

AMS 205 – Homework 8
due in class, Tuesday November 30

1. Let $f(x; \theta) = \frac{1}{2\theta}$, $-\theta < x < \theta$, $0 < \theta < \infty$, zero elsewhere. Show that this family is not complete by finding at least one nonzero function $u(x)$ such that $E[u(x)] = 0$ for all $\theta > 0$.
2. Write the pdf $f(x; \theta) = \frac{1}{6\theta^4} x^3 \exp(-x/\theta)$, $0 < x < \infty$, $0 < \theta < \infty$, zero elsewhere, in the exponential form. If X_1, X_2, \dots, X_n is a random sample from this distribution, find a complete sufficient statistic Y_1 for θ and the unique function $\phi(Y_1)$ of this statistic that is the unbiased minimum variance estimator for θ . Is $\phi(Y_1)$ itself a complete sufficient statistic? *Hint:* to find $E[Y_1]$, use what you already know about this distribution, you don't need to re-derive it.
3. Let X_1, X_2, \dots, X_n be a random sample from the gamma distribution with parameters α and β . Find a minimal sufficient statistic.
4. Let $X_1, X_2, \dots, X_n \stackrel{iid}{\sim} N(\theta, 1)$. Find the unbiased minimum variance estimator for θ^2 .