CMPS 101
Summer 2009
Homework Assignment 6

1. (1 Point) p.551: 22.4-1
   Show the ordering of vertices produced by TOPOLOGICAL-SORT when it is run on the dag of Figure 22.8, under the assumption of Exercise 22.3-2.

2. (1 Point) p.557: 22.5-2
   Show how the procedure STRONGLY-CONNECTED-COMPONENTS works on the graph of Figure 22.6. Specifically, show the finishing times computed in line 1 and the forest produced in line 3. Assume that the loop of lines 5-7 of DFS considers vertices in alphabetical order and that the adjacency lists are in alphabetical order.

3. (1 Point) p.1091: B.5-4
   Use induction to show that a nonempty binary tree with n nodes has height at least $\lfloor \lg n \rfloor$. Hint: use the recursive definition of height discussed in class:

   $$h(T) = \begin{cases} -\infty & n(T) = 0 \\ 0 & n(T) = 1 \\ 1 + \max(\, h(L), h(R)\,) & n(T) > 1 \end{cases}$$

   Here $n(T)$ denotes the number of nodes in a binary tree T, $h(T)$ denotes its height, L denotes its left subtree, and R its right subtree. Note that this proof can be phrased equally well as an induction on $n(T)$ or on $h(T)$.

   Hint: use (and prove) the following fact: $\lfloor \lg(2k+1) \rfloor = \lfloor \lg(2k) \rfloor$ for any positive integer k.

4. (1 Point) p.132: 6.2-5
   The code for Max-Heapify is quite efficient in terms of constant factors, except possibly for the recursive call in line 10, which might cause some compilers to produce inefficient code. Write an efficient Max-Heapify that uses an iterative control construct (a loop) instead of recursion.