CMPS 101
Midterm 2 Review Problems

Study problem 19 from the Midterm 1 review sheet, as well as solutions to homework assignments 4, 5 and 6. Also do the problems on the Master Theorem Problems handout.

Figure 1:

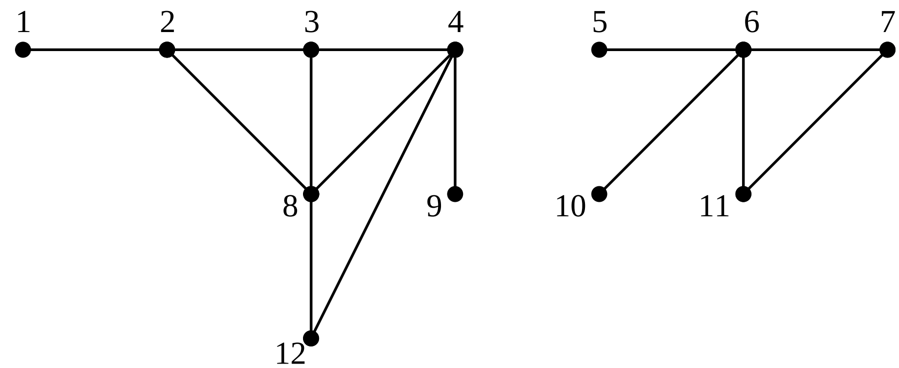


Figure 2:

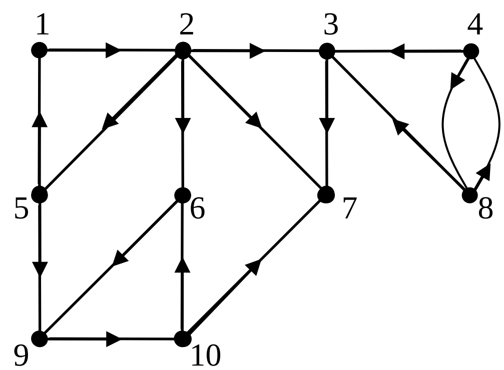
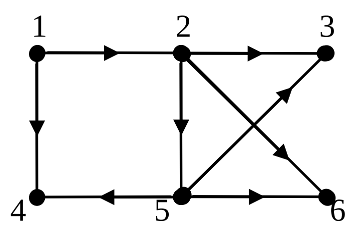


Figure 3:



Problems

1. Trace BFS on the following graphs. For each vertex, record its color, parent, and distance fields, draw the resulting BFS tree, and determine the order in which vertices are added to the Queue. Process adjacency lists in ascending numerical order.
   a. The graph in figure 1, with 1 as the source.
   b. The directed graph in figure 2 with 1 as source.

2. Trace DFS on the following graphs. For each vertex, record its color, parent, discover, and finish times, and draw the resulting DFS forest. Classify each edge as tree, back, forward, or cross. Process adjacency lists in ascending numerical order.
   a. The graph in figure 1. Process vertices in the main loop of DFS in (ascending) numerical order.
   b. The graph in figure 2. Process vertices in the main loop of DFS in (ascending) numerical order.
   c. The transpose of the graph in figure 2. Process vertices in the main loop of DFS in order of descending finish times from part b. Determine the strongly connected components of the graph in figure 2, and draw its component graph. Also determine a topological sort of the strong components.

   d. The graph in figure 3. Process vertices in the main loop of DFS in (ascending) numerical order. Show that this graph is acyclic and determine a topological sort of the vertices.
   e. The graph in figure 3. Process vertices in the main loop of DFS in descending order. Determine a topological sort of the vertices which is different from that in part d.

3. Write an algorithm called isBipartite(*G*) that takes as input a connected (undirected) graph *G* and returns *true* or *false* according to whether *G* is or is not bipartite. Hint: see the solutions to hw5 problem 4 (the wrestler problem).

4. Let $G = (V, E)$ be a connected (undirected) graph. Prove $|E| \geq |V| - 1$. Hint: use induction on $|E|$. Observe that this is Lemma 3 from page 4 of the Graph Theory handout.

5. Prove Lemmas 1, 2, 4, 5, and 6 on the Graph Theory handout.

6. Let *G* be a directed graph. Determine whether, at any point during a Depth First Search of *G*, there can exist an edge of the following kind. (No justification is required.)
   a. A tree edge that joins a white vertex to a gray vertex.
   b. A back edge that joins a black vertex to a white vertex.
   c. A forward edge that joins a gray vertex to a black vertex.
   d. A cross edge that joins a black vertex to a gray vertex.

7. a. State the parenthesis theorem.
   b. State the white path theorem.

8. Let *G* be a directed graph. Prove that if *G* contains a directed cycle, then *G* contains a back edge. (Hint: use the white path theorem.)

9. Let *G* be a connected graph, and suppose that $|E(G)| = |V(G)|$. Show that *G* is unicyclic, i.e. *G* contains exactly one cycle. (Hint: use lemma 1 and lemma 3 from the graph theory handout, and note that this is lemma 7.)