

## Sample Final Problems

1. Show that  $\text{Accept}(\Lambda)$  is undecidable.
2. Show that if  $\mathcal{P} = \mathcal{NP}$ , a polynomial time algorithm exists that, given a Boolean formula  $\phi$ , actually produces a satisfying assignment for  $\phi$ .
3. Show that  $\mathcal{P}$  is closed under Kleene star. That is if  $L \in \mathcal{P}$  then  $L^* \in \mathcal{P}$  as well. Do the same for  $\mathcal{NP}$ .
4. Show that the Hamiltonian Path problem is NP-complete by reducing the Hamiltonian Cycle problem to it.
5. Give the definition of NP-completeness
6. Let  $L = \{e(T)x1^t : \text{where } T \text{ is a deterministic TM that accepts } x \text{ within } t \text{ steps}\}$ . Show that  $L$  is NP-complete.
7. Show that TSP is NPC by reduction for HC to TSP.
8. Show that the following problem is in  $\mathcal{P}$ :  
k-CLIQUE:  
INSTANCE: An undirected graph  $G = (V, E)$ .  
QUESTION: Does  $G$  have a clique of size  $k$ , where  $k$  is a constant.

9. Show that the following problem is NP-complete.

- FEEDBACK VERTEX SET

INSTANCE: Directed Graph  $G = (V, A)$ , positive integer  $K \leq |V|$ .

QUESTION: Is there a subset  $V' \subseteq V$  such that  $|V'| \leq K$  and such that every directed circuit in  $G$  includes at least one vert from  $V'$ ? (Hint: reduce from Vertex Cover)

- DOMINATING SET

INSTANCE: Graph  $G = (V, E)$ , positive integer  $K \leq |V|$ .

QUESTION: Is there a subset  $V' \subseteq V$  such that  $|V'| \leq K$  and such that every vertex  $v \in V - V'$  is joined to at least one member of  $V'$  by an edge in  $E$ ? (Hint: reduce from Vertex Cover)

10. Give a definition of the Post Correspondence Problem. What is the complexity of this decision problem?

11. A real number  $x$  in the interval  $(0..1)$  is called *computable* if there is a TM that enumerates the digits of  $x$  after the decimal point. Are all reals in  $(0..1)$  computable? Give reasons for your answer!
12. Is there an unrestricted grammar generating the following language:

$$SA = \{e(T) : T \text{ accepts } e(T)\} ?$$

What is language class generated by unrestricted grammars?

13. Give a definition of the Busy Beaver Function. Somebody claims that this function can be computed with a new kind of computer that uses light to send messages. What is your opinion of this claim?
14. Recall Collatz's conjecture:

Each natural number is associated with a sequence as follows: if it is even, then divide the number by two and if it is odd, multiply it by three and add one.

Example: 7 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1 4 2 1 ...

Conjecture: All natural number go to 1.

This conjecture is open!

Show that if the Halting Problem was decidable then this conjecture could be resolved one way or the other.

Hint: Assume the existence of a TM  $T$  that halts on  $n$  iff  $n$  goes to 1. Further assume that  $T_H$  decides the halting problem, i.e.  $T_H$  on inputs  $e(T)$  and  $e(n)$  decides whether  $n$  goes to 1.

Now sketch a new TM that halts iff there is a number that does not go to one. Put it all together and show how this outline can be used to resolve the conjecture.

15. Show that  $\mathcal{P}$  is closed under polynomial time reductions.
16. Show that if an  $\mathcal{NP}$ -complete problem is in  $\mathcal{P}$ , then  $\mathcal{P} = \mathcal{NP}$ .