

CMPS 290c Topics - Spring '08

1. Convex Sets

- Affine sets
- Convex sets
- Convex combinations and convex hull
- cones and conic combinations
- hyperplanes and half-spaces
- Norms and Norm balls
- polyhedra
- Convexity preserving operations: intersection, image of convex set under affine transformation
- Perspective function
- dual cones, generalized (conic) inequalities, minimum/minimal elements
- Separating hyperplane theorem
- Supporting hyperplane theorem

2. Convex functions

- Definitions of convexity, concavity, strict convexity
- convex if every restriction to a line is convex
- First order condition
- Second order condition
- Epigraphs and sub-level sets, Jensen's inequality
- Closure under: non-negative weighted sums, composition with affine functions, point-wise max and sup, composition with scalar function, minimization over convex set
- conjugate function
- quasi-convexity

3. Optimization problems

- Problems in standard form
- Domain; feasible, optimal, and locally optimal points

- Implicit vs. Explicit constraints
- convex optimization problems
- optimality conditions for differentiable f_0
- Transforming problems and equivalent problems: slack variables, adding/eliminating equality constraints, epigraph form
- linear programming
- quad. programming, positive semi-definite matrices, Hessians, and local/global minima
- Multicriterion optimization and Pareto optimal values

4. Duality

- Lagrangian $L(x, \lambda, \nu)$, Lagrange dual function $g(\lambda, \nu)$
- Lower bound property of $g(\lambda, \nu)$
- Soft constraint interpretation of $g(\lambda, \nu)$
- Dual problem
- Dual of Dual (for linear programming)
- Weak and strong duality
- Slater's constraint qualification
- complementary slackness
- (KKT conditions)