AMS 205 – Homework 4

due in class, Tuesday October 26

1. Let $\overline{X}$ be the mean of a random sample of size 5 from a normal distribution with $\mu = 0$ and $\sigma^2 = 125$. Determine c so that $P(\overline{X} < c) = 0.90$.

2. Let $X_1, \ldots, X_{25}$ and $Y_1, \ldots, Y_{25}$ be two independent random samples from two normal distributions, $N(0, 16)$ and $N(1, 9)$ respectively. Let $\overline{X}$ and $\overline{Y}$ denote the corresponding sample means. Compute $P(\overline{X} > \overline{Y})$.
   Hint: Note that $P(\overline{X} > \overline{Y}) = P(\overline{X} - \overline{Y} > 0)$, and determine the distribution of $\overline{X} - \overline{Y}$.

3. Let S^2 be the variance of a random sample of size 6 from the normal distribution $N(\mu, 12)$. Find $P(2.30 < S^2 < 22.2)$.

4. Let $X_1, X_2, \ldots, X_n$ have a multivariate normal distribution, where $\boldsymbol{\mu}$ is the matrix of means and $\boldsymbol{V}$ is the positive definite covariance matrix. Let $Y = \boldsymbol{c'X}$ and $Z = \boldsymbol{d'X}$ where $\boldsymbol{X}$'$=$ $[X_1\ X_2\ \ldots\ X_n]$, $\boldsymbol{c'} = [c_1\ c_2\ \ldots\ c_n]$, and $\boldsymbol{d'} = [d_1\ d_2\ \ldots\ d_n]$ are real matrices.

   (a) Find the joint MGF $M(t_1, t_2) = E[\exp(t_1 Y + t_2 Z)]$ and see that Y and Z have a bivariate normal distribution. (*Note*: Y and Z are scalars.)

   (b) Prove that Y and Z are independent if and only if $\boldsymbol{c'Vd} = 0$.

5. If $X_i \sim Bin(n_i, p)$ for $i = 1, 2, \ldots, k$, and the X_i's are mutually independent, find the distribution of $Y = \sum_{i=1}^{k} X_i$.

6. Let X_n have a gamma distribution with parameters $\alpha = n$ and β, where β is not a function of n. Let $Y_n = \frac{X_n}{n}$. Find the limiting distribution of Y_n (as $n \to \infty$).