1. Give a pushdown automaton for accepting the following language. State your acceptance criterion.

$$\{a^i b^j : i \neq j \text{ and } i, j \geq 0\}.$$

Hint: First construct a 2-state (non-deterministic) pushdown automaton for accepting

$$\{a^i b^j : i = j \text{ and } i, j \geq 0\} \quad = \quad \{a^n b^n : n \geq 0\} \ .$$

Then modify this automaton in two ways for accepting the languages $\{a^i b^j : i > j \geq 0\}$ and $\{a^i b^j : 0 \leq i < j\}$, respectively. Now that is enough hints! Don't forget the acceptance criterion!

2. Give a complete proof that the following language is not context free:

$$\{a^n b a^n b a^n : n \geq 0\}.$$

Hint: Use the following version of the pumping lemma for context free languages:

If L is context free, then there exists N such that: If $u \in L$ and $|u| \geq N$, then u can be written as $vwxyz$ for which

- $|wxy| \leq N$,
- $|wy| > 0$, and
- $vw^m xy^m z \in L$ for all $m \geq 0$.

3. Give a concise description of the language generated by the following grammar.

$$
\begin{aligned}
S &\to ZN \\
Z &\to 00ZM \mid \Lambda \\
N &\to MN1 \mid \Lambda \\
M &\to a \mid b
\end{aligned}
$$

(The non-terminals are upper case and $\{0, 1, a, b\}$ are terminal.)

Hint: Give a concise description of the language generated by M, N, Z and S, respectively.

The kind of descriptions we want are regular expressions, or mathematical formulas such as $\{a^n b^n c^{2n} : n \geq 0\}$.

4. Convert the following context-free grammar into Chomsky Normal Form. Show your steps!

$$
\begin{aligned}
S &\to ABC \\
A &\to aA \mid C \mid \Lambda \\
B &\to Bb \mid b \\
C &\to bCc \mid \Lambda
\end{aligned}
$$

5. Can the following grammar

$$S \to AB \mid BC$$
$$A \to BA \mid a$$
$$B \to CC \mid b$$
$$C \to AB \mid a$$

generate the string *bbab*? Use the CYK algorithm. Show your work by giving the table. How can the table be used to answer the question whether the string can be generated or not!

6. Show that the following CFG grammar is ambiguous and find an equivalent unambiguous CFG.

$$S \to aSb \mid aaSb \mid \lambda \ .$$

7. Give an algorithm that decides whether a given Finite State Automaton accepts a finite language. Does your algorithm have polynomial running time?

8. Give an algorithm that decides whether a given Context Free Grammar generates the empty language. Does your algorithm have polynomial running time?