

CMPS 101
Winter 2009 Homework Assignment 4

1. (3 Points)

Consider the function $T(n)$ defined by the recurrence formula

$$T(n) = \begin{cases} 6 & 1 \leq n < 3 \\ 2T(\lfloor n/3 \rfloor) + n & n \geq 3 \end{cases}$$

- a. (1 Point) Use the iteration method to write a summation formula for $T(n)$.
- b. (1 Point) Use the summation in (a) to show that $T(n) = O(n)$
- c. (1 Point) Use the Master Theorem to show that $T(n) = \Theta(n)$

2. (6 Points)

Use the Master theorem to find asymptotic solutions to the following recurrences.

- a. (1 Point) $T(n) = 7T(n/4) + n$
- b. (1 Point) $T(n) = 9T(n/3) + n^2$
- c. (1 Point) $T(n) = 6T(n/5) + n^2$
- d. (1 Point) $T(n) = 6T(n/5) + n \log(n)$
- e. (1 Point) $T(n) = 7T(n/2) + n^2$
- f. (1 Point) $S(n) = aS(n/4) + n^2$ (Note: your answer will depend on the parameter a .)

3. (1 Point) p.75: 4.3-2

The recurrence $T(n) = 7T(n/2) + n^2$ describes the running time of an algorithm A . A competing algorithm B has a running time of $S(n) = aS(n/4) + n^2$. What is the largest integer value for a such that B is asymptotically faster than A ?

4. (1 Points)

Let G be an acyclic graph with n vertices, m edges, and k connected components. Show that $m = n - k$. (Hint: use the fact that $|E(T)| = |V(T)| - 1$ for any tree T , from the induction handout.)

5. (1 Point) (Appendix B.4 problem 3)

Show that any connected graph G satisfies $|E(G)| \geq |V(G)| - 1$. (Hint: use induction on the number of vertices.)

6. (1 Point) p. 538: 22.2-2

Show the d and π values that result from running breadth-first search on the undirected graph of Figure 22.3, using vertex u as the source.

