Class Exercise on Perceptron Learning [Slide No: 14]

DATA

x_1 = "win the vote" = $\begin{bmatrix} \text{BIAS} : 1 \\ \text{win} : 1 \\ \text{game} : 0 \\ \text{vote} : 1 \\ \text{the} : 1 \end{bmatrix}$ $y^* = \text{Politics}$

x_2 = "win the election" = $\begin{bmatrix} \text{BIAS} : 1 \\ \text{win} : 1 \\ \text{game} : 0 \\ \text{vote} : 0 \\ \text{the} : 1 \end{bmatrix}$ $y^* = \text{Politics}$.

x_3 = "win the game" = $\begin{bmatrix} \text{BIAS} : 1 \\ \text{win} : 1 \\ \text{game} : 1 \\ \text{vote} : 0 \\ \text{the} : 1 \end{bmatrix}$ $y^* = \text{Sports}$.

LEARNING

For x_1

$$w_{SPORT} \cdot x_1 = 1$$
$$w_{POLITICS} \cdot x_1 = 0 \quad \Rightarrow y = \text{SPORT} \quad y^* = \text{POLITICS}.$$
$$w_{TECH} \cdot x_1 = 0$$

Update weights.

$$w_{SPORT} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ -1 \end{bmatrix}$$

$$(w'_{SPORT} - x = w_{SPORT})$$

$$w_{POLITICS} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad (w'_{POLITICS} + x = w_{POLITICS})$$

For x_2

$$w_{SPORT} \cdot x_2 = -2$$
$$w_{POLITICS} \cdot x_2 = 3 \quad \Rightarrow \quad y = \text{POLITICS} \ \& \ y^* = \text{POLITICS}.$$
$$w_{TECH} \cdot x_2 = 0$$

No change to weights.

For x_3

$$w_{SPORT} \cdot x_3 = -2$$
$$w_{POLITICS} \cdot x_3 = 3 \quad \Rightarrow \quad y = \text{POLITICS} \ \& \ y^* = \text{SPORTS}$$
$$w_{TECH} \cdot x_3 = 0$$

Update weights

$$w_{SPORT} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$w_{SPORT}' + x = w_{SPORT}$$

$$w_{POLITICS} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

$$w_{POLITICS}' - x = w_{POLITICS}$$

Class Exercise on Variable Elimination [Slide No: 100]

From the graph we have

$$P(Sm, St, Pr, F, Pa) = P(Sm)P(St)P(Pr|Sm, St)P(F)P(Pa|Sm, Pr, F).$$

$$P(Pa) = \sum_{Sm, St, Pr, F} P(Sm, St, Pr, F, Pa)$$

$$= \sum_{Sm, St, Pr, F} P(Sm)P(St)P(Pr|Sm, St)\underbrace{P(F)\,P(Pa|Sm, Pr, F)}_{f_F = \sum_F P(F)P(Pa|Sm, Pr, F)}.$$

where

$f_F =$

Sm	Pr	Pa	$f_F(Sm, Pr, Pa)$
T	T	T	$0.9\times0.9+0.1\times0.1 = 0.82$
T	T	F	$0.9\times0.1+0.1\times0.9 = 0.18$
T	F	T	$0.9\times0.7+0.1\times0.1 = 0.64$
T	F	F	$0.9\times0.3+0.1\times0.9 = 0.36$
F	T	T	$0.9\times0.7+0.1\times0.1 = 0.64$
F	T	F	$0.9\times0.3+0.1\times0.9 = 0.36$
F	F	T	$0.9\times0.2+0.1\times0.1 = 0.19$
F	F	F	$0.9\times0.8+0.1\times0.9 = 0.81$

$$P(Pa) = \sum_{Sm, Pr, St} P(Sm)f_F(Sm, Pa, Pr)\underbrace{P(St)P(Pr|Sm, St)}_{f_{St} = \sum_{St} P(St)P(Pr|Sm, St)}.$$

where

$f_{St} =$

Sm	Pr	$f_{St}(Pr, Sm)$
T	T	$0.6\times0.9+0.4\times0.5 = 0.74$
T	F	$0.6\times0.1+0.4\times0.5 = 0.26$
F	T	$0.6\times0.7+0.4\times0.1 = 0.46$
F	F	$0.6\times0.3+0.4\times0.9 = 0.54$

$$P(Pa) = \sum_{Pr, Sm} \underbrace{P(Sm)\, f_F(Sm, Pa, Pr)\, f_{St}(Pr, Sm)}$$

$$f_{Sm} = \sum_{Sm} P(Sm)\, f_F(Sm, Pa, Pr)\, f_{St}(Pr, Sm)$$

where

$f_{Sm} =$

Pr	Pa	$f_{Sm}(Pr, Pa)$
T	T	$0{\cdot}8 \times 0{\cdot}82 \times 0{\cdot}74 + 0{\cdot}2 \times 0{\cdot}64 \times 0{\cdot}46 = 0{\cdot}5443$
T	F	$0{\cdot}8 \times 0{\cdot}18 \times 0{\cdot}74 + 0{\cdot}2 \times 0{\cdot}36 \times 0{\cdot}46 = 0{\cdot}1396$
F	T	$0{\cdot}8 \times 0{\cdot}64 \times 0{\cdot}26 + 0{\cdot}2 \times 0{\cdot}19 \times 0{\cdot}54 = 0{\cdot}1536$
F	F	$0{\cdot}8 \times 0{\cdot}36 \times 0{\cdot}26 + 0{\cdot}2 \times 0{\cdot}81 \times 0{\cdot}54 = 0{\cdot}1623$

$$P(Pr) = \sum_{Pa} f_{Sm}(Pr, Pa)$$

Pr	P(Pr)
T	$0{\cdot}5443 + 0{\cdot}1396 = 0{\cdot}6839$
F	$0{\cdot}1536 + 0{\cdot}1623 = 0{\cdot}3158$

$$P(Pr = True) = 0{\cdot}6979$$