

# Higher-Order Polymorphism

CMPS 290G: Spring 2005

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We motivated our discussion of higher order polymorphism by encoding the *pair* function using the kinds construct discussed earlier. This looked as follows:

$$\lambda X :: *. \lambda Y :: *. \forall Z :: *. (X \rightarrow Y \rightarrow Z) \rightarrow Z$$

A concrete example of this expression was given as the following:

$$\text{PAIR NAT BOOL} = \forall Z :: *. (\text{NAT} \rightarrow \text{BOOL} \rightarrow Z) \rightarrow Z$$

Proceeding onward the *cons* and *car* expressions and their types were defined as follows:

$$\text{cons} : \forall X. \forall Y. X \rightarrow Y \rightarrow \text{PAIR } X \ Y$$

$$\Lambda X :: *. \Lambda Y :: *. \lambda x : X. \lambda y : Y. \Lambda Z :: *. \lambda f : X \rightarrow Y \rightarrow Z. fxy$$

$$\text{car} : \forall X. \forall Y. \text{PAIR } X \ Y \rightarrow X$$

$$\Lambda X :: *. \Lambda Y :: *. \lambda p : \text{PAIR } X \ Y. p[X](\lambda x : X. \lambda y : Y. x)$$

From these examples we notice that the following has been added to our syntax:

$$\begin{aligned} T & ::= \forall X :: K. T \\ v & ::= \Lambda X :: K. t \\ t & ::= t[T] \mid \Lambda X :: K. t \end{aligned}$$

To summarize, we have the following table which describes what we are able to map between (including one that is foreshadowing of what is to come), the representation of that mapping, and the symbol we use to express that mapping:

Mapping	Representation	Symbol
terms to terms	$\lambda x : T. t$	$\rightarrow$
types to terms	$\lambda X :: K. T$	$\Rightarrow$
types to terms	$\Lambda X :: K. t$	$\forall$
terms to types	$\lambda x : T. T'$	$\Pi x : T. T'$

With our mappings defined we produced a graphical representation of a few examples. What resulted is the following illustration shown in figure 1.

We concluded by re-examining the  $\lambda$  cube, filling in all but two corners. Thus, we arrived at the  $\lambda$  cube shown in figure 2.

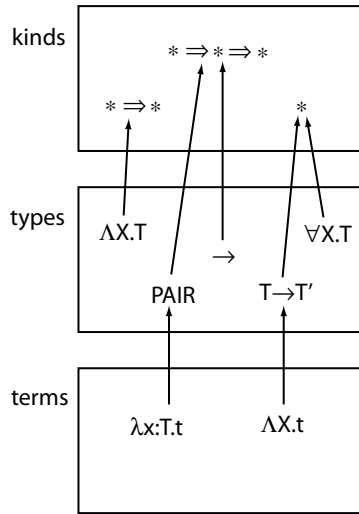


Figure 1: A mapping between kinds, terms and types

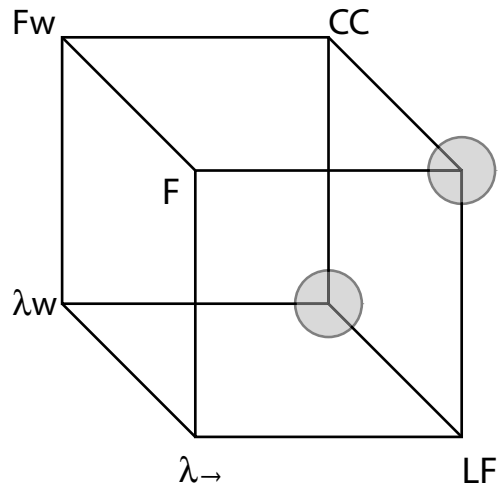


Figure 2: The  $\lambda$ -Cube