AMS 205 – Homework 6

due in class, Tuesday November 16

1. Let $X_1, \ldots, X_n$ be a random sample from each of the distributions having the following probability density functions:

   (a) $f(x;\theta) = \theta x^{(\theta-1)}$, $0 < x < 1$, $0 < \theta < \infty$, zero elsewhere

   (b) $f(x;\theta) = \frac{1}{2}\exp(-|x-\theta|)$, where the ranges of x and θ are the real line

   In each case, find an estimator of θ by the method of moments and show that it is consistent (hint: to show consistency, consider using Slutsky's theorem and the fact that if a sequence converges in distribution to a constant, then it converges in probability to that constant).

2. Let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics of a random sample of size n from the uniform distribution of the continuous type over the closed interval $[\theta - \rho, \theta + \rho]$. Find the maximum likelihood estimators for θ and ρ. Are these two unbiased estimators?

3. Let the observed value of the mean $\overline{X}$ of a random sample of size 18 from a distribution that is $N(\mu, 80)$ be 81.6. Find a 95% confidence interval for μ.

4. Let a random sample of size 17 from the normal distribution $N(\mu, \sigma^2)$ yield $\bar{x} = 4.7$ and $s^2 = 5.76$. Determine a 90 percent confidence interval for μ.

5. Suppose it is known that a random variable X has a Poisson distribution with parameter λ. A sample of 200 observations from this population has a mean equal to 3.4. Construct an approximate 90% confidence interval for λ.

6. Let two independent random samples, each of size 10, from two normal distributions $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\bar{x} = 4.8$, $(s_1)^2 = 8.64$, $\bar{y} = 5.6$, $(s_2)^2 = 7.88$. Find a 95% confidence interval for $\mu_1 - \mu_2$.