CMP 132 Homework 4 (modified)

To be done in groups of 2. Please work with different partners than in the other assignments
6 problems, 23 pts, due Thursday February 16

1. (4 pts) Prove that whenever language L is Turing recognizable, but not decidable then any TM recognizing L must run forever on infinitely many inputs. (Hint: what if there was a TM M that recognizes L, and only runs forever on finitely many inputs?).

2. (3 pts) Consider the $3x + 1$ question described in problem 5.31 of the text:

   > Does every $n \in \mathcal{N}$, have sequence $n, f(n), f(f(n)), \ldots$ that eventually includes a 1?

   Construct a TM T that ignores its input and accepts if the answer to $3x+1$ question is "yes" and rejects if the answer to the $3x+1$ problem is "no". You may assume that you have a TM R (and its description $\langle R \rangle$) that decides the language $\text{ALL}_{TM} = \{\langle M \rangle | L(M)\Sigma^*\}$.

   Hint: TM T will probably write a TM description $\langle M \rangle$ and then use the result of R on input $\langle M \rangle$ to help get its answer.

   You may solve the problem as originally stated for extra credit. In the original statement you can assume that a TM H that decides A_{TM} (as well as its description $\langle H \rangle$) is available. (My solution to this actually uses H in two different ways, once as part of M and once as part of T.)

3. (4 pts) Call a language L *reversible* if for all $w \in L$ the string $w^{\mathcal{R}}$ is also in L. Prove that the language: $\text{Rev}_{TM} = \{\langle M \rangle | M \text{ is a TM and } L(M) \text{ is reversible}\}$ is undecidable. This is a restatement of problem 5.9 in the text. Use a reduction from an undecidable language, do *not* use Rice's theorem.

4. (4 pts) Fix an alphabet $\Sigma = \{0, 1, (,), \#\}$. Prove that for every recursively enumerable language L over Σ, language L reduces to the language $A_{TM} = \{\langle Tw \rangle | T \text{ is a TM and } T \text{ accepts } w\}$ (i.e. for each recursively enumerable language L over Σ, show that there is a computable function f_L from Σ^* to Σ^* such that for each $w \in \Sigma^*$, $f_L(w) \in A_{TM}$ if and only $w \in L$.

5. (4 pts, new) Prove that $L = \{\langle M\rangle |$all strings w accepted by M have an even number of symb
is undecidable.

   (Hint: Recall that in the proof of theorem 5.2 in the text, TM S accepts when R rejects and visa versa).

6. (4 pts, new) Problem 5.13 in the text. (Hint: you can use transitions that don't move the head, and it might be helpful to have a special symbol that causes most states to not move the head and transition to the next state.)

Recommended: Exercises 5.1 through 5.4; problems 5.12 and 5.13.