CMPS 101
Winter 2009
Homework Assignment 2

1. (1 Point) p.50: 3.1-1
   Let $f(n)$ and $g(n)$ be asymptotically non-negative functions. Using the basic definition of Θ-notation, prove that $f(n)+g(n)=\Theta(\max(f(n),g(n)))$.

2. (1 Point) p.50: 3.1-3
   Explain why the statement "The running time of algorithm A is at least $O(n^2)$" is meaningless.

3. (2 Points) p. 50: 3.1-4
   Determine whether the following statements are true or false.
   a. (1 Point) $2^{n+1}=O(2^n)$
   b. (1 Point) $2^{2n}=O(2^n)$

4. (6 Points) p.58: 3-2abcdef
   Indicate, for each pair of expressions (*A*, *B*) in the table below, whether *A* is *O*, *o*, Ω, ω, or Θ of *B*. Assume that $k \geq 1$, $\varepsilon > 0$, and $c > 1$ are constants. Place 'yes' or 'no' in each of the empty cells below, and justify your answers.

	A	B	O	o	Ω	ω	Θ
a. (1 Point)	$\lg^k n$	n^{ε}					
b. (1 Point)	n^k	c^n					
c. (1 Point)	$\sqrt{n}$	$n^{\sin n}$					
d. (1 Point)	2^n	$2^{n/2}$					
e. (1 Point)	$n^{\lg c}$	$c^{\lg n}$					
f. (1 Point)	$\lg(n!)$	$\lg(n^n)$					

5. (4 Points) p.58: 3-4cdeh
   Let $f(n)$ and $g(n)$ be asymptotically positive functions (i.e. $f(n) > 0$ and $g(n) > 0$ for sufficiently large *n*.) Prove or disprove the following statements.

   c. (1 Point)
      Assume $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for all sufficiently large *n*. Then $f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$.

   d. (1 Point)
      $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.

   e. (1 Point)
      $f(n) = O((f(n))^2)$.

   h. (1 Point)
      $f(n) + o(f(n)) = \Theta(f(n))$.

6. (10 Points)
   Let $f(n) = \Theta(n)$. Prove that $\sum_{i=1}^{n} f(i) = \Theta(n^2)$. (See the hint at bottom of p.4 of the handout on asymptotic growth rates.)