

**CMPS 101**  
**Summer 2009**  
**Homework Assignment 2**

1. (1 Point) p.50: 3.1-1

Let  $f(n)$  and  $g(n)$  be asymptotically non-negative functions. Using the basic definition of  $\Theta$ -notation, prove that  $f(n) + g(n) = \Theta(\max(f(n), g(n)))$ .

2. (1 Point) p.50: 3.1-3

Explain why the statement “The running time of algorithm A is at least  $O(n^2)$ ” is meaningless.

3. (2 Points) p. 50: 3.1-4

Determine whether the following statements are true or false.

a. (1 Point)  $2^{n+1} = O(2^n)$

b. (1 Point)  $2^{2n} = O(2^n)$

4. (6 Points) p.58: 3-2abcdef

Indicate, for each pair of expressions ( $A$ ,  $B$ ) in the table below, whether  $A$  is  $O$ ,  $o$ ,  $\Omega$ ,  $\omega$ , or  $\Theta$  of  $B$ . Assume that  $k \geq 1$ ,  $\varepsilon > 0$ , and  $c > 1$  are constants. Place 'yes' or 'no' in each of the empty cells below, and justify your answers.

	A	B	$O$	$o$	$\Omega$	$\omega$	$\Theta$
a. (1 Point)	$\lg^k n$	$n^\varepsilon$					
b. (1 Point)	$n^k$	$c^n$					
c. (1 Point)	$\sqrt{n}$	$n^{\sin n}$					
d. (1 Point)	$2^n$	$2^{n/2}$					
e. (1 Point)	$n^{\lg c}$	$c^{\lg n}$					
f. (1 Point)	$\lg(n!)$	$\lg(n^n)$					

5. (4 Points) p.58: 3-4cdeh

Let  $f(n)$  and  $g(n)$  be asymptotically positive functions (i.e.  $f(n) > 0$  and  $g(n) > 0$  for sufficiently large  $n$ .) Prove or disprove the following statements.

c. (1 Point)

Assume  $\lg(g(n)) \geq 1$  and  $f(n) \geq 1$  for all sufficiently large  $n$ . Then  $f(n) = O(g(n))$  implies  $\lg(f(n)) = O(\lg(g(n)))$ .

d. (1 Point)

$f(n) = O(g(n))$  implies  $2^{f(n)} = O(2^{g(n)})$ .

e. (1 Point)

$f(n) = O((f(n))^2)$ .

h. (1 Point)

$f(n) + o(f(n)) = \Theta(f(n))$ .

6. (1 Point)

Let  $f(n) = \Theta(n)$ . Prove that  $\sum_{i=1}^n f(i) = \Theta(n^2)$ . (See the hint at bottom of p.4 of the handout on asymptotic growth rates.)