HW #2, CMPS 130, spring '08

100 pts

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3.3) → **Part e** if r_1 & r_2 don't represent equivalent languages, then

$(r_1 + r_2)^* r_1 r_2 (r_1 + r_2)^* + r_2^* r_1^*$
$((r_1 + r_2)^* - r_2^* r_1^*) + r_2^* r_1^*$
$(r_1 + r_2)^* r_1 r_2 (r_1 + r_2)^*$ can form any combination of r_1 and r_2 except $r_2^* r_1^*$.
$(r_1 + r_2)^*$

Otherwise, if r_1 and r_2 are equivalent regular expression. then obviously

$$r_2^* r_1^* = (r_1+r_2)^* r_1 r_2 (r_1+r_2)^* = (r_1+r_2)^*$$

So, for any arbitrary regexes we have

$$(r_1+r_2)^* r_1 r_2 (r_1+r_2)^* + r_2^* r_1^* = (r_1+r_2)^* \quad ///$$

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3.9 part i) Note that this language is similar to

$$L = \{ \text{strings containing substring X} \} \quad (3)$$
$$= (0+1)^* X (0+1)^* \quad (4)$$

except that there's two substrings ("11" and "010") which could appear in either order.

$$L = \{ \text{strings containing both 11 and 010 as substrings} \} \quad (5)$$
$$= \left((0+1)^* 11 (0+1)^* 010 (0+1)^*\right) + \left((0+1)^* 010 (0+1)^* 11 (0+1)^*\right) \quad (6)$$

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3.17 part e) **(i)**

Solving this problem is facilitated by doing state-elimination from the DFA.

State	RegExp	Description
I	$(ab+ba)^*$	number of as and bs are balanced pairwise
II	$(ab+ba)^* a$	one extra a
III	$(ab+ba)^* b$	one extra b
IV	$(ab+ba)^*(aa+bb)(a+b)^*$	unbalanced

20

4) 3.46 b)

$L = \{0^n 1^n \mid n \geq 0\}$

$=$ {strings which are some number of 0's followed by the same number of 1's}

Claim: any two strings in the set $S = \{0^m \mid m \geq 1\}$ are distinguishable w/ respect to L

Proof:

Let j and k be distinct positive integer.

$(j, k \geq 1$ and $j \neq k)$

0^j and 0^k are members of S

$0^j 1^j \in L$ but because $j \neq k$ we know

$0^k 1^j \notin L$

so $0^k, 0^j$ are distinguishable w/ respect to L because the string 1^j (and 1^k) distinguish them w/ respect to L

So the Clam is proven and we have shown any pair of strings in S are distinguishable with respect to L.

It follows that L is not a regular language

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5) Show that if L is a non-regular language and
F a finite language, then L - F is non-regular
(where - denote the set minus).
Hint: Do a proof by contradiction and use the closure
properties of regular languages.

Proof:

Let L be any non-regular language
and let F be any finite language

To work towards a contradiction,
assume that L-F is a regular language

Because F is a finite language,
F is a regular language; as is any subset of F.

In particular, $F \cap L$, which is a subset of F
must be regular.

$(L-F) \cup (F \cap L) = L$ because $L-F = L-(L \cap F)$

Now, because regular languages are closed under union,
we come to the conclusion that L is regular.

This is a contradiction because
L is a non-regular language by
hypothesis.
Then it must be that our assumption
that L-F is regular must be incorrect
Thus we have shown
If L is a non-regular language and
F is a finite language,
then L-F is non-regular.

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