Monty Hall Problem

"lets make a deal!"

3 doors…

- There are three doors 1, 2, 3
- Hosts puts good stuff behind one door *g*, junk behind other two
- Contestant picks a door
- Host shows junk behind a different door (call it *s*)
- Contestant can switch, should they?

What is experiment/Atomic events?

- Assume host picks good door *g* uniformly at random so

  $p(g=1) = p(g=2) = p(g=3) = 1/3$
- If one unpicked junk door the host shows it, if two unpicked junk the host randomly pickes one (50/50) to show
- Let's try it: Pick partners, everybody play contestent twice: switch once and don't switch once

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•Assume contestant initially picks door 1 (to simplify)

Outcome Space
(contestant initially picks 1)

Prob's	g=1	g=2	g=3	Marginal prob *s*'s
s=1	0	0	0	0
s=2	1/6	0	1/3	1/2
s=3	1/6	1/3	0	1/2
Marginal prob *g*'s	1/3	1/3	1/3	1

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Conditional distribution

Prob's	g=1	g=2	g=3	Marg
s=1	0	0	0	0
s=2	1/6	0	1/3	1/2
s=3	1/6	1/3	0	1/2
Marg.	1/3	1/3	1/3	1

P(g=3|s=2) = P(g=3 and s=2) / P(s=2) = 2/3,
P(g=1|s=2) = P(g=1 and s=2) / P(s=2) = 1/3
So switching is twice as good as not switching

Note: P(g|s=2) and P(s|g=1) are conditional distributions on {1,2,3}

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Exercise:

- Repeat analysis for the following modification to host:
  - whenever doors 2 and 3 both junk, always show door 2 rather than picking randomly

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For Learning:

hypotheses

Data

Prob's	[illegible]	[illegible]	[illegible]	Marg
s=1	0	0	0	0
=2	1/6	0	1/3	1/2
s=3	1/6	1/3	0	1/2
Marg.	1/3	1/3	1/3	1

But joint probabilities usually not known..

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For Learning 2

Prob's	h_1	h_2	…	Marg
d_1	?	?	?	?
d_2	?	?	?	?
…	?	?	?	?
Marg. (priors)	$P(h_1)$	$P(h_2)$	…	1

Have priors $P(h_i)$ and (usually) conditional probabilities $P(d_j \mid h_i)$; want $P(h_i \mid d_j)$

For Learning 3

	h_1	h_2	…	Marg
d_1	?	?		?
d_2	?	?	?	$P(d_2)$
…	?	?	?	?
Marg. (priors)	$P(h_1)$	$P(h_2)$	…	1

Have $P(d_2|h_2)$

Exists but Not known

$P(h_2 \mid d_2) = P(d_2 \mid h_2)\, P(h_2) / P(d_2)$ (Bayes' rule)

Although $P(d_2)$ not know, it is the same constant for every h_i: $P(h_2 \mid d_2)$ proportional to $P(d_2 \mid h_2)\, P(h_2)$

Posterior:

$P(h_2 \mid d_2) = P(d_2 \mid h_2)\, P(h_2) / \Sigma_i(P(d_2 \mid h_i)\, P(h_i))$