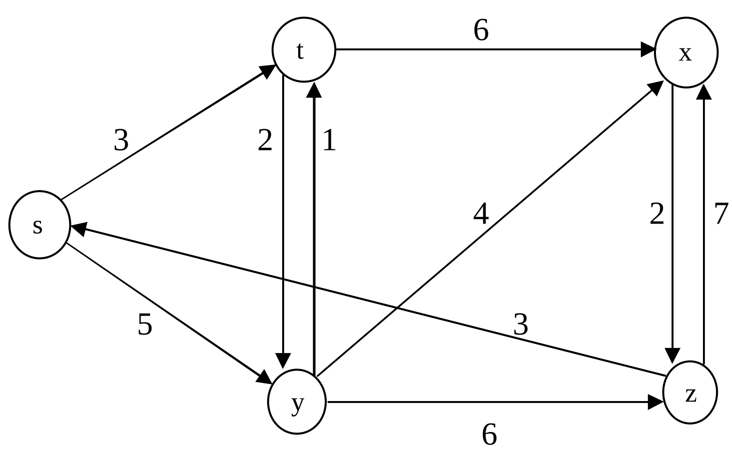
CMPS 101
Summer 2009
Homework Assignment 7

1. (1 Point)
Let $x \in V(G)$ and suppose that after INITIALIZE-SINGLE-SOURCE(*G*, *s*) is executed, some sequence of calls to Relax( ) causes $d[x]$ to be set to a finite value. Then *G* contains an *s*-*x* path of weight $d[x]$. (Hint: Use induction on the length of the Relaxation sequence, and recall that this result was proved in class.)

2. (1 Point) p.591: 24.1-3
Given a weighted, directed graph $G = (V, E)$ with no negative-weight cycles, let *m* be the maximum over all pairs of vertices $u, v \in V$ of the minimum number of edges in a shortest path from *u* to *v*. (Here, the shortest path is by weight, not the number of edges.) Suggest a simple change to the Bellman-Ford algorithm that allows it to terminate in $m+1$ passes.

3. (1 Point) p.600: 24.3-1
Run Dijkstra's algorithm on the directed graph of Figure 24.2, first using vertex *s* as the source and then using vertex *z* as the source. In the style of Figure 24.6, show the *d* and π values and the vertices in set *S* after each iteration of the **while** loop.



4. (1 Points) p.600: 24.3-4
We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \le r(u, v) \le 1$ that represents the reliability of a communication channel from vertex *u* to vertex *v*. We interpret $r(u, v)$ as the probability that the channel from *u* to *v* will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.

5. (1 Point) p.613: 24.5-4
Let $G = (V, E)$ be a weighted, directed graph with source vertex *s* and let *G* be initialized by INITIALIZE-SINGLE-SOURCE(*G*, *s*). Prove that if a sequence of relaxation steps sets $\pi[s]$ to a non-NIL value, then *G* contains a negative-weight cycle.