WE SAY THAT TWO ENCODINGS e_1, e_2 OF A ARE <u>POLYNOMIALLY RELATED</u> IFF THERE EXIST FUNCTIONS

$$f_{12}, f_{21} : \{0,1\}^* \to \{0,1\}^*$$

SUCH THAT $f_{12}(e_1(\alpha)) = e_2(\alpha)$ AND $f_{21}(e_2(\alpha)) = e_1(\alpha)$ FOR ALL INSTANCES α OF THE PROBLEM A, AND FURTHER BOTH f_{12} AND f_{21} ARE COMPUTABLE IN POLYNOMIAL TIME.

WE ASSUME FROM NOW ON THAT ALL DECISION PROBLEMS COME WITH A CLASS OF POLYNOMIALLY RELATED "GOOD" ENCODINGS.

<u>LEMMA</u>:
LET Q BE A DECISION PROBLEM WITH TWO POLYNOMIALLY RELATED ENCODINGS e_1 AND e_2. WRITE $e_1(Q)$ FOR THE CORRESPONDING CONCRETE PROBLEM, AND SIMILARLY $e_2(Q)$. THEN

$$e_1(Q) \in P \text{ iff } e_2(Q) \in P.$$

PROOF IN BOOK (Lemma 34.1, P. 975)

THUS WE CAN TALK OF A PROBLEM Q BEING POLYNOMIAL TIME SOLVABLE WITHOUT REFERENCE TO ANY PARTICULAR ENCODING.

A LANGUAGE L IS A SUBSET

$$L \subseteq \{0,1\}^* = \{\text{bit strings}\}$$

WE WRITE ε FOR THE EMPTY STRING.

LET A BE AN ALGORITHM WHOSE INPUT IS ANY $x \in \{0,1\}^*$ AND WHOSE OUTPUT $A(x)$ IS 0, OR 1, OR NO OUTPUT (i.e. A DOES NOT HALT ON x.)

WE SAY

A ACCEPTS x IF $A(x) = 1$

A REJECTS x IF $A(x) = 0$

THE <u>LANGUAGE ACCEPTED BY A</u> IS THE SET

$$L = \{x \in \{0,1\}^* \mid A(x) = 1\}$$

NOTE $x \in \{0,1\}^* - L$ DOES NOT IMPLY $A(x) = 0$, SINCE IT IS POSSIBLE A DOES NOT HALT ON INPUT x.

WE SAY A LANGUAGE L IS <u>DECIDED</u> BY AN ALGORITHM A IFF

$$A(x) = 1 \text{ IFF } x \in L$$
AND $$A(x) = 0 \text{ IFF } x \in \{0,1\}^* - L.$$

WE SAY $L \subseteq \{0,1\}^*$ IS <u>ACCEPTED IN POLYNOMIAL TIME</u> BY A IFF THERE EXISTS CONSTANT K SUCH THAT

$\forall x \in L$: A RETURNS WITH 1 IN TIME $O(n^k)$ WHERE $n = |x|$.

WE SAY L IS <u>DECIDED IN POLYNOMIAL TIME</u> BY A IFF THERE EXISTS K S.T.

$\forall x \in L$: A RETURNS WITH 1 IN TIME $O(n^k)$
AND $\forall x \notin L$: A RETURNS WITH 0 IN TIME $O(n^k)$

WHERE $n = |x|$.

NOTICE TO ACCEPT A LANGUAGE AN ALGORITHM NEED ONLY WORRY ABOUT $x \in L$, WHEREAS TO DECIDE A LANGUAGE, IT MUST CORRECTLY ACCEPT OR REJECT EVERY $x \in \{0,1\}^*$.

WE NOW GIVE AN ALTERNATE DEFINITION OF THE COMPLEXITY CLASS P.

$$P = \{ L \subseteq \{0,1\}^* \mid \text{THERE EXIST A POLY-TIME ALGORITHM THAT } \underline{\text{ACCEPTS}}\ L \}$$

<u>THEOREM</u>:

$$P = \{ L \subseteq \{0,1\}^* \mid \text{THERE EXIST A POLY-TIME ALGORITHM THAT } \underline{\text{DECIDES}}\ L \}$$

Proof ON P. 977 (THM 34.2).