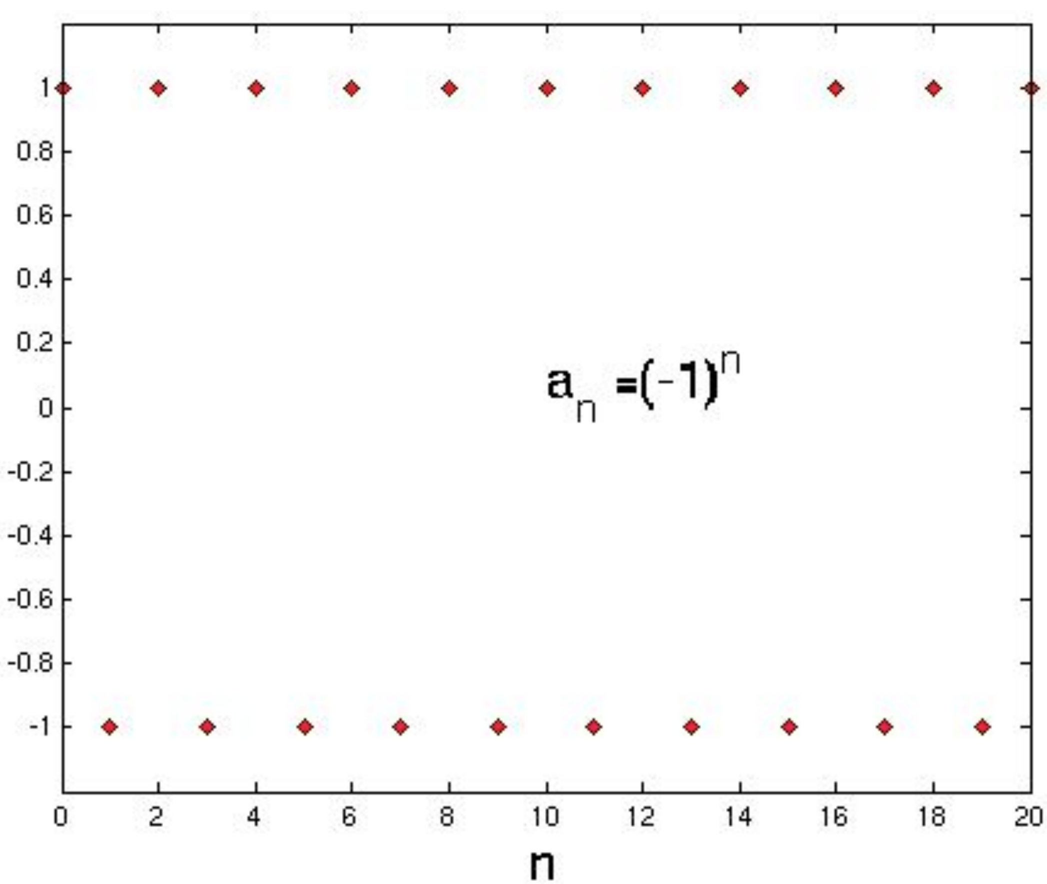
$\lim_{n \to \infty} a_n = +\infty$

Examples of diverging sequences



The sequence diverges; it has no real limit.

A sequence that neither converges nor diverges

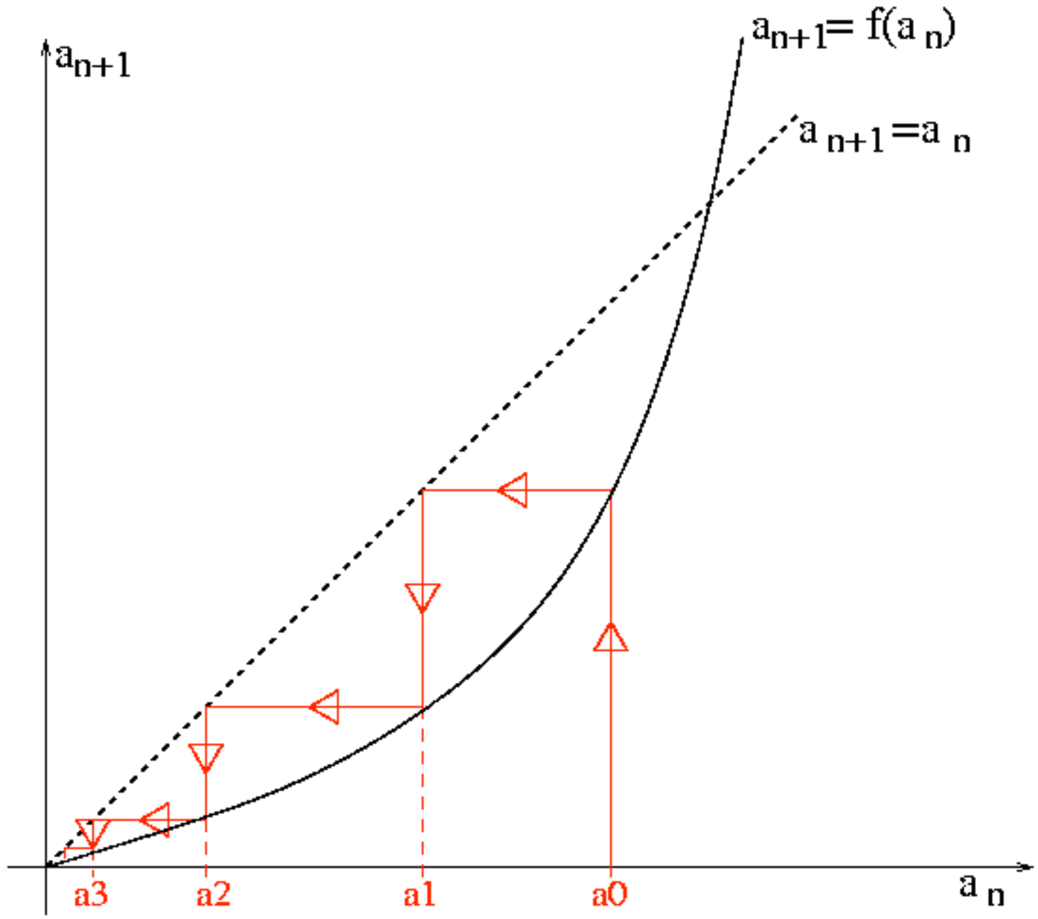


This sequence neither converges nor diverges.

In this particular case, it is periodic.

Graphical interpretation of limits for implicit sequences

A recursion that converges



This sequence is defined by an implicit formula

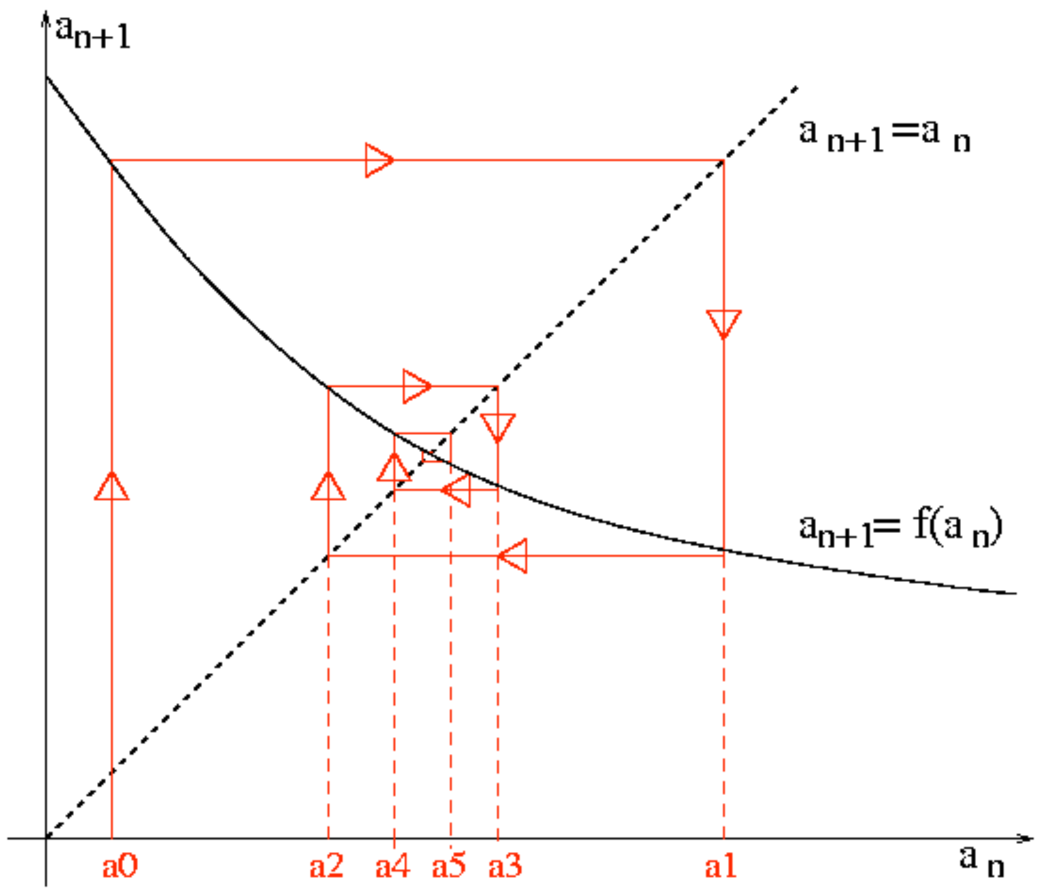
$a_{n+1} = f(a_n)$

From the cobweb diagram, we see that it converges to 0:

We write

$\lim_{n \to \infty} a_n = 0$

A recursion that converges



This sequence is defined by an implicit formula

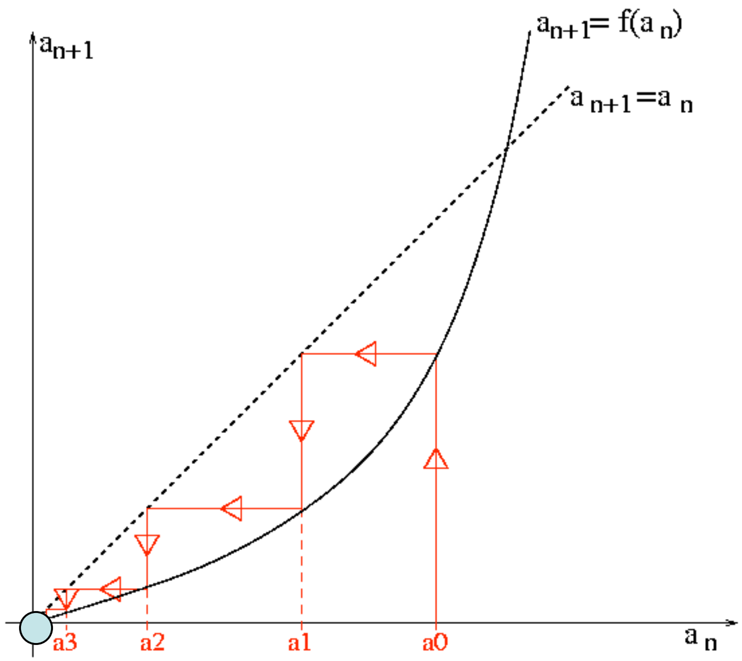
$a_{n+1} = f(a_n)$

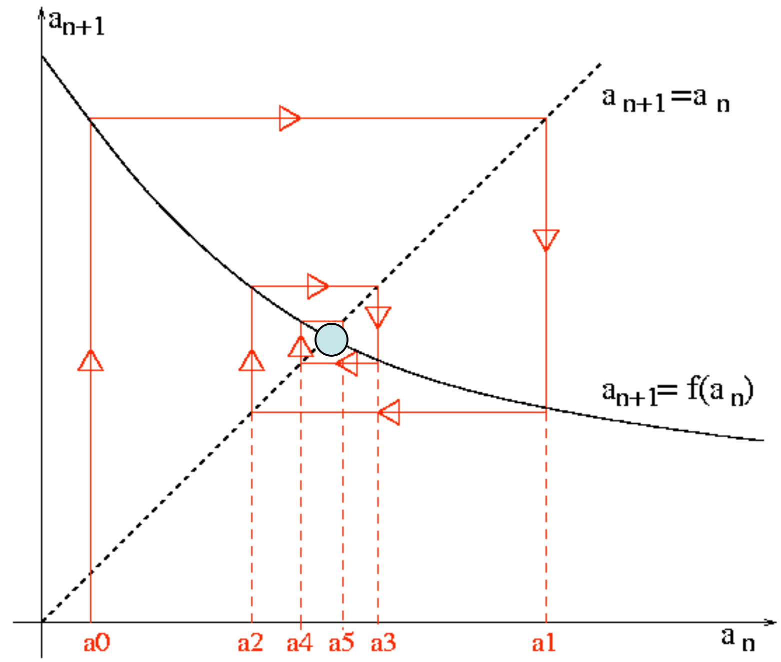
From the cobweb diagram, we see that it converges to a value a:

We write

$\lim_{n \to \infty} a_n = a$

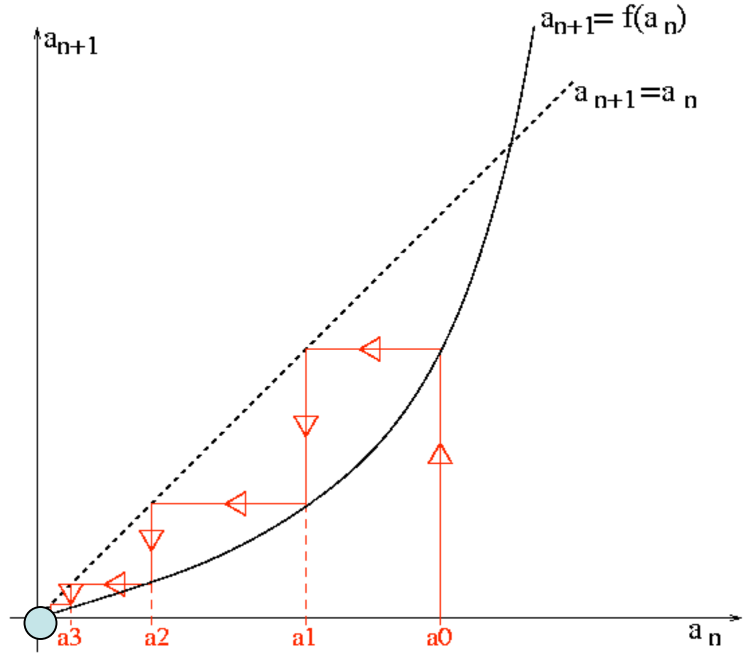
A recursion that converges

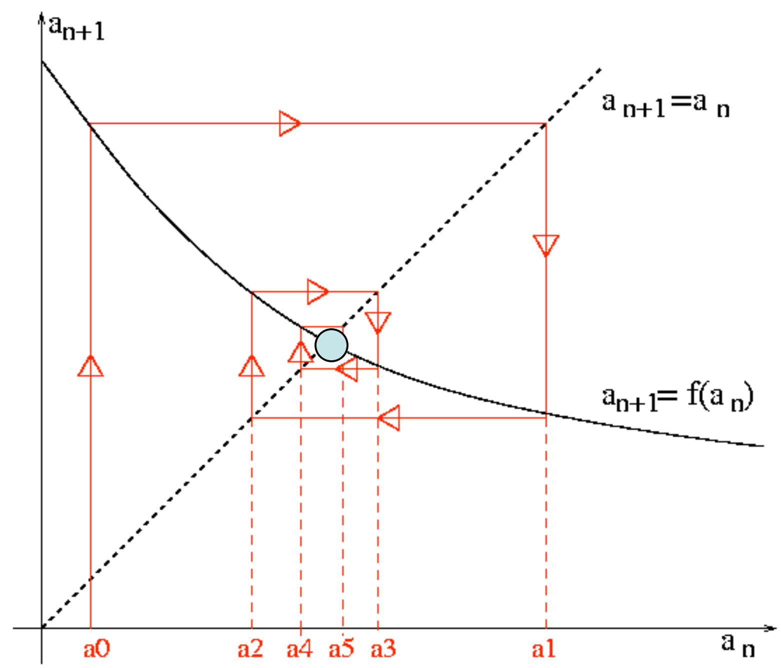




When the sequence converges, it always converges to
A point at the intersection of the $a_{n+1} = a_n$ line (dashed), and the $a_{n+1} = f(a_n)$ curve (solid)

A recursion that converges

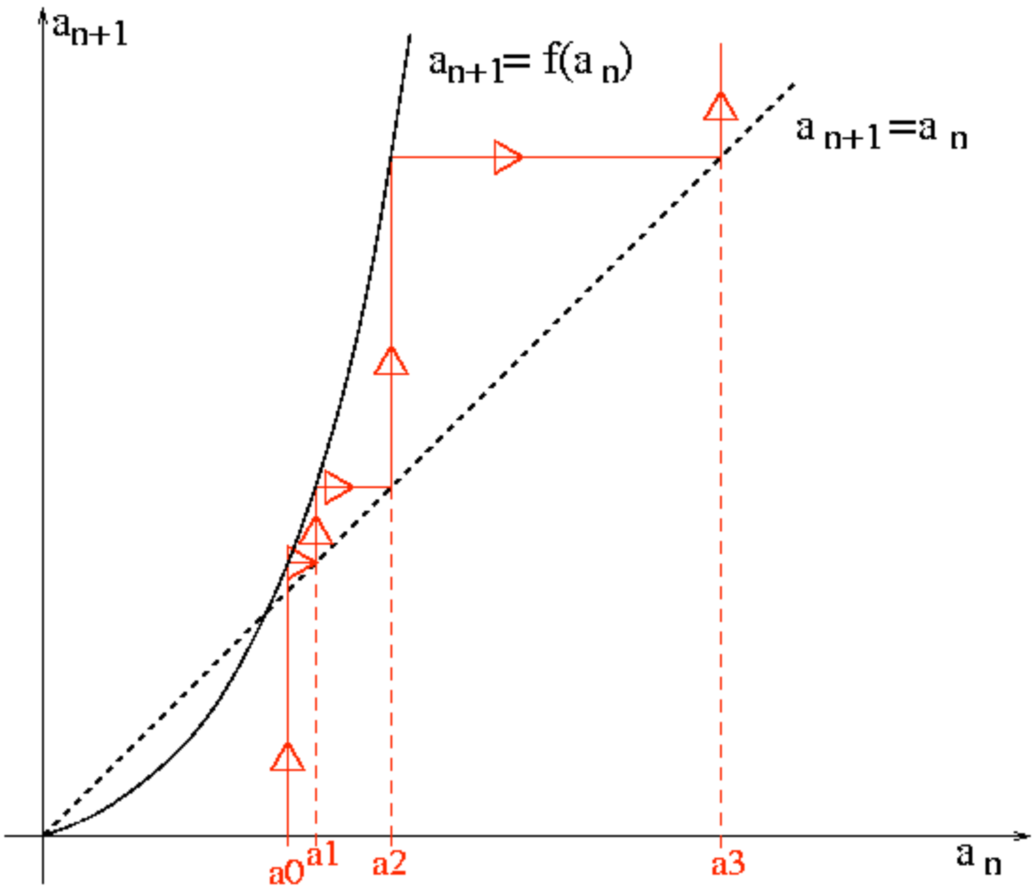




These points are called fixed points

Fixed points satisfy the equation a_{n+1} = $a_n = f(a_n)$

A recursion that diverges

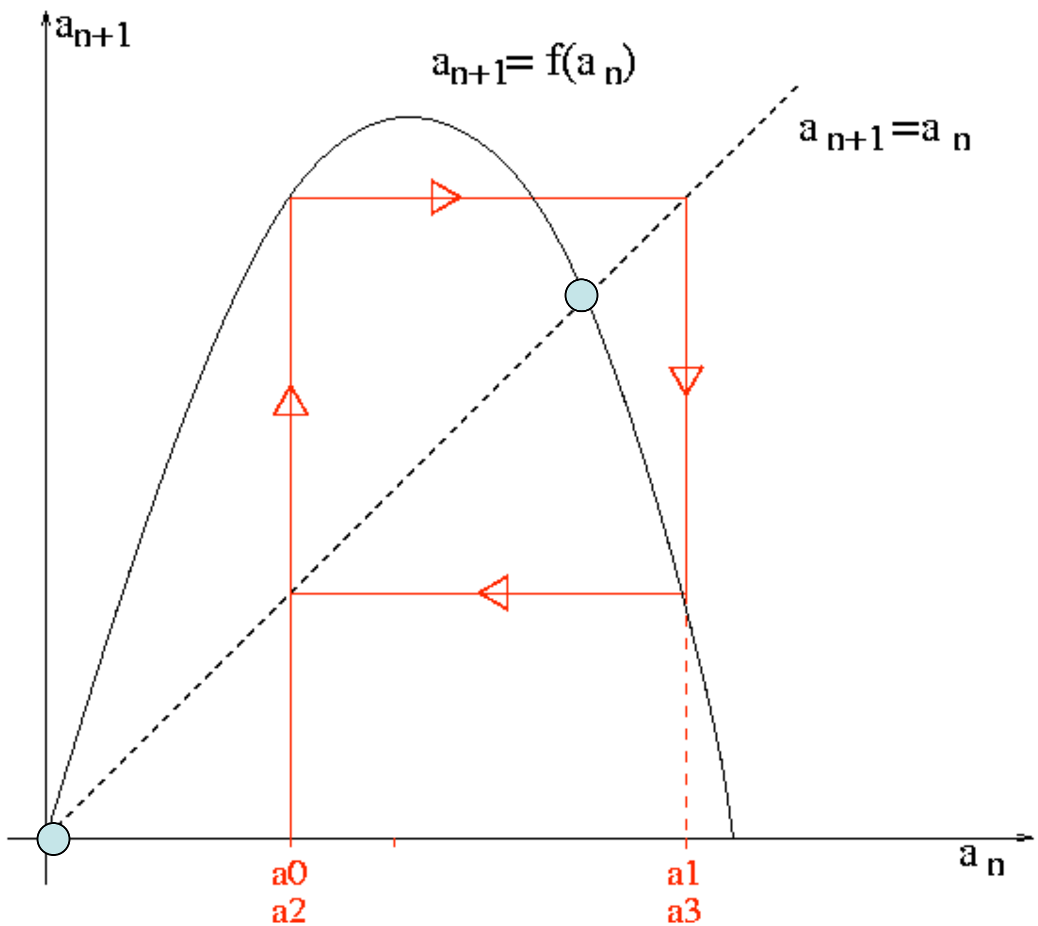


This sequence diverges.

We can write

$\lim_{n\to\infty} a_n = \infty$

A recursion that neither converges nor diverges

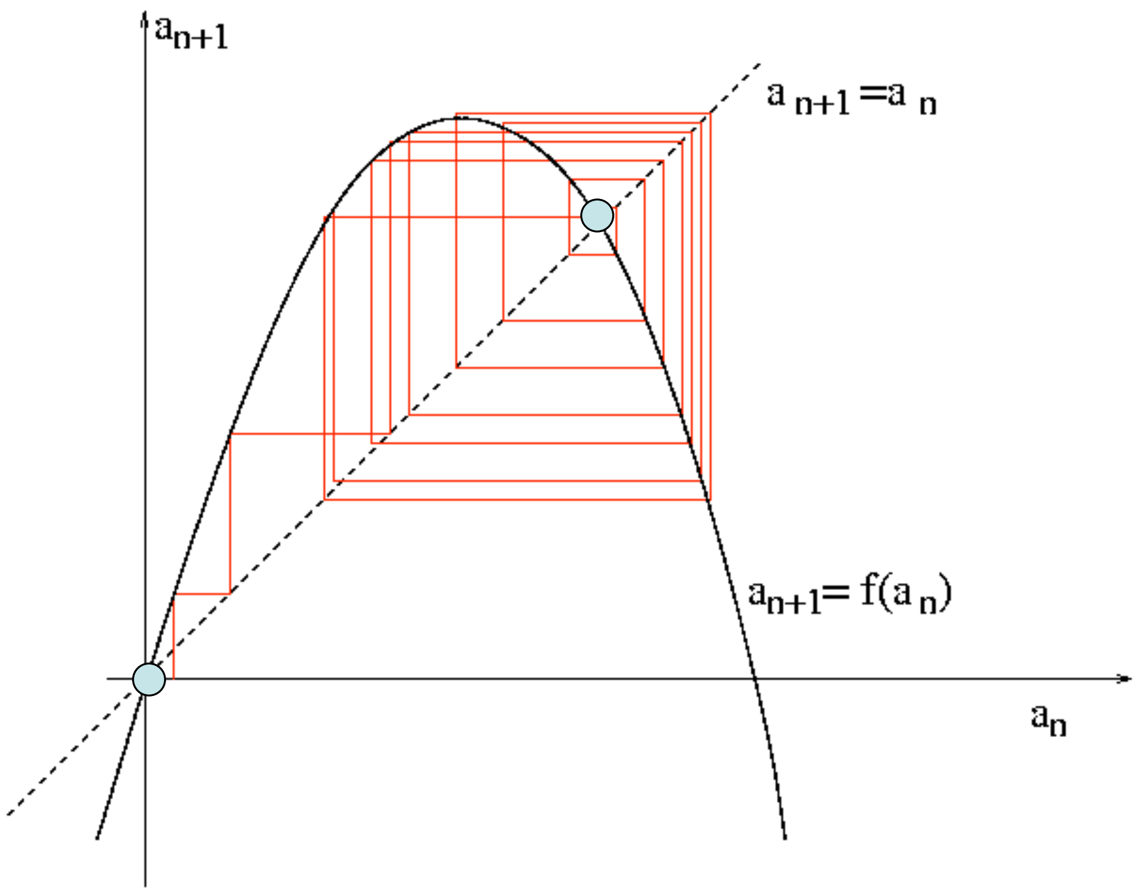


This sequence neither converges not diverges.

It is periodic.

The fixed points exists but they are not the limit of the sequence

A recursion that neither converges nor diverges

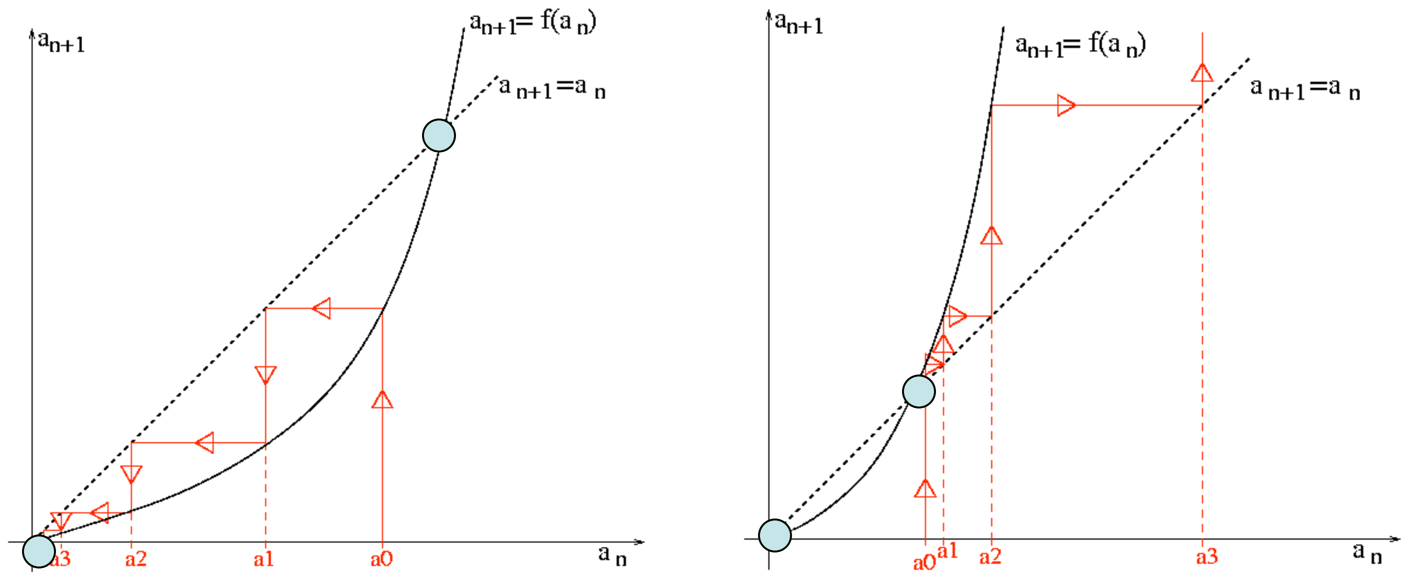


This sequence neither converges not diverges.

It is chaotic.

The fixed points exists but they are not the limit of the sequence

Not all fixed points are limits



Not all fixed points are limits