CMPS 102 Homework 2

due at the start of class, Tuesday April 12, 2004

Reading: Sections 4.1-4.3 and 5.1-5.2 (plus any sections of appendix C that you need).

1. Give a formal proof by induction that every binary tree with n nodes has exactly $n-1$ edges. Pay particular attention to your proof methodology. You should use the following definition which disallows empty trees:

   A binary tree is either:

   (a) a single node (the root) with no edges, or

   (b) a node (the root) connected to the root of a binary tree, or

   (c) a node (the root) connected to the roots of two disjoint binary trees.

   You may use without proof that (with this definition) every binary tree has at least one node.

2. Let $T(n)$ be defined for $n \geq 1$ by:

$$T(n) = \begin{cases} 0 & \text{for } 1 \leq n \leq 2 \\ 3T(\lfloor n/3 \rfloor) + 2 & \text{for } n \geq 3 \end{cases}$$

   Prove by (strong) induction that $T(n) \leq n$ for all $n \geq 1$. (Hint, it is much easier to use a slightly stronger inductive hypothesis.)

3. Prove that $\ln(\ln n)$ is in $o(\ln n)$. You may use the limit lemmas and l'Hopital's rule.

Recommended problems (not to be turned in): Exercises 2.3-3 (pg 36), 3.2-1 (pg 57), 3.2-3 (pg 57), 4.3-1 (pg 75), 4.3-4 (pg 75), 5.2-1 (pg 98), 5.2-3 (pg 98) and Problem 3-4 on page 59.